ERROR ESTIMATES FOR IN SITU PROBE SYSTEMS AND WAVE DETECTIONS

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ABSTRACT

Data from the ionospheric plasma would be best studied by probe configurations having scale sizes small compared to the characteristic scales of the plasma disturbances. This condition makes it possible to effectively treat the results as originating from a point measurement. Unfortunately, such a condition is only rarely fulfilled. Aim of our study is to illustrate the problems related to finite probe separations on the rocket.

INTRODUCTION

Ionosphere is rich source for plasma waves and instabilities and sounding rocket experiment has provided plasma parameters along with ground based radar measurements. The electric field magnitude and direction estimated from the probe measurements will generally be different from the true values, and we discuss these errors. These discussions will have a general nature, and the conclusions will be relevant for other similar probe configuration. In order to exemplify the general idea of error analysis, we use the data obtained by four spherical probes placed at two booms from Rose rocket [1]. By this construction, the probes can give information of all three vector components of electric fields in the ionosphere.

PROBE COMBINATIONS

We use a combination of Langmuir probes to approximate the three components of the electric fields. The potential differences between the selected two probes and spatial distance will give approximated component of the field signals. It is expected that these signals would exactly recover the field-components for constant electric fields.

In the case of wavelengths longer than the probe separations, we assume this probe combination to give a good approximation for the magnitude and direction of the fluctuating electrostatic field.

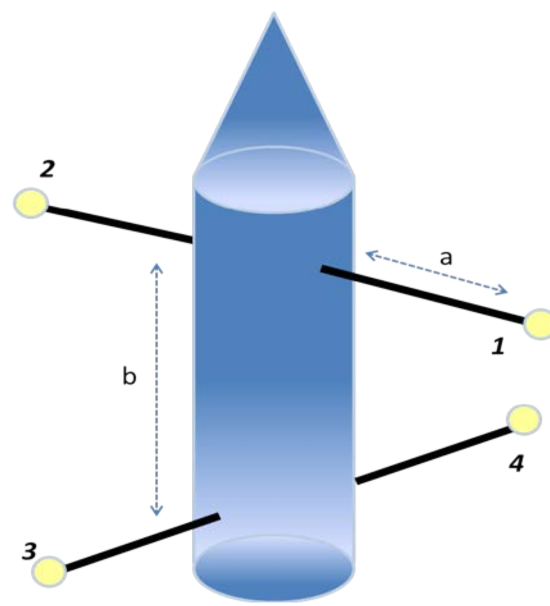

Figure 1. Schematic diagram for positioning of the four probe system. Probes are mounted on two pair of booms which gives probe separation 2a. The separation along the rocket axis is denoted by b.

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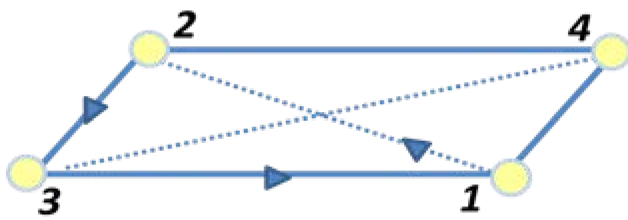

Figure2. An illustration of graphical understanding of closed loops of probes. All the four probes from Fig.1 are mapped to corners of a rectangle.

Figure 1 shows our simple model of sounding rocked with two-boom system where the each probes are separated vertically along the rocket body. The boom length a and the separation distance b determine effective rocket size as point measurement. We obtain electric potential value ϕ for each probe with respect to a suitably defined common ground. We analyse the fluctuating signals $U6(t) = \phi1(t) - \phi2(t)$; $U5(t) = \phi4(t) - \phi3(t)$; $U4(t) = \phi1(t) - \phi4(t)$; $U3(t) = \phi2(t) - \phi3(t)$; $U2(t) = \phi1(t) - \phi3(t)$; and $U1(t) = \phi2(t) - \phi4(t)$, where $\phi j(t)$ for $j = 1, 2, 3, 4$ denotes j-th probe potential.

Many basic tests can be carried out to determine the reliability of the data. The one of the simplest analysis consists of basic check sums: inspection of Figure 1 shows that sums of selected signals should ideally vanish such as, for instance, $U6(t) + U3(t) - U2(t) = 0$. The number of vanishing signal selection can be obtained by making closed loop of probes.

The idea of vanishing sum is easily illustrated when the probes are mapped to 2D plane as shown in Figure 2 where four probes create corners of a rectangle. The arrows denote an example of closed loop for the selection mentioned above. This mapping and loop selection can be valid when the number of probes is increased.

We compare values of single probe signal and closed loop form Rose rocket experiment. One data example is shown in Fig3. It shows a fluctuating potential signal values are reasonably vanished at selected sum. We have made these checks and find them to be satisfied within 3% accuracy. The deviations have no correlations with the amplitudes of the probe signals.

With configuration of rocket boom system in Fig.1, our electric field estimates are based on the following probe combinations: $Ex = - U6/2a$, $Ey = - U5/2a$. Ideally, we could use $Ez = -(U3 + U4)/2b$ just as well as $Ez = -(U1 + U2)/2b$, the two signals being identical. However, due to imperfections in the setup there can be small differences up to at most a few presence , so we here use the average value $Ez = -(U3 + U4 + U1 + U2)/4b$, with the z axis being along the rocket axis. These combinations would give the exact result for a constant electric field (like the ambient electric field) in an arbitrary direction.

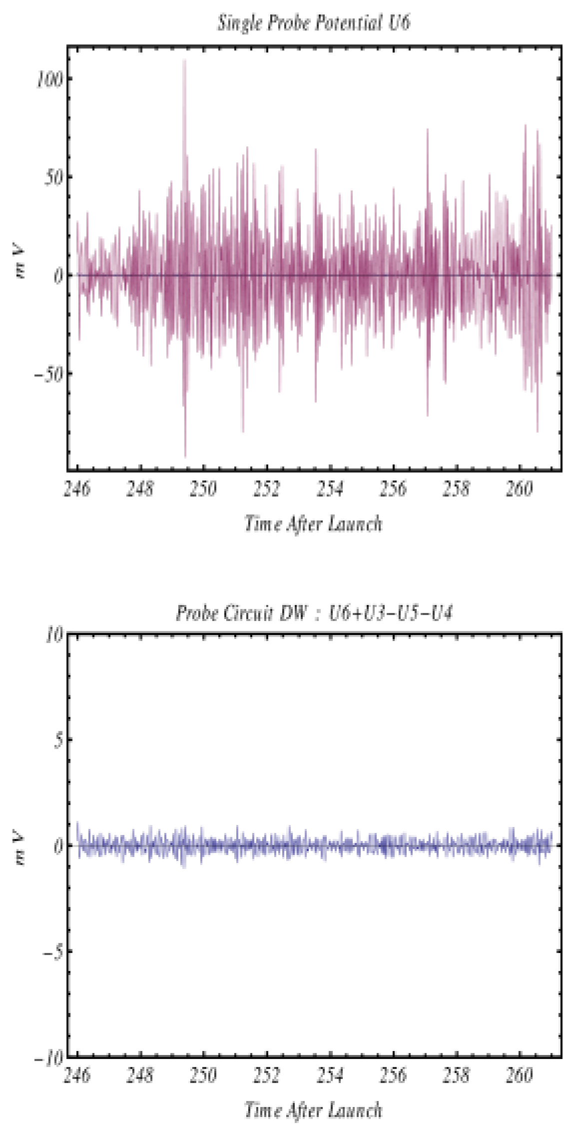

Figure 3. Comparison of signals from fluctuating single probe potential and a closed loop including the same probe from ROSE rocket. The rocket has the same probe system as Figure 1

PLANE WAVE ASSUMPTION

Ionospheric waves are in many cases propagating in the direction approximately perpendicular to the local magnetic field, i.e., $k \perp B_0$. Thus numbers of rocket experiments have been designed to place the rocket body parallel to the magnetic field either for up leg or down leg flight part. For a general mathematical model of a wavefield composed of many plane electrostatic waves, we have

$$\mathbf{E}(\mathbf{r},\mathrm{t}) = \int\int\int_{-\infty}^{\infty} \mathrm{E}(\mathbf{k}) \frac{\mathbf{k}}{k} e^{-i(\omega(\mathbf{k})t - \mathbf{k}\cdot\mathbf{r})} \qquad (1)$$

and the dispersion relation $\omega = \omega\,(\mathbf{k})$ is assumed to be known. The integral gives the weighted average of the electric field vector at a space-time position (r,t) and at the same time also a correspondingly averaged direction of propagation, where $(E)k$ enters as a weight function for the direction of the unit vector $k\,/\,/k$.

The local Boltzmann equilibrium is justified for long wavelengths and low frequencies [2]. Our assumption thus is $n_0 + \tilde{n} \approx n_0 \exp(e\tilde{\phi}/T_e)$ which implies $\widetilde{E} \sim kn$ for this limit.

ERROR ESTIMATE FOR PROPAGATION DIRECTION

We now apply the assumption of electric field propagation (1) to corresponding potential variation which will be observed by rocket probes. The model for the electric static potential in arbitrary direction can be written as

$$\varphi(\mathrm{x,y,z,t}) = \mathrm{A}\cos(\mathrm{k}_x\mathrm{x} + \mathrm{k}_y\, y + \mathrm{k}_z\mathrm{z} - \omega t - \psi) \qquad (2)$$

where ψ is a phase. The total phase addition to $\mathbf{k}\cdot\mathbf{r}$ is $-\omega t + \psi$, which allows us to take $t = 0$ and let Ψ represents all of the phase without loss of generality. Inserting position of the all the four probes to spatial variables, we obtain all three components of estimated electric field.

$$E_i = \frac{A}{a}\sin(\frac{\mathrm{k}_z\, b}{2} \pm \psi)\sin(\mathrm{k}_i\mathrm{a}) \qquad \mathrm{i = x, y}$$

$$E_z = \frac{A}{a}\left(\cos(\frac{\mathrm{k}_z\, b}{2} - \psi)\cos(\mathrm{k}_y\mathrm{a}) - \cos(\frac{\mathrm{k}_z\, b}{2} + \psi)\cos(\mathrm{k}_z\mathrm{a}) \right) \qquad (3)$$

By the definitions in (3) we effectively consider the rocket as a point probe. The information regarding phase differences from the probe sets giving U_1 and U_2 is lost. Similarly, they are lost for the probe sets giving U_3 and U_4. This phase information can be, when available, utilized to estimate the components of the propagation velocity that is perpendicular to the rocket axis.

ERROR ESTIMATE FOR PROPAGATION DIRECTION

The plane wave model (1) gives components $E^t_{x,y,z} = A\,k_{x,y,z}\sin(\mathbf{k}\cdot\mathbf{r} + \psi)$, which can be named as the true electric field. The propagation direction is given by the k vector. At the rocket reference position (the geometrical center of the probes) we have $E^t_{x,y,z} = A\,k_{x,y,z}\sin(\psi)$. It is easily seen that the differences between the two fields E and E_t vanish in the limit where $a \to 0$ and $b \to 0$. Since an arbitrary electric field variation can be described by a superposition of plane waves, we can use this single wave as an adequate model. In particular, we can give results for the error that we make concerning the electric field direction and magnitude by using the estimates (3) instead of the true electric field. We here define the error inn direction by the angle $\arccos(\mathbf{E}^t \cdot \mathbf{E}^t \,/\, |E^t| \cdot |E|)$.

Figure 4 shows the average error in the direction of propagation when using (3) to represent the true electrostatic electric field E_t. The model is applied to the boom systems of ROSE rocket where boom length is 180cm and the vertical separation is 185cm. The sphere corresponds to one wave number, here wavelength 17.5 m or frequency 20Hz by use of a characteristic phase velocity of 350 m/s with an averaging over all ψ. A point on the spheres correspond to a direction for the wave propagation given by k and the color coding gives an indication of the error, with scales given by the color bar. In general case, it should be noted that the frequencies need to be within the range of band pass filter for particular waves of interest for individual experiments.

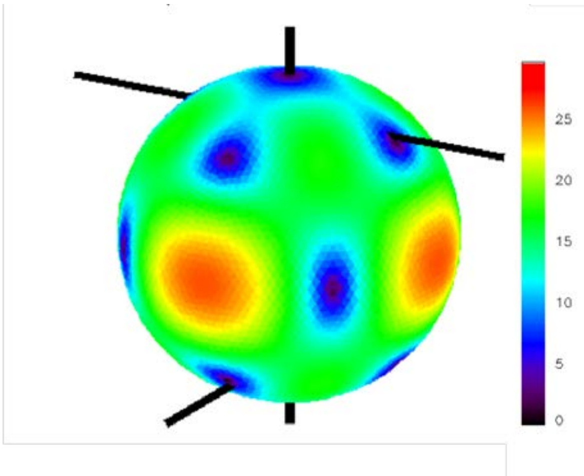

Figure 4. Error estimate for direction of propagation for one wavelength17.5m. The black lines indicate the relative positions of the boom and probes in Rose rocket system where a=180cm and b=185cm.The color scale represents degree.

The results in Fig.4 give the average over all phases for a particular wavelength. The variation of error estimate and its dependence in frequency are obtained by applying different frequencies in the range concerned for the analysis. The error can be larger for individual phase values. It is particularly seen when the phase approaches to 0. For vanishing electric fields the field direction is thus undefined and the error becomes large which may lead to experience that a local small can be detected as having the opposite direction. The further analysis of the same data and errors in amplitude are seen in [3].

SUMMARY AND OUTLOOK

In present study we analyzed the error estimate for rocket measurement of three dimensional wave fields. By assuming propagation of plane waves, the errors in propagation directions are shown. The future study will include further development of general idea to variation of in-situ probe combinations and design to detect selective wave phenomena.

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