Frame Synchronization for Next Generation Uplink Coding in Deep Space Communications

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***Abstract*—In this paper we develop two new approaches for frame synchronization in the binary-input AWGN channel, in which we account for the sign ambiguity of the received symbols and exploit knowledge of an alternating sequence which precedes the synchronization word. We present an approach based on an extended sliding window and the appropriate decision metric. For the common case that the synchronization word is followed by encoded data we present a solution which exploits the error detection capability of the channel decoder and applies a list decoding approach for frame synchronization. The proposed methods are validated through computer simulations in the deep-space communication uplink and show significant performance gains compared to current solutions.**

I. Introduction

Frame synchronization is an important receiver function which has to be performed before decoding of the transmitted data can begin. It consists in finding the position of a known synchronization word in the incoming symbol stream. Common engineering practise is to compute the correlation of a part of the received sequence with the known sync word at each symbol position and compare it to a threshold. This approach is optimum for the binary symmetric channel, but not for AWGN or fading channels. For the case of a periodically inserted sync word, Massey derived the optimum frame synchronizer [1] while Chiani presented the solution for a single sync word [2], [3].

In this paper, we consider the case of a single sync word in the AWGN channel with BPSK modulation and consider that even after perfect carrier and phase synchronization an ambiguity about the sign of the received antipodal symbols remains. We also exploit that in many communication systems, as it is the case for deep-space communications [4], [5], [6], the sync word is preceded by a sequence of alternating symbols, which is used for time and frequency acquisition. Several strategies are proposed to enhance frame synchronization. We first consider an extended sliding window where the observation window, for which the decision metric is computed, is longer than the synchronization sequence. The second improvement considers the presence of a buffered span of received symbols and a channel decoder with error detection capability [7], [8] and implements a list decoding approach. Both schemes bring significant gains to the receiver performance, decreasing the minimum SNR required to ensure a given frame synchronization error, or in turn reducing the overall frame error rate.

The rest of the article is organized as follows. Section II reviews the current approaches to frame synchronization. Sections III and IV propose new frame synchronization schemes that account for a preceding alternating sequence before the sync word. All these proposals are validated through computer simulations in a deep-space telecommand link, where the potential benefits of using these type of approaches become apparent.

II. Frame Synchronization for BPSK

A. Frame Synchronization by Hypothesis Testing

We consider a communication system in which a transmitter sends BPSK-modulated data frames which are preceded by a sync word which consists of a known sequence of N BPSK symbols. The task of the frame synchronizer is to find this sync word in a stream of received noisy symbols. The typically applied procedure takes the last N received symbols $\mathbf{r} = [r_1, r_2, \ldots, r_N]$ and compares them to the known sync word and takes a decision according to the two hypotheses:

$$\mathcal{H}_0 : \mathbf{r} \text{ does not correspond to the sync word}$$
$$\mathcal{H}_1 : \mathbf{r} \text{ corresponds to the sync word}$$

and the corresponding decisions $\mathcal{D}_0$ or $\mathcal{D}_1$. The optimum approach for this hypothesis testing problem is described in [2] and is given by the likelihood ratio test (LRT) [9]

$$\Lambda(\mathbf{r}) \triangleq \frac{p(\mathbf{r} \mid \mathcal{H}_1)}{p(\mathbf{r} \mid \mathcal{H}_0)} \underset{\mathcal{D}_0}{\overset{\mathcal{D}_1}{\gtrless}} \lambda. \tag{1}$$

In other words, a sliding observation window of the same length as the sync word takes N symbols out of the received noisy symbol stream and computes a metric $\Lambda(\mathbf{r})$ which is compared to a threshold. If the metric exceeds this threshold, the receiver declares the observation window $\mathbf{r} = [r_1, r_2, \ldots, r_N]$ to be the sync word.

For binary signaling over an AWGN channel, the received symbols are given by

$$r_n = x_n + w_n,\ x_n \in \{-1, 1\},\ w_n \sim \mathcal{N}(0, N_0/2) \tag{2}$$

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and we denote the known sync word by $\mathbf{s} = [s_1, s_2, \ldots, s_N]$ with $s_n \in \{-1, 1\}$, while $\mathbf{d} = [d_1, d_2, \ldots, d_N]$ with $d_n \in \{-1, 1\}$ denotes a random data sequence. With this model, the hypotheses can be formulated as

$$\begin{aligned} \mathcal{H}_0 &: \mathbf{r} = \mathbf{d} + \mathbf{w} \\ \mathcal{H}_1 &: \mathbf{r} = \mathbf{s} + \mathbf{w} \end{aligned} \tag{3}$$

where $\mathbf{w} = [w_1, \ldots, w_N]$ is AWGN. As shown in [2], this leads to the metric

$$\Lambda_{\mathsf{MC},1}(\mathbf{r}) = \frac{2}{N_0} \sum_{n=1}^{N} s_n r_n - \ln\cosh\left(\frac{2}{N_0} r_n\right) \tag{4}$$

The attentive reader will have observed that this approach neglects the “mixed data” case in which the observation window $\mathbf{r}$ constains both data and a part of the sync word. This simplification is valid for any reasonably designed sync word and is confirmed in [2]. It is remarkable to observe that the metric $\Lambda_{\mathsf{MC},1}$ is equivalent to equations (5) and (6) of Massey’s classical paper on frame synchronization for the case of a periodically repeated sync word [1], which has also been noted by Chiani [3, Section V]. For this reason, in the following we refer to $\Lambda_{\mathsf{MC},1}$ as the *Massey-Chiani (MC)* metric.

B. The Massey-Chiani Metric for the Binary-Input AWGN Channel with Sign Ambiguity

In the following, we consider an extension of the above model. While we still assume that timing, frequency and phase synchronization have been accomplished perfectly, we account for the unkown sign of the received BPSK symbols: even with perfect timing, frequency and phase synchronization, an ambiguity about the polarity of the received symbols r_n remains. In the next Section, we will exploit the case that the sync word is preceded by an alternating ± 1 sequence, which is inserted for time and frequency acquisition. This is e.g. the case for the uplink in deep space communications [4], [10].

As a reference, we will first derive the MC metric for the BI-AWGN channel with sign ambiguity before we develop a new metric based on an extended observation window, which exploits knowledge about the preceding alternating sequence.

The channel with sign ambiguity can be modeled by

$$r_n = h \cdot x_n + w_n,\ x_n \in \{-1, 1\},\ w_n \sim \mathcal{N}(0, N_0/2) \tag{5}$$

where $h \in \{-1, 1\}$, $P[h = -1] = P[h = 1]$ accounts for the unknown sign and this coefficient is constant but unknown for each synchronization attempt. We can rewrite the hypotheses for this case as

$$\begin{aligned} \mathcal{H}_0 &: \mathbf{r} = h \cdot \mathbf{d} + \mathbf{w} \\ \mathcal{H}_1 &: \mathbf{r} = h \cdot \mathbf{s} + \mathbf{w} \end{aligned} \tag{6}$$

where we can omit the coefficient h for the null hypothesis since it does not change the statistics of the random data sequence. With the signal model (5), we obtain for the likelihood of the null hypothesis the same conditional likelihood as if the sign was known,

$$\begin{aligned} p(\mathbf{r} \mid \mathcal{H}_0) &= \prod_{n=1}^{N} \frac{1}{2} \left(p(r_n \mid d_n = -1) + p(r_n \mid d_n = 1)\right) \\ &= K_N(\mathbf{r}) \prod_{n=1}^{N} \cosh(\tilde{r}_n) \end{aligned}$$

where we define

$$K_N(\mathbf{r}) \triangleq \prod_{n=1}^{N} \frac{1}{\sqrt{\pi N_0}} \exp\left(-\frac{r_n^2 + 1}{N_0}\right)$$

and $\tilde{r}_n \triangleq \frac{2}{N_0} r_n$.

For the other hypothesis, we find

$$\begin{aligned} p(\mathbf{r} \mid \mathcal{H}_1) &= \frac{1}{2} \left(p(\mathbf{r} \mid \mathbf{x} = -\mathbf{s}) + p(\mathbf{r} \mid \mathbf{x} = \mathbf{s})\right) \\ &= \frac{1}{2} \left(\prod_{n=1}^{N} p(r_n \mid x_n = -s_n) + \prod_{n=1}^{N} p(r_n \mid x_n = s_n)\right) \\ &= K_N(\mathbf{r}) \cdot \cosh\left(\frac{2}{N_0} \sum_{n=1}^{N} r_n s_n\right) = K_N(\mathbf{r}) \cdot \cosh\left(\tilde{\mathbf{r}} \mathbf{s}^{\mathrm{T}}\right) \end{aligned}$$

This leads to the MC metric for sign ambiguity

$$\Lambda_{\mathsf{MC},2}(\mathbf{r}) = \ln\cosh\left(\tilde{\mathbf{r}} \mathbf{s}^{\mathrm{T}}\right) - \sum_{n=1}^{N} \ln\cosh(\tilde{r}_n) \tag{7}$$

Comparing the MC metrics with and without sign ambiguity, we observe that the correction term is the same in both cases, whereas expression (7) cannot be obtained by simple intuition from (4). A simple intuitive step could be to take the absolute value of the first term in (4) to account for both possible signs. This intuition is valid for high SNR since $\ln\cosh(x) \approx |x| - \ln(2)$ for $|x| \gg 1$.

C. Correlation Metrics

The correlation of the received samples with the known sync word is still a popular metric despite its sub-optimality and the only marginally lower computational complexity compared to a simplified version of the optimum metric. Since these metrics do not have a rigorous theoretical justification, we apply the correlation to the received sequence and its inverse and define the maximum of both as the correlation metric for the Binary-Input-AWGN channel with sign ambiguity.

For the *hard correlation (HC)* metric, we first make a hard decision on each bit and then correlate it with the know sync word:

$$\begin{aligned} \Lambda_{\mathsf{HC}}(\mathbf{r}) &\triangleq \frac{1}{2} \max\left\{\mathrm{sgn}(\mathbf{r})\, \mathbf{s}^{\mathrm{T}}, -\mathrm{sgn}(\mathbf{r})\, \mathbf{s}^{\mathrm{T}}\right\} \\ &= \frac{1}{2} \left|\mathrm{sgn}(\mathbf{r})\, \mathbf{s}^{\mathrm{T}}\right| \in \left\{0, 1, \ldots, \frac{N}{2}\right\} \end{aligned} \tag{8}$$

We introduced the factor $\frac{1}{2}$ in order to obtain a range of contiguous integers as possible values for this metric. Naturally, any other constant factor (or monotonic function) can be applied as well.

In analogy to correlating with the hard-decided signal, another natural metric is the *soft correlation (SC)*, which applies the correlation directly on the noisy BPSK signal,

$$\Lambda_{\mathsf{SC}}(\mathbf{r}) \triangleq \frac{1}{2}\left|\mathbf{r}\mathbf{s}^{\mathrm{T}}\right| \tag{9}$$

The factor $1/2$ is again introduced for convenience and comparability with (8). Note that, in contrast to decoding, there is no reason why soft correlation should be superior to hard correlation. While the correlation metric is optimum on the binary symmetric channel, for the AWGN channel both correlations are only heuristic metrics.

III. Metrics with an Extended Observation Window

A. Alternating Acquisition Sequence and Extended Observation Window

In the following, we assume that the sync word is preceded by an alternating sequence, which is typically used for time and frequency acquisition and consists of alternating ± 1 symbols. We denote this sequence by $\mathbf{a} = [a_1, a_2, \ldots, a_A]$ with $a_n \triangleq (-1)^n$ and length A which is not known at the receiver. We denote the partial sync word by $\mathbf{s}_n \triangleq [s_1, s_2, \ldots, s_n]$ for $n \leq N$.

In order to better exploit the known properties of the preceding alternating sequence, we extend the sliding observation window to a length $M \geq N$, as depicted in Fig. 1. We denote the entire noiseless sequence by

$$\mathbf{x} = [h_1\mathbf{a}, h_2\mathbf{s}, \mathbf{d}] \tag{10}$$

where $\mathbf{d}$ denotes an unknown data sequence. The random coefficients $h_1, h_2 \in \{-1, 1\}$ model the sign ambiguity of the received signal and the sign ambiguity of the acquisition sequence. Although we assume that at the receiver side, the sign ambiguity is the same for the entire received sequence, we need the two factors h_1 and h_2 to account also for the uncertainty on whether the acquisition sequence ends with a -1 or $+1$. This uncertainty could be easily removed at the transmitter side. However, since e.g. the recommendation [4] allows for both options, we account for this detail in the following.

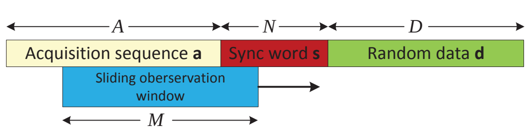

Figure 1. Search for sync word with extended sliding observation window

We define a noiseless observation window $\mathbf{x}_m$ at position m as

$$\mathbf{x}_m \triangleq \left[h_1 \cdot \mathbf{a}_{M+1-m}, h_2 \cdot \mathbf{s}_{m-1}\right], \; m = 1, \ldots, N+1 \tag{11}$$

Figure 2 and Table I indicate the meaning of the index m, which determines the position of the sliding window relative to the position of the sync word. The index m refers to the last symbol position of the sliding window, counted from the last symbol of the alternating sequence, whereas the index n refers to the first symbol of the sliding window, counted from the start of the alternating sequence $\mathbf{a}$. Both indices are related by $n = A - M + m$.

We consider only window positions in which the observation window ends before or at the same bit interval as the sync word, and therefore the random data sequence $\mathbf{d}$ has no effect.

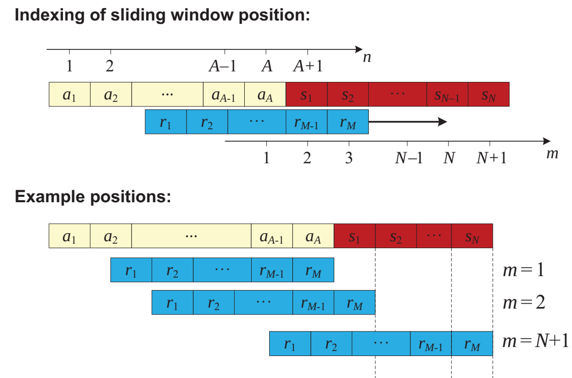

Figure 2. Indexing for sliding window operation

Table I
Relation between indices n and m and the observed vector $\mathbf{x}_m$

n	m	$\mathbf{x}_m$
$1 \cdots A-M+1$	1	$h_1 \cdot \mathbf{a}_M$
$A-M+2$	2	$[h_1 \cdot \mathbf{a}_{M-1}, h_2 \cdot s_1]$
$A-M+3$	3	$[h_1 \cdot \mathbf{a}_{M-2}, h_2 \cdot \mathbf{s}_2]$
$\vdots$	$\vdots$	$\vdots$
$A-M+N$	N	$[h_1 \cdot \mathbf{a}_{M-N+1}, h_2 \cdot \mathbf{s}_{N-1}]$
$A-M+N+1$	$N+1$	$[h_1 \cdot \mathbf{a}_{M-N}, h_2 \cdot \mathbf{s}]$

The received signal in the observation window is hence

$$\mathbf{r} = \mathbf{x}_m + \mathbf{w}, \; w_n \sim \mathcal{N}(0, N_0/2)$$

One of the key aspects when considering the acquisition sequence is that, in contrast to a sync word preceded by random data, the mixed data case cannot be neglected. For this reason, we need to consider all positions of the observation window for the null hypothesis. With the indexing of Fig. 2, we can reformulate the two hypotheses as

$$\mathcal{H}_0 : m \in \{1, 2, \ldots, N\}$$
$$\mathcal{H}_1 : m = N+1$$

For the null hypothesis, we have

$$p(\mathbf{r} \mid \mathcal{H}_0) = \sum_{\mu=1}^{N} \rho_m p(\mathbf{r} \mid m = \mu) \tag{12}$$

where $\rho_\mu \triangleq P[m = \mu]$ denotes the a priori probability that the sliding window is in position $m = \mu$. We assume

$$\rho_\mu = \frac{1}{A+N-M-1}\begin{cases} A-M & \text{for } \mu = 1 \\ 1 & \text{for } \mu = 2, \ldots, N \end{cases}$$

and the same probability for the four sign ambiguities, i.e.

$$p(\mathbf{r} \mid m=\mu) = \frac{1}{4} \sum_{h_1,h_2} p(\mathbf{r} \mid m=\mu, h_1, h_2)$$

then

$$\begin{aligned} p(\mathbf{r} \mid m, h_1, h_2) &= \prod_{n=1}^{M-m+1} p(r_n \mid x_{mn} = h_1 a_n) \\ &\quad \cdot \prod_{n=M-m+2}^{M} p(r_n \mid x_{mn} = h_2 s_{n-M+m-1}) \\ &= K_M(\mathbf{r}) \cdot \prod_{n=1}^{M-m+1} \exp(h_1 a_n \tilde{r}_n) \\ &\quad \cdot \prod_{n=M-m+2}^{M} \exp(h_2 s_{n-M+m-1} \tilde{r}_n) \end{aligned}$$

With $\tilde{\mathbf{r}}_n^m \triangleq [\tilde{r}_n, \tilde{r}_{n+1}, \ldots, \tilde{r}_m]$, we can write

$$\begin{aligned} p(\mathbf{r} \mid m, h_1, h_2) = K_M \cdot \exp\left(h_1 \tilde{\mathbf{r}}_1^{M-m+1} \mathbf{a}_{M-m+1}^{\mathrm{T}}\right) \\ \cdot \exp\left(h_2 \tilde{\mathbf{r}}_{M-m+2}^{M} \mathbf{s}_{m-1}^{\mathrm{T}}\right) \end{aligned}$$

and hence

$$\begin{aligned} p(\mathbf{r} \mid m) = K_M \cdot \cosh\left(\tilde{\mathbf{r}}_1^{M-m+1} \mathbf{a}_{M-m+1}^{\mathrm{T}}\right) \\ \cdot \cosh\left(\tilde{\mathbf{r}}_{M-m+2}^{M} \mathbf{s}_{m-1}^{\mathrm{T}}\right) \end{aligned}$$

and

$$\begin{aligned} p(\mathbf{r} \mid \mathcal{H}_0) = K_M \sum_{m=1}^{N} \rho_m \cosh\left(\tilde{\mathbf{r}}_1^{M-m+1} \mathbf{a}_{M-m+1}^{\mathrm{T}}\right) \\ \cdot \cosh\left(\tilde{\mathbf{r}}_{M-m+2}^{M} \mathbf{s}_{m-1}^{\mathrm{T}}\right) \end{aligned}$$

For the other hypothesis, we obtain

$$p(\mathbf{r} \mid \mathcal{H}_1) = K_M \cdot \cosh\left(\tilde{\mathbf{r}}_1^{M-N} \mathbf{a}_{M-N}^{\mathrm{T}}\right) \cdot \cosh\left(\tilde{\mathbf{r}}_{M-N+1}^{M} \mathbf{s}^{\mathrm{T}}\right)$$

which leads to the LRT-A (the "A" stands for the acquisition sequence) in logarithmic domain,

$$\begin{aligned} \Lambda_{\mathsf{LRT-A}}(\mathbf{r}) &= \ln\cosh\left(\tilde{\mathbf{r}}_1^{M-N} \mathbf{a}_{M-N}^{\mathrm{T}}\right) + \ln\cosh\left(\tilde{\mathbf{r}}_{M-N+1}^{M} \mathbf{s}^{\mathrm{T}}\right) \\ &- \ln \sum_{m=1}^{N} \rho_m \cosh\left(\tilde{\mathbf{r}}_1^{M-m+1} \mathbf{a}_{M-m+1}^{\mathrm{T}}\right) \cdot \cosh\left(\tilde{\mathbf{r}}_{M-m+2}^{M} \mathbf{s}_{m-1}^{\mathrm{T}}\right) \end{aligned}$$

This expression simplifies slightly for $M = N$, but does not become identical to (7). The difference comes from the fact that here we explicitly account for the mixed data case.

B. Probabilities of False Alarm and Missed Detection

The application of the LRT (1) at every symbol position can lead to two types of error: a *false alarm* occurs if the presence of the sync word is indicated by $\Lambda(\mathbf{r}) \geq \lambda$ at another position than the true one, while a *missed detection* occurs if the observation window is a the true position but the metric $\Lambda(\mathbf{r})$ is below the threshold λ. For the false alarm probability, we additionally distinguish the error events by the window position given by the index n of Table I. The probabilities for false alarm $P_{\mathsf{fa}}(\nu)$ and missed detection P_{md} are hence given by

$$\begin{aligned} P_{\mathsf{fa}}(\nu) &= P[\Lambda \geq \lambda, n = \nu], \ \nu = 1, \ldots, A - M + N \\ P_{\mathsf{md}} &= P[\Lambda < \lambda, m = N + 1] \end{aligned} \tag{13}$$

For the overall false alarm probability $\bar{P}_{\mathsf{fa}}$, since the events $\{n=1\}, \{n=2\}, \ldots, \{n = A - M + N\}$ are mutually exclusive, we have

$$\bar{P}_{\mathsf{fa}} = \sum_{\nu=1}^{A-M+N} P_{\mathsf{fa}}(\nu) \tag{14}$$

Since in each failed synchronization attempt, either a false alarm or a missed detection occurs, the probability of a *frame synchronization error (FSE)* is given by the sum of both probabilities

$$P_{\mathsf{FSE}} = \bar{P}_{\mathsf{fa}} + P_{\mathsf{md}} \tag{15}$$

C. Simulation Paramters for Deep Space Uplink

In the following, we use the system parameters for deep space telecommand (uplink) as a running example, being the most important aspect the length of the sync word. The sync word is defined in [4] and has a length of $N = 16$ bits. For the length of the acquisition sequence, we assumed $A = 512$ which corresponds to the length of the longest codeword following the sync. This choice is motivated by the reasoning that for longer acquisition sequences, eventually occuring false alarms can be detected after the first decoding attempt. Simulations showed an insignificant impact of the choise of A on the FSE.

In Figure 3, the false alarm and missed detection probabilities, as well as the resulting FSE are plotted as a function of the decision threshold λ for two metrics at $E_{\mathrm{S}}/N_0 = 0\,\mathrm{dB}$. From the definition of the LRT (1) and the error probabilities (14), (13) it is clear that $\bar{P}_{\mathsf{fa}}$ is a decreasing function of the threshold λ, while P_{md} is increasing. The parameter of interest, however, is the FSE which simplifies the problem of finding the optimum threshold to a simple one-dimensional minimization which can be solved numerically by simulation.

The FSEs for all considered metrics as a function of the decision threshold at a fixed SNR are plotted in Figure 4. From this diagram, we can find the optimum threshold for each metric for a given SNR, as listed in Table II. We can observe that, at least within this range, only the SC and the MC metrics depend on the SNR, while for the HC and the LRT-A the same threshold can be applied for all SNR values. This aspect is important in practical receivers where an accurate SNR estimation is often not viable.

The achieved FSE with the presented metrics for different SNRs is plotted in Figure 5. We can observe that, while soft correlation performs very poorly, the hard correlation metric comes comes close to the Massey-Chiani metric for high SNR. We can also see that the proposed LRT-A metric achieves a significant performance improvement for all SNR values, even without extending the window length. This gain comes from the exploitation of the structure of the alternating sequence, in

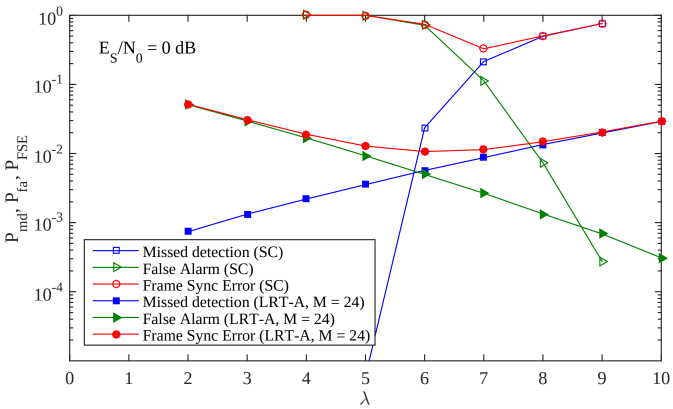

Figure 3. Missed detection, false alarm and frame synchronization error probabilities as a function of the detection threshold for the soft correlation (SC) and the LRT-A metrics with $M = 24$

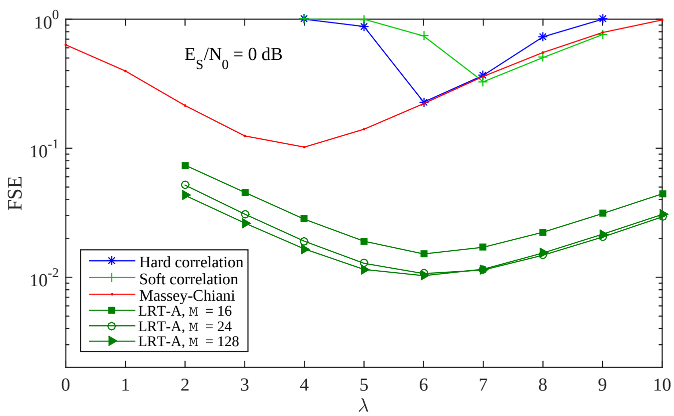

Figure 4. FSE as a function of the detection threshold

particular in the mixed data case. The performance improves slightly by extending the observation window from 16 to 24 bits, while a further extension to 128 bits does not lead to a further improvement.

IV. Peak Detection in Long Observation Window

A. Single Peak Detection

While for one-shot detection of the sync word, for every window position a metric is compared to a threshold, for periodically inserted sync words with known periodicity, the receiver can search for the maximum metric within a frame

Table II
Detection thresholds for minimum FSE

E_S/N_0	HC	SC	MC	LRT-A
−3 dB	6	9	5	6
−2 dB	6	8	4	6
−1 dB	6	7	4	6
0 dB	6	7	4	6
1 dB	6	6	3	6
2 dB	6	6	2	6
3 dB	6	6	1	6
4 dB	6	6	0	6

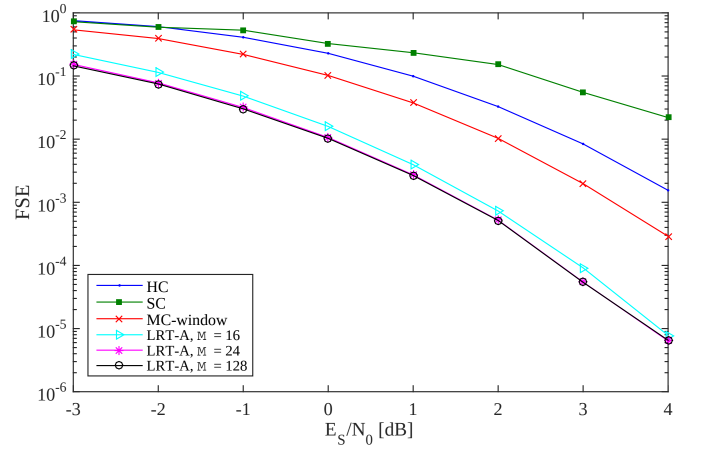

Figure 5. FSE with (extended) sliding window

(peak detection) and there is no need to determine any threshold [1], [3, Section IV, V].

Nevertheless, we can apply the method of peak detection even for a single sync word with the following approach. We partition the incoming symbol stream into long overlapping observation windows. The overlap is as long as the sync word to avoid that this falls in between two windows. Then, we apply peak detection on the long observation window. This will inevitably lead to false alarms in windows which do not contain the sync word. These false alarms can be detected after decoding of the first code word after the sync word, provided that the undetected error probability of the channel coding scheme is lower than the target FSE. This is an additional requirement which, however, is typically satisfied anyway.

We therefore assume that the long observation window contains $B = A + N + D \gg N$ symbols and contains the acquisition sequence, the sync word and data, as depicted in Figure 1. The entire noiseless sequence in the buffer of length B is given in (10) and the received sequence is denoted by $\mathbf{y} = \mathbf{x} + \mathbf{w}$. The maximum likelihood rule to determine the index of the first bit of the sync word is given by

$$n^* = \arg\max_m \{p(\mathbf{y} \mid A = m)\} + 1$$

Similar to the derivation for the extended observation window, we start with $p(\mathbf{y} \mid A = m) = \frac{1}{4}\sum_{h_1,h_2} p(\mathbf{y} \mid A = m, h_1, h_2)$. Since we are considering the entire buffer, we factor the conditional probability of $\mathbf{y}$ as

$$p(\mathbf{y} \mid m, h_1, h_2) = \prod_{n=1}^{m} p(y_n \mid h_1 a_n)$$

$$\prod_{n=m+1}^{m+N} p(y_n \mid h_2 s_{n-m}) \prod_{n=m+N+1}^{B} \frac{p(y_n \mid -1) + p(y_n \mid 1)}{2}$$

$$= K_B(\mathbf{y}) \exp\left(h_1 \tilde{\mathbf{y}}_1^m \mathbf{a}_m^{\mathrm{T}}\right) \exp\left(h_2 \tilde{\mathbf{y}}_{m+1}^{m+N} \mathbf{s}^{\mathrm{T}}\right) \prod_{n=m+N+1}^{B} \cosh(\tilde{y}_n)$$

which leads to

$$p(\mathbf{y} \mid A = m) = K_B \cdot \cosh\left(\tilde{\mathbf{y}}_1^m \mathbf{a}_m^{\mathrm{T}}\right) \cdot \cosh\left(\tilde{\mathbf{y}}_{m+1}^{m+N} \mathbf{s}^{\mathrm{T}}\right) \cdot \prod_{n=m+N+1}^{B} \cosh\left(\tilde{y}_n\right)$$

and, finally, we define the metric to be maximized as

$$\begin{aligned}\Lambda_{\mathsf{LW}}(m) &\triangleq \ln\left(\frac{1}{K_B} p(\mathbf{y} \mid A = m)\right) \\ &= \ln\cosh\left(\tilde{\mathbf{y}}_1^m \mathbf{a}_m^{\mathrm{T}}\right) + \ln\cosh\left(\tilde{\mathbf{y}}_{m+1}^{m+N} \mathbf{s}^{\mathrm{T}}\right) \\ &+ \sum_{n=m+N+1}^{B} \ln\cosh\left(\tilde{y}_n\right)\end{aligned}$$

The most likely position of the first symbol of the sync word is then found by

$$n^* = \arg\max_m \left\{\Lambda_{\mathsf{LW}}(m)\right\} + 1$$

B. Multiple Peak Detection: List Decoding

The fact that the sync word is followed by codewords can be exploited further, provided the code provides sufficient error detection capability and multiple decoding attempts are affordable. These are rather mild assumptions, since the probability of undetected error is usually required to be significantly lower than the FSE and bit rates for telecommand operations are typically moderate, hence multiple decoding attempts within the observation window, which is at least as long as a codeword, are not unrealistic.

For multiple peak detection, we order the indices $n \in \{1, 2, \ldots, B\}$ in order of decreasing metric,

$$\Lambda_{\mathsf{LW}}(m_1) \geq \Lambda_{\mathsf{LW}}(m_2) \geq \cdots \geq \Lambda_{\mathsf{LW}}(m_B)$$

and perform L successive decoding attempts for the indices $m_1, m_2, \ldots, m_L$. In coding theory, this approach is known as list decoding. For $L = 1$, we have the simple peak detection as described above in Section IV-A, while for the unrealistic value $L = B$, the FSE is limited only by the undetected word error probability of the channel coding scheme. This extreme case is similar to the approach proposed in [11], which focusses on the synchronization of single codewords when no sync marker is available.

Figure 6 shows the achieved FSE with multiple peak detection (PD) for different list lengths L. A short value of additional decoding attempts already provides very significant gains for frame synchronization. As a reference, we also applied the Massey-Chiani metric, computed in a sliding window operation as in the original work of Massey [1]. This metric suffers from an error floor which is due to false alarms which are unavoidable if the 16-bit sync word appears in the data.

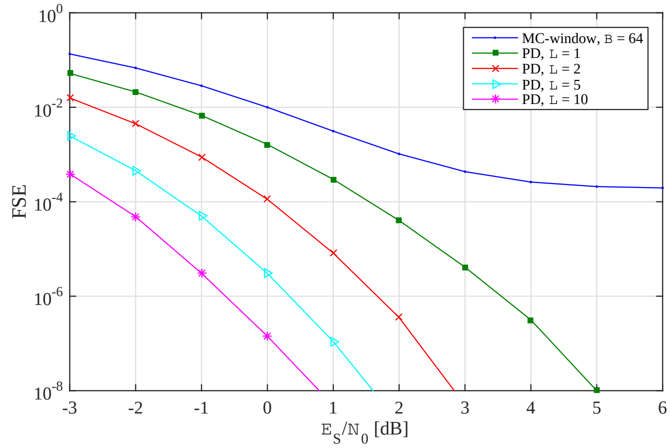

Figure 6. FSE with buffer-based frame synchronization

V. Conclusion

We presented two solutions for frame synchronization of frame formats in which the known sync word is preceded by a sequence of alternating symbols which are typically used for timing and carrier acquisition. The first approach applies the likelihood ratio test on a sliding observation window which may exceed the length of the sync word and does account for the case of mixed data. The second solution makes use of the error detection capability of the employed channel decoder and works on a longer span of received symbols. This approach uses an ordered list of candidate positions and provides a substantial performance gain.

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