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Bachelor Thesis

Application of a network reduction approach on an energy system model and the impact of different node aggregations on the curtailment of renewable energy generation

– the case of REMix

submitted by
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Abstract

Along with the increasing demand for energy system analysis, they become ever more detailed in terms of analyzed technologies, load situations, temporal resolution or power grid representation.

This thesis observes the representation of load and generation locations — so called nodes — in an energy system model. For that purpose, research on existing solutions for network reduction approaches was reviewed and their applicability on the optimization model "REMix" (Renewable Energy Mix) was tested. The integration of a power flow model into an energy system optimization model imposes particularly high requirements on its execution time. While calculation time for a large number of nodes should be kept at a minimum, the network model's power flow and factors like line loss and transmission capacity must not differ too much from those of the real power network. As a result, algorithms known from electrotechnical appliances need to be modified. Several methods for power network reduction that meet these requirements have been evaluated. An approach by Di Shi and Daniel Tylavski has been adopted that makes use of Power Transfer Distribution Factors (PTDF).

In the course of this, a method to sum up transmission lines between aggregated nodes was implemented. Using this implementation, different zone layouts for the German power grid were examined. As a conclusion, significant variations of dispatch depending on the layout of regions appear. Moreover, this thesis identifies transmission capacities and positions of power injection as two important influence factors on power flow.

Zusammenfassung

Durch den steigenden Bedarf an Energiesystemanalysen werden diese auch immer detaillierter bei der Betrachtung von Technologien, Lastsituationen, zeitlicher Auflösung oder des Stromnetzes.

Diese Arbeit befasst sich mit der Abbildung von Last und Erzeugern an den Übertragungsnetzknoten eines Energiesystemmodells. Dazu wird eine Recherche zu bereits vorhandenen Lösungen der Lastflussberechnung durchgeführt und untersucht, ob sich die Ansätze auf das Optimierungsmodell „REMix“ (Renewable Energy Mix) anwenden lassen. Die Integration eines Lastflussmodells in ein Modell zur Energiesystemoptimierung stellt dabei besonders hohe Anforderungen an die Berechnungszeit. Während die Berechenungszeit für eine hohe Anzahl Knoten so niedrig wie möglich bleiben soll, dürfen Lastfluss und Eigenschaften wie Übertragungsverluste und -kapazitäten des Netzmodells nicht zu stark von denen des realen Netzes abweichen. Aus diesem Grund müssen aus der Elektrotechnik bekannte Anwendungen abgewandelt werden. Mehrere Methoden zur Netzwerkreduktion, die die obigen Anforderungen erfüllen, wurden evaluiert. Gewählt wurde ein Ansatz von Di Shi und Daniel Tylavski, der Gebrauch von Power Transfer Distribution Factors (PTDF) macht.

Im Zuge dessen wird eine Methode für die Zusammenfassung von Übertragungsleitungen zwischen zusammengefassten Knoten implementiert. Mithilfe der Methode werden unterschiedliche Zoneneinteilungen des deutschen Stromnetzes untersucht. Ein Ergebnis sind signifikante Änderungen des Kraftwerkseinsatzes in Abhängigkeit der Zonenanordnung. Zudem identifiziert die Arbeit Übertragungskapazitäten und die Positionen der Stromeinspeisung als zwei wichtige Einflussfaktoren des Lastflusses.

Contents

1. Introduction

The Energiewende has been declared by the German government in 2011 to design a much more sustainable energy system until the year 2050. Subsequently, the topic has come into European focus and structurally substantial changes have been proclaimed [European Commission 2012]. More efficient ways to transform primary energy into heat and current ought to be found while at the same time its use should be reduced. By employing renewable energy technologies like photovoltaic, wind or hydro turbines, overall CO_2 emissions will decline in the long term. Inevitably the necessity for some kind of power buffer arises, e.g. water pumped hydro storage, air pressure, hydrogen tanks or lithium batteries. Beside this trend toward a sustainable energy mix, the (European) Energiewende aims for decoupling gross domestic product growth and energy consumption, e.g. the reduction of power used in electrical devices, mobility or facility climate control. In this strategic shift, every country has to take its own actions but having a European energy exchange in mind, not only segregated "island" networks must be analyzed. As a whole the shift from fossil to renewable energy sources (RES) poses complex challenges while minor changes of the energy supply system lead to unforeseeable follow-ups.

With the long-term plan set, scenarios for different systems can be created to discover the best combination of technologies for specific situations. It could be sufficient to use fluctuating energy sources like photovoltaics and wind power together with storage facilities like hydro pumping storage or lithium-ion batteries. Whereas under some weather conditions the integration of dispatchable renewable energy sources like Concentrated Solar Power or biomass might be better. Regulations like CO_2 emission costs or reimbursements, as used for photovoltaic power plant installations, change the system's input parameters frequently.

The energy system model REMix approaches this problem by minimizing overall system costs, taking into account techno-economic input factors, weather and demand. As a bottom-up model, REMix optimizes the power plant complex under constraint of equality between energy demand and generation [Scholz 2012].

Possible future technologies can be conceptually included and their influence on factors like green house gas emissions, grid stability and other storage technologies can be estimated. An example of this use case is the integration of electrical vehicles in REMix as energy storage [Luca de Tena 2014].

Regarding distributed as opposed to central power production, an energy system with high ratio of RES will most likely be more reliant on energy distribution than current systems with large fossil-fueled power plants [Eßer-Frey 2012]. As a consequence, distribution of energy is an essential component of system models that focus on scenarios for a future energy mix.
Most models therefore integrate a DC load flow simulation which represents a high-voltage power grid as an electrical circuit. In such circuits, energy is transmitted from point A to point B via an electrical (transmission) line with respect to its impedance and some loss factor. The circuit is often downsized, i.e. its nodes are reduced.
Each transmission line also imposes a limit on how much power can be transmitted. This "capacity" is a major factor in decisions about grid expansion. As an example one might imagine a power network having enough transmission capacities to handle peak loads within several regions. Nowadays however, in case of high wind occurrence, wind power plants in North Germany need to be curtailed because the energy cannot be transferred to South Germany where it could be used [Bundesnetzagentur 2015, pp. 13-14]. If power was previously bought on the European spot market and cannot be transferred to its destination, a redispatch takes place: the energy needs to be produced and injected in another place where transmission capacities last out.
When redispatch is needed, the cheapest power plant can not be used. The additional costs of the compensatory power plant have to be payed by the general public. For this reason, the rectification of congestions in the European network is another strategic goal of the EU. Consequently, the modeling of power networks in energy system models is a very important factor.

To keep the calculated power flow in a network of reduced buses as accurate as possible, several approaches exist. Some of them are evaluated in section 3.2.
Using the selected power flow algorithm, the influence of different aggregations on optimization outcome is examined. More precisely, six different partitions of the German power grid are embedded in a scenario that executes an optimization for the year 2050. Subsequently, differences in the model output are observed.

2. Motivation & Background

In order to reduce computation time of energy system models, three alternatives are possible:

1. Reduction of temporal resolution, i.e. fewer or larger time steps.
2. Reduction of spatial resolution, i.e. fewer and larger nodes.
3. Reduction of technologies, i.e. less different components in the power system.

It is obvious that the range of technologies should not be restricted for a model that is built to find an optimal mix of energy sources. In the following, the other options are discussed.

One application of an energy system model is the integration of concepts for energy balancing like energy storage or demand side management (DSM). To analyze efficiency of the aforementioned concepts, time steps of 24 hours do not suffice in the simulation. Small storages installed in private houses may be active for only a few hours a day, heavily dependent on external factors like the weather. Actors in DSM also change their energy injection and consumption levels multiple times a day. To account for this behavior, the usual temporal unit of an energy system model is set to hours. In most cases time frame for the optimization is one year or several years, resulting in 8760 time steps and above. However, with increasing spatial and temporal resolution, calculation time increases significantly.

Because the resolution of time and the amount of technologies can't be substantially reduced and are essential for REMix, lower precision in spatial resolution must be tolerated. As described in the introduction, accurately modeling energy distribution is an important requirement especially with many RES in the system. To fulfill this requirement, one can not represent large regions like nations as nodes in a power flow simulation. Doing so would hide the effects of energy peaks on specific parts of the transmission network. For instance, there are situations of high power injection in the north of Germany and at the

same time high demand in Switzerland. If each country was modeled as a single node, the injected power could be transported to Switzerland without any restriction from actual transmission lines and load situation in the south of Germany. In other words, transmission capacities would not be followed.
For a highly accurate load flow simulation, every network node would have to be integrated into the power grid simulation. This would be unfeasible with regard to model run time in scenarios where the whole European region is optimized as well as for single countries. This thesis observes whether different aggregations have different effects on the results of curtailment or dispatch of power plants.

Most energy system models that focus on Germany and adjacent countries use the same aggregation of regions. Here, these regions are used as "data nodes" to form six different zones whereon load flow is calculated using an adopted approach from [Di Shi et al. 2012]. Aggregation of lines between zones should be done automatically once the mapping of nodes to zones is given.

3. State of the Art

This chapter gives an overview on power flow calculation methods that are currently used regarding energy system modeling.

3.1. Load Flow Simulation

By modeling the power grid as an electric circuit, one can use Ohm's and Kirchhoff's laws to calculate power flows thereby simulating the real power grid. Transmission lines transferring direct current (DC) are most often installed for long distances, e.g. connections over sea.

A majority of the European transmission lines carry alternating current (AC), which is why only AC transmission is covered in this thesis. Since the energy system model REMix is implemented utilizing linear programming, the power flow calculation must base on linear calculations likewise. This is fulfilled by the DC load flow approach [Oeding et al. 2011]. Applying assumptions like constancy of voltage magnitude in all nodes (which is nearly true for high voltage grids), the AC load flow equations become linear. It follows a form of Ohm's law for the relation between active power flow and voltage angel:

$$P = B\ \theta \tag{3.1}$$

In this thesis, the terms node and bus are used interchangeably, originating from the fact that one of them is preferred in traditional and the other is favored in market based power flow studies. Transmission lines between nodes are represented as electrical lines in the circuit. Load corresponds to received power whereas generation is equivalent to infeed of power at a specific bus.

3.1.1. Transmission capacity

A common term in power network calculations is transmission capacity. This generic name refers to the maximal amount of power that is allowed to be carried by a transmission line due to its maximal thermal stress limit. Fine-grained definitions exist depending on the context [entsoe 2011]. Total Transfer Capacity (TTC) is closest to the physical limit of the line, depending on a specific state of the grid.
A legal requirement for setting the TTC is adherence to $N-1$ and $N-2$ criteria. With N being the total number of components in the network, the system must tolerate the outage of 1 and 2 lines, respectively. To further lessen probability to reach physical line limits and preserve capacities for emergency transfers, a buffer exists: the Transmission Reliability Margin (TRM), which also averts measuring errors. This figure determines the amount of capacity that has to be reserved in a normally operating grid. Subtracting TRM from TTC, the result is Net Transfer Capacity (NTC). Transmission System Operators (TSO), who maintain the power grid, usually report NTC values when asked for transmission capacities. In different contexts, the concepts of Already Allocated Capacity (AAC) and Available Transfer Capacity (ATC), can be useful.
Energy system models mostly use the term capacity and refrain from a more detailed classification.

3.1.2. Power Transfer Distribution Factors

The idea of Power Transfer Distribution Factors (PTDF) is to analyze by what percentage power flow through every line l varies when power injection at node i is changed:

$$PTDF_{l,i} = \frac{\Delta P_l}{\Delta P_i}. \tag{3.2}$$

For instance, if the amount of current in line 4 raises by 100 MW and power injection at bus 2 was raised by 400 MW, then

$$PTDF_{4,2} = \frac{100}{400} = 25\%.$$

A big advantage of PTDF is the possibility to calculate load flows by simple matrix multiplications once the PTDF values are available. On the other hand, PTDF are static. They suffer from significant deviations if power injection positions or topology change [Duthaler et al. 2008].

As PTDF values are mostly looked at for the complete network, they are represented by an $L \times N$ matrix, where N is the number of buses and L the amount of transmission lines in the network. The matrix can be filled by an iterative process where power flow for the network is simulated and power injection is varied for every bus. Although the difference to DC PTDF does not seem to be high, the effects on load flow calculations can be significant according to [Duthaler et al. 2008]. Depending on the load flow calculation used, one speaks of AC PTDF or DC PTDF.
On a side note, the overhead of load flow calculation to construct a PTDF matrix can be circumvented if power flow data of an existing network is taken over. While this is an option for analysis of present networks, it is not always sufficient to be used for future scenarios.

In energy system models which allow the building of new generation capacities (and therefore the gross power with them) could change in almost every step of the system optimization. This circumstance does not only decrease accuracy of PTDF but also of other methods (see chapter 3.2.1). Therefore, the PTDF matrix would have to be calculated after every change of the system state.
Some authors try to solve this problem by so-called generation shift key (GSK) matrices [van den Berg et al. 2014; Kurzidem 2010]. A rather classical approach and one resembling the GSK approach that try to solve the above problem are introduced in chapters 3.2.2 and 3.2.3, respectively.

3.2. Network Reduction Methods

Multiple techniques exist to reduce the number of nodes in the representation of a power network. A concise comparison of some of them can be found in [Papaemmanouil et al. 2011]. In the process of reduction, the original nodes (“data nodes”) are aggregated to result in a network whose nodes will be called “model nodes”.

3.2.1. Ward, Kron and REI

An early approach for bus aggregation in load flow analysis was first described by [Ward 1949]. Ward examined circuits which had at least one generator and load while most buses were “passive” buses without load and generators connected to them. These passive buses were aggregated[1] by an algorithm known as Kron's Reduction [Dörfler et al. 2013; Kron 1959]. By applying the algorithm x times, the network is reduced by x buses. To demonstrate the procedure, it is explained on four nodes which can be seen as a section of any larger network (see figure 3.1).

Each line has some resistance R, in conjunction with its reactance X formulating complex impedance Z. Its reciprocal admittance is proportional to conductance G and susceptance B:

$$\begin{aligned} Z &= R + jX \\ Y &= G + jB = Z^{-1} \end{aligned}$$

with j being the imaginary unit. Note the reciprocity between resistance and conductance as well as between reactance and susceptance.

By Kirchhoff's law, the injection of current into bus l equals the amount of current that flows away from it. Thus, current I_l at this bus is [Brown 1985]:

$$I_l = V_l \sum_{m \neq l} y_{l,m} - \sum_{m \neq l} V_m \, y_{l,m} \tag{3.3}$$

where V_l is voltage at bus l and $y_{l,m}$ is the transmission admittance between buses l and m.

Finally, the following matrix equation holds for every node i in a network with N buses:

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} \begin{bmatrix} Y_{1,1} & Y_{1,2} & \dots & Y_{1,N} \\ Y_{2,1} & Y_{2,2} & \dots & Y_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{N,1} & Y_{N,2} & \dots & Y_{N,N} \end{bmatrix} \tag{3.4}$$

with

$$y_{k,k} = \sum_{k=1, k \neq i}^{N} y_{k,i} \quad \text{and} \quad Y_{k,i} = -y_{k,i}.$$

The rightmost matrix above is called the Y-Bus.
For electrical lines in the high voltage grid, resistance is very low compared to their reactance [Oeding et al. 2011]. Tolerating some loss of calculation accuracy, just the real part

[1] usually, the term "reduced" was used

of admittance is considered:

$$R << X \iff G >> B \tag{3.5}$$

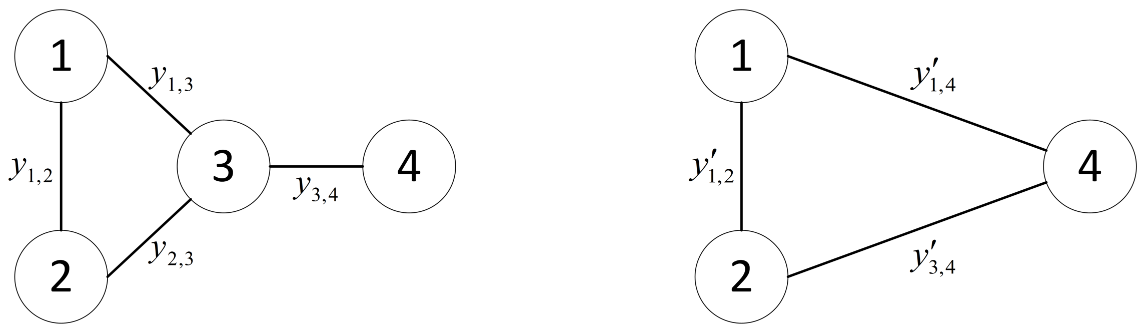
$$\implies Y = G \tag{3.6}$$

Assuming $I_k = 0$ for a specific bus, Kron's reduction can be applied as a single calculation to remove bus k.

On removal of bus 3 which is connected to at least two other buses, its adjacent lines disappear (see figure 3.1). The admittance of these reduced lines are inherited by one replacing line. When node k is removed, the replacement admittance $y'_{l,m}$ for edges (l, k) and (k, m) is calculated by [Kron 1959]:

$$y'_{l,m} = y_{l,m} - \frac{y_{l,k}\, y_{k,m}}{y_{k,k}}. \tag{3.7}$$

This formula essentially computes a wye-delta [Kennelly 1899] transformation on condition that injection current at bus k is zero.
The replacement admittance needs to be calculated for every neighbor bus of k. Since only this equation consisting of basic arithmetic is needed, the algorithm scales well for large networks.



(a) Before reduction: Y-shape

(b) Delta shape

Figure 3.1.: An example for Kron's reduction

However, there is an important limitation. Removing a bus changes the power flow between its neighbor buses, if either load or power injection is located at the removed bus. This has been tested by the author and others [Shayesteh et al. 2015; Liacco et al. 1978]. Over the last decades, attempts have been made to minimize load flow errors, one of them called REI.

REI is an acronym for "Radial Equivalent Independent" which denotes one class of electrical network reduction, first mentioned in [Dimo 1975]. The idea is that one section of the power network stays the same, that should be analyzed in detail. It is called the internal system. All the remaining buses are reduced by Kron's Reduction and collectively called the external system.
For generators and load that exist in the external system, artificial buses are created. These artificial buses are connected between the boundary buses of the internal system and active buses of the external system. By linearizing admittance, all load and generators from the external zone are transferred to the artificial buses [Savulescu 1981]. This means that for each reduced external system, several boundary buses have to be retained.
There have also been approaches that define multiple external zones and interconnect these zones. In these methods, indeterminate iterative techniques [Min et al. 2006; Zhao et al. 2004; Granada Echeverri et al. 2010] or some kind of coordination between zones [Zhang et al. n.d.] are used to minimize errors in subsequent load flow calculations.

All of the above algorithms require additional computations after network reduction to calculate power flows with sufficient accuracy. In energy system models, power flow calculations can not be as sophisticated as in pure power flow calculators.
An important argument against these REI methods is the fact that the locations of power injection changes numerous times in energy systems. Re-computation of the external zones and their associated boundary buses would be required for every such relocation. In addition to that, external and internal system have to be redefined.

Kron's reduction itself can be useful to process big data resources. If the data at hand contains many buses with guaranteed absence of load and generation (passive buses), the algorithm is an effective tool to decrease data complexity and size. In energy system models, Kron's reduction is only useful for data preprocessing. Most of the time, there are only active nodes in the optimization phase.

3.2.2. Consideration of transmission capacities

Many power flow simulations give no limit on the amount of power that can flow through a line. In more recent simulation methods a limit of power flow for specific lines is considered. In this section, a method from [Wonhyeok, S. Mohapatra, et al. 2013] is presented. Like several others it combines Kron's reduction with the calculation of Power Transfer Distribution Factors. In addition to that, an attempt to calculate transmission line limits

is made.
Certain advantages for the integration of capacities in energy system models unfold. For instance, it enables compliance to N-1 reliability and implementation of political or economic constraints, e.g. transmission limits determined by contracts. For this reason, algorithms integrating PTDF and transmission capacities are sometimes called market-based.

The authors begin with a network in which some buses have generators attached to them. Now for a bus k that should be removed, power transactions to each of its neighbor buses ("sinks") are calculated, using a lossless dc approximation. Dividing the amount of power that flows through every line on the path to the sink by the total power flow of the transaction gives the PTDF value for every line on the path. Subsequently, bus k is removed and post-reduction admittance is calculated for the bus, using Kron's formula 3.7 from above. Repeating the steps before reduction yields post-reduction PTDF values for the lines near to k.

Afterwards, the appropriate replacing line limits $\tilde{F}_{l_i}$ for each line l_i are calculated. The lines that replace the previous lines are also called equivalent lines. As can be seen in figure 3.2 removing a bus results in its adjacent lines being removed. The figure also shows a transition state where bus k was removed and its adjacent lines were joined, now situated between buses 1 and 3.

To get from figure 3.2a to figure 3.2a the lines {1,2} and {2,3} are joined[2]. Whenever two lines in series are joined, the limit $\tilde{F}_l$ of the equivalent line equals [Wonhyeok, S. Mohapatra, et al. 2013, equation 4]:

$$\tilde{F}_l = \min_{l_i} F_{l_i} \tag{3.8}$$

for all lines l_i in a series.

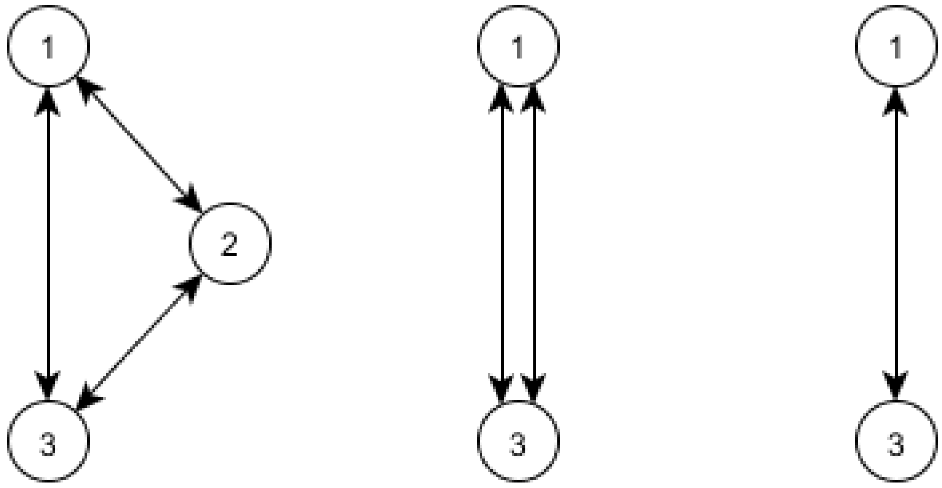
The equation can be explained easily: if electrical current flows through i lines in a series, the amount of current is bounded by the line l_i with the lowest capacity.

Similarly, the bounding limit for transitioning into state 3.2c can be calculated (equation 5 of the above):

$$\tilde{F}_l = \min_{l_i}\{F_{l_i} \frac{y_l'}{y_l}\} \tag{3.9}$$

where l_i are the parallel lines to be replaced by one equivalent line. y_l is admittance of a line to be replaced and y_l' is admittance of the equivalent line. The final line's admittance

[2] {i,j} denotes an indirected line between buses i and j



(a) Before reduction (b) Transition state (c) After reduction

Figure 3.2.: Aggregating three nodes. Each node represents a bus as part of a power grid.

is known in this calculation because bus k was reduced by Kron's reduction before which yields admittance of all adjacent lines.

Lastly, lower and upper bounds for transmission capacity per transaction can be calculated. Computational complexity is stated to be $\mathrm{O}\left(\binom{e}{2}^3\right)$ where e is the (average) number of neighbor buses of the reduced buses. While computation time has become better recently from code optimizations [Wonhyeok, Saurav Mohapatra, et al. 2015], there is still other work to do, according to S. Mohapatra et al. [2014]:

> Work is in progress [...], which will result in equivalents exhibiting lesser sensitivity to their original operating point and greater preservation of transmission limits.

Meaning, that the quality of this reduction depends largely on the real system's state and is still being improved on. In another section, the authors mention the problem with many power injecting (active) buses already discussed in the section before.

3.2.3. An adjusting PTDF approach

Because of the problems of other methods that neither reduce active buses by significant amounts nor output satisfactory line limits, another approach [Di Shi et al. 2012] is examined. The method is suitable for market based power flow implementations, although it ignores limits. In addition to that, the authors claim to produce a quality of results almost independent of the system operating point.

The following is assumed in the lossless DC approach:

$$P_{\text{inj}} = B_{\text{bus}}\theta \tag{3.10}$$

$$P_{\text{flow}} = B_{\text{branch}}\theta \tag{3.11}$$

Power injection vector P_{inj} defines the amount of power injection for all buses and is of dimensions $(N \times 1)$. B_{bus} $(N \times N)$ denotes the vector containing all bus susceptances and θ contains the bus voltage angles.
Power flow for the branches is represented by vector P_{flow}, proportional to branch susceptance matrix B_{branch} $(L \times L)$ and bus voltage angles.

Combination of equations 3.10 and 3.11 gives

$$P_{flow} = B_{\text{branch}}B_{\text{bus}}^{-1}P_{\text{inj}} \tag{3.12}$$

From this equation, the PTDF matrix Φ is derived:

$$\Phi = B_{\text{branch}}B_{\text{bus}}^{-1} \tag{3.13}$$

Based on the previously used simplification of resistance being significantly less than reactance in electrical wires, further rearranging of formulas enables calculation of the PTDF matrix by two variables:

$$\Phi = diag(1/x)K[K^T diag(1/x)K]^{-1} \tag{3.14}$$

where $diag(1/x)$ is a matrix whose entries are the inverse reactances on the diagonal axis and 0 for all other indices.
The incidence matrix K is an $(N \times L)$ matrix[3] with:

$$K_{ij} = \begin{cases} 1 & \text{if node i is incident to line j} \\ 0 & \text{else} \end{cases} \tag{3.15}$$

[3]The incidence's dimensions in [Di Shi et al. 2012] are given transposed, probably by mistake

Until here all has been prior art and was already implemented in REMix. The prominent equation is the one to calculate the PTDF matrix of the reduced network ("zonal PTDF")[Di Shi et al. 2012]:

$$\Phi_R = \Pi_{\text{flow}}\, \Phi\, diag(P_{\text{inj}})\, \Gamma\, [diag(\Pi_g\, P_{\text{inj}})]^{-1} \tag{3.16}$$

The matrices Π_{flow} $(L \times L_r)$ and Π_g $(N \times N_r)$ map lines and nodes to their corresponding lines and nodes in the reduced network while N_r is the number of nodes in the reduced network and L_r is the number of lines in the reduced network, so

$$\Pi_{\text{flow},ij} = \begin{cases} 1 & \text{if line i corresponds to zonal line j} \\ 0 & \text{else} \end{cases}$$

Analogously for Π_g.

Computation of Φ_R is very efficient due to sparse matrices and matrix to be inverted being a diagonal matrix.

Finally, power flowing over the lines is retrieved by a simple matrix multiplication:

$$(P_{\text{flow}}^{inter-zonal})_R = \Phi_R (P_{\text{inj}})_R \tag{3.17}$$

Besides having a computation complexity of O (1) which scales well for large systems, there is another big advantage. If only power injections change, the computational cost is low compared to other approaches. Only the zonal PTDF matrix must be recomputed to be able to calculate power flows in one more calculation.

Moreover, Shi and Tylavsky report better results regarding load flow deviations than other methods. In a selected scenario, power flow remains very close to the results of an exact power flow model. Almost no error is evoked even if power injection is varied by up to 100% (see figure 3.3).

3.2.4. Summary

The classical load flow calculations, originating from electrical research, have been complex dedicated computation methods. They achieve good results for static systems if enough computation time is available. Because of this they are still more suitable for operational applications like security analysis.

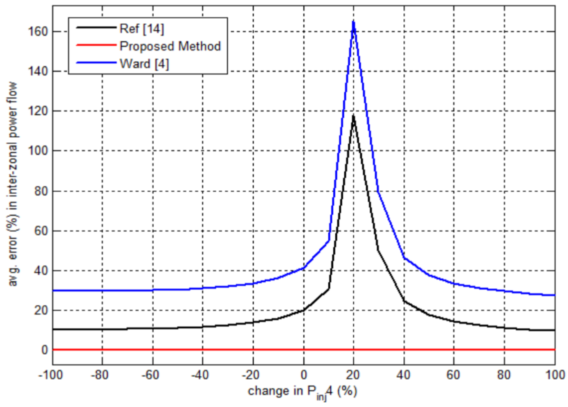


Figure 3.3.: Average error in inter-zonal power flow on variation of power injection at an active node [Di Shi et al. 2012]

Energy system analysis, which include market analysis, put high demands on computation speed and accuracy. Furthermore, capacity constraints for power transmission must be implemented to reflect non-technical conditions of real power grids.

Instead of investing much effort and computation time in calculation of proper line capacities, an easily implementable and satisfyingly accurate approach for load flow simulation by Di and Tylavsky is preferred in this thesis.

4. Method

In order to analyze model output for different node aggregations, the node reduction approach explained in the previous chapter (3.2.3) is adapted. The objective of this thesis is to find out whether the output dispatch of the model changes if the mapping of nodes to zones is changed. Dispatch describes the composition of power plants that are operated by the energy system model in the optimization result. Further details on the method are described in the sections below the following listing of my procedure.

1. Implementation of the approach in GAMS by utilization of JavaScript and Python.
2. Validation of the implementation by comparison with the previously existing method.
3. Execution on a scenario for the year 2050. Different node aggregations were used for a one-year period in hourly time steps.
4. Comparison of the scenarios regarding the resulting dispatch.

4.1. REMix model

At the institute of Engineering Thermodynamics at the German Aerospace Center DLR, the bottom-up energy system model REMix was created. Using a linear optimization solver served by GAMS[1], an objective function to minimize investment costs and annual payments is compiled.
Weather and load data for Europe are available in hourly resolution to design scenarios that simulate various mixes of energy generation technologies. Heat and power producing power plants can be parameterized in high detail and local capacities for installation of the plants are inputted. Generally, for every hour of the scenario energy power generation needs to be equal to power consumption. Other constraints can be added on a modular basis, for instance taxes on the generation of green house gas. Integration of concepts

[1] http://www.gams.com

like energy storage and demand side management is optional, but is not examined in this thesis. The main focus lies on the network module that calculates AC power flows and provides capacity constraints. Further details on the model can be looked up in [Scholz et al. 2014; Scholz 2012].

Three different types of starting points for the scenarios can be chosen regarding the capacity of power plants. Either a network without any power plants is given, in which case upper bounds for the expansion of power plants in each zone have to be given. Or a network with some power plants already installed is given. In this instance, expansion of new power plants can be allowed or deactivated. Here, some power plants are already given as an input and capacity expansion is allowed to be able to compare scenarios better.

4.1.1. OMaT

Easy parameterization of the model is accomplished by a graphical user interface that has been written specifically for REMix, called OMaT. Within a project-based structure, OMaT allows to manage multiple (sub-)scenarios. Inside each scenario, model input parameters can be entered, reordered and filtered. The data is stored in human-readable text format that can be parsed by GAMS. Modules for the modeling of technologies and other constraints can be included without modification of the GAMS code with which they are implemented.
After parameterization, GAMS execution can be launched from within OMaT. Results can either be quickly inspected with the GAMS integrated development editor or other data formats can be generated for a more detailed analysis.
The interface has been written in C++ with the Qt4 library.

All parameters and options that can be entered are defined in the modules which are implemented in GAMS and JavaScript. Since GAMS features are limited with respect to data preparation and conversion, other tools and programming languages like Python are used for post-processing purposes.

4.1.2. JavaScript

JavaScript is used in close connection with GAMS code to enable a flexible input of parameters in OMaT depending on specific project requirements. It enables automatic injection of these parameters into the modules. Data between OMaT and JavaScript is exchanged via a script engine offered by the Qt4 library, called QScriptEngine. This script engine is also responsible to run the JavaScript code. The execution of JavaScript is launched inside OMaT by a simple button or automatically, if preferred.

Every parameter and option that's offered to the user of a module must be declared in JavaScript code. The JavaScript programmer can also pass validation callbacks that are used to validate user input. In the network module, a validation check is performed to verify that every line has exactly one start and end node. This way, the user of a module does not need a long introduction phase to understand all possible input parameters for any new module.

As a whole, the combination of JavaScript with GAMS is powerful for automating all kinds of preparations to be executed before optimization. However, because Javascript in its current form follows a functional programming paradigm rather than an object-oriented one, the code is prone to become unmaintainable. It is challenging to structure the code for large projects that use many modules.

4.2. Data

All data that's used for the scenarios in this module resembles the one used in past applications of REMix [Cebulla et al. 2015; Scholz 2012]. Except that in this work, only data for Germany is used.
Currently, the German transmission network is owned and maintained by four transmission system operators (TSO): Tennet, 50hertz, Amprion and TransetBW. One of them, Amprion, published a regional model of the German power grid [Amprion 2011]. This data set divides Germany's power grid into 20 regions and specifies transmission capacities. Although the data set does not have a high spatial resolution, it is recommended by the dena Grid Study II [dena German Energy Acency 2010]. This grid study is intended as a guideline for the integration of the power grid in strategic energy system analysis.

When model data for REMix is parameterized, two types of nodes can be defined: "data nodes" and "model nodes". Data nodes are the representation of the nodes in high resolution, e.g. 20 regions in Germany. Model zones are the aggregated data nodes, e.g. all regions can be mapped to one "Germany"-node. With this design, the mapping of data to model nodes is versatile and can be easily configured for sub-scenarios. The optimization

is only to be done on model nodes.
Before the network module was modified as part of this work, there were only one type of transmission lines, which had to be defined manually for every set of model nodes. Now, data of the transmission lines only needs to be entered once and is automatically aggregated according to the mapping of nodes.

Minor adjustments to the network was made because the scenarios of this work are calculating on the year 2050. E.g., the offshore wind farms in the north of Germany are connected by capacities expected to be installed by the year 2020.

Here, the maximal number of zones has been limited to be able to optimize one year in several hours of real time.
Aggregation of the zones can not be completely arbitrary because every node in a zone must be connected to at least one other node in the zone, i.e. the zones may not be disseminated. Lastly, every node must belong to at least one zone.

4.3. The AC network module

The calculation of AC power flows that's implemented in REMix is mainly based on the equations explained in 3.2.3, including the formulas for the (model) PTDF matrix.

Until now, only transmission line lengths were factored into the load flow calculation. With the new implementation, power injection at one specific time ("injection state") is also integrated in the PTDF calculation (see equation 3.16).
For a full implementation of the network reduction approach, the power generation in the original network is required to get the time dependent variable P_{inj} (see eq. 3.16). Therefore, all variables representing the power injection of every technology need to be extracted from every module for every time step. Because of the large number of different modules this requires considerate effort and coordination between all people who maintain the modules. Another hurdle when implementing P_{inj} for every module were concerns about calculation time, which would rise.

As mentioned in chapter 4.1.1, not every module's functionality is known when executing the optimization. Another hurdle when implementing P_{inj} for every module were concerns about calculation time, which would rise. Because of that, the implementation of formula 3.16 is simplified in this thesis by using a static value for P_{inj}. Therefore, only the installed capacities are extracted from every module.

Implementation of the PTDF matrix requires the calculation of a matrix' pseudoinverse. GAMS does not offer a built-in function to calculate the pseudo inverse and complex mathematical calculations are extremely tedious to write in GAMS language. Because of that it has been decided to implement the PTDF computation in Python. While the implementation of the calculation itself was trivial, integration in the REMix environment was not. Every necessary parameter has to be extracted using JavaScript, and written to files. The calculated PTDF matrix then needs to be returned to GAMS.

4.3.1. Capacities

Since the implemented power flow method does not handle the aggregation of transmission capacities with aggregation of nodes, a simple custom method is used. Limits between two nodes that are mapped to the same zone are set to be infinite, i.e. they are ignored. This means that inside a defined zone, it must be assured that transmission capacities are sufficient. For all other lines between zones, their limits are just added up and applied to one equivalent "inter zonal line". This is is easy to compute once Π_{flow} is known. Because Π_{flow} can not be calculated by simple matrix multiplications, an algorithm has been implemented in JavaScript which can be found in A.2).

This means the user only has to input Π_g, i.e. the nodal to zonal mapping, and the aggregation of lines is calculated automatically. The capacities of these lines are then just added up to have acceptable upper boundaries for transmission limits. If no boundary checks are in place, assuming the load flow calculation does not precisely match real power flows, deviations ("errors") would be more grave.

4.3.2. Other constraints

In addition to the other calculations, losses are implemented. For transmission lines, resistance R is proportional to line length l, cross sectional area A and the dielectric constant κ, varying for different materials:

$$R = \kappa * \frac{l}{A} \tag{4.1}$$

Since conductivity variation is assumed to be roughly the same for similar transmission lines, κ is set to be the equal for all lines in the model. Just the same is assumed for A, so line loss is only dependent on its length.

To complete the transmission module, one more constraint is added. According to Kirchoff's Current Law, energy production and consumption are always in equilibrium to each other:

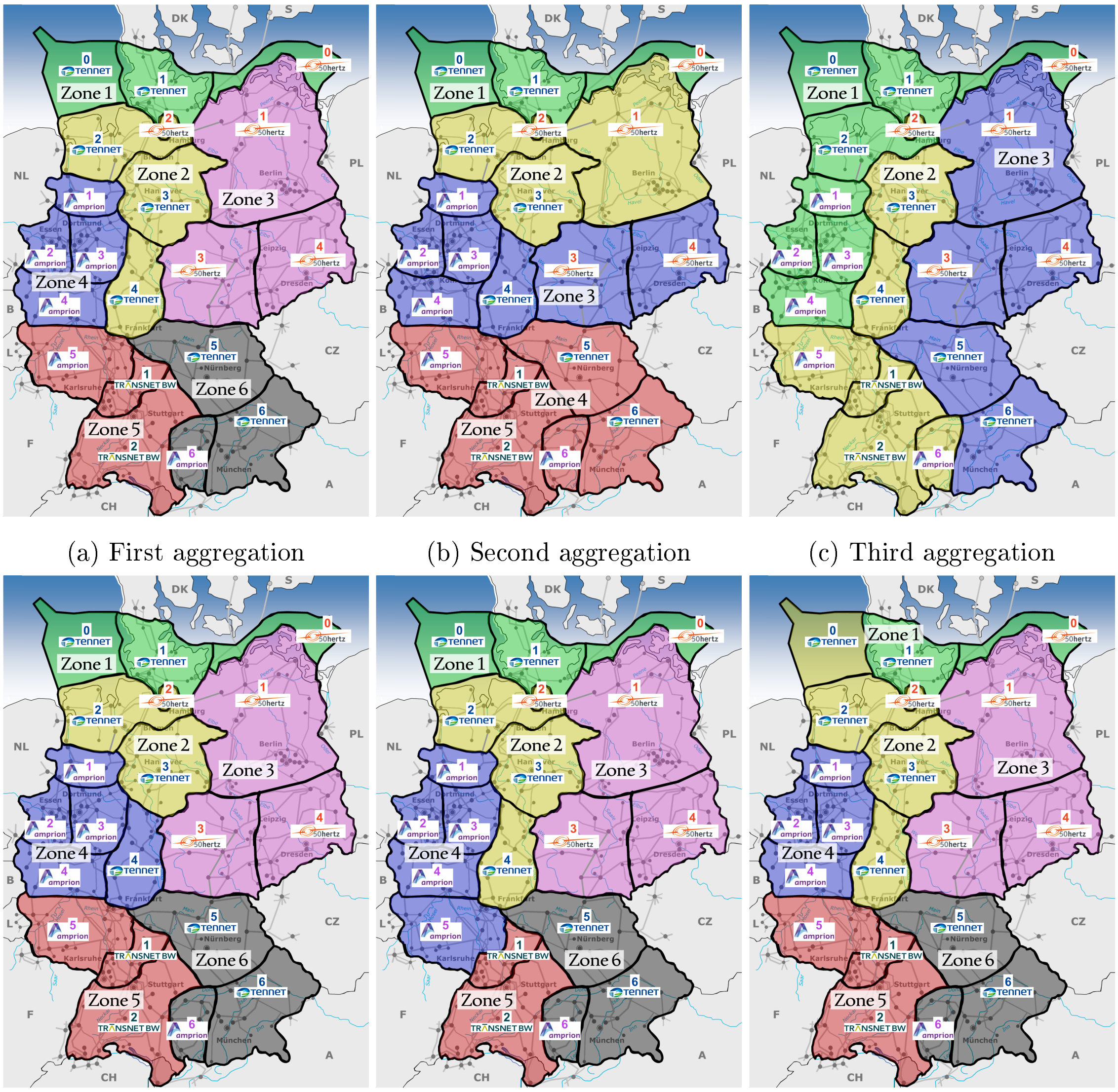
$$\sum_{i=1}^{N} P_i = 0 \tag{4.2}$$

Simply put, every injection of power needs to be taken off the grid somewhere.

4.4. Node aggregations

Using the regions from the TSO's region model as data nodes, a total of six different node aggregations have been arbitrarily constructed:

- The reference scenario "6Zones_0" (A.1) aggregates 20 nodes to six zones so that approximately every zone matches a region that's controlled by a TSO.
- The second aggregation "4Zones_0" (A.2) divides Germany into four zones of horizontally spread shape, each of them having at most 2 neighbors.
- In the third aggregation "3Zones_0" (A.3), three vertically spread zones exist with high generation capacity in North Germany (offshore wind) and high demand in the west.
- Scenarios "6Zones_1", "6Zones_2" and "6Zones_3" (A.4, A.5 and A.6) all have six zones and for each, exactly one node belongs to a different zone than in the reference scenario.



(a) First aggregation (b) Second aggregation (c) Third aggregation

(d) Fourth aggregation (e) Fifth aggregation (f) Sixth aggregation

Figure 4.1.: Node aggregations

5. Results

5.1. Curtailment of fluctuating renewable power generation

The optimization output is summarized in this chapter. Mainly curtailment is analyzed since it is a good indicator for dispatch. The feed-in of renewable energy sources is prioritized by law in Germany. Curtailment is therefore prevented in the energy system model whenever possible. If it is not possible, the curtailed power needs to be generated somewhere else, which results in higher costs for power generation. However, the power generation by fluctuating renewable energy source can not be anticipated. Contrary to non fluctuating sources, they are dependent on the weather and therefore indeterministic. In REMix, this indeterministic behavior is achieved by the input of hourly time series for weather and load.

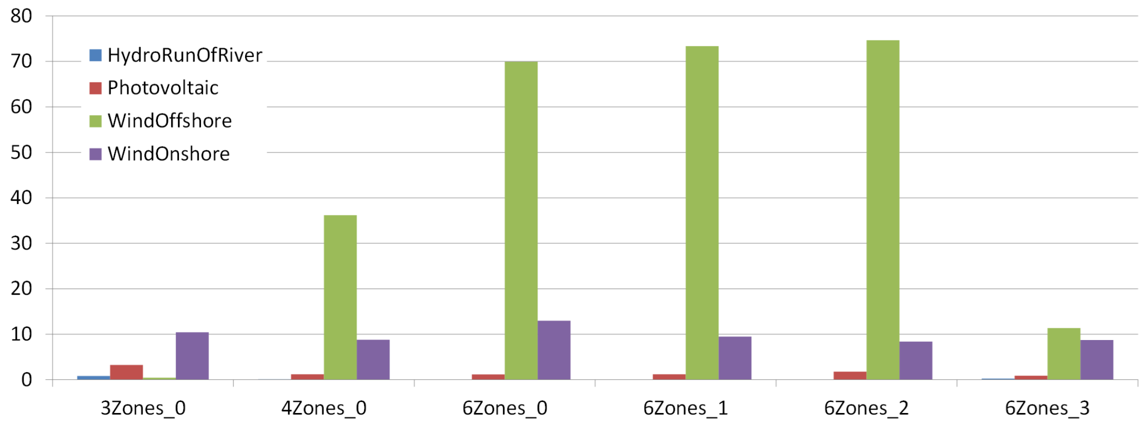


Figure 5.1.: Scenario comparison of annual curtailed energy from fluctuating renewable power generation in Germany in 2050

In the above figure, curtailment per technology for the computed scenarios can be compared[1]. Curtailment of all zones is summed up, so the data represents Germany as a whole. Technologies that are not displayed here, but in figure 5.3, don't experience any curtailment.

The green bars show the curtailment of offshore wind power. They show the highest amount of curtailment (above 70 TWh per year) as well as the highest differences between the scenarios. This shows that offshore wind curtailment is significantly influenced by how zone aggregations are laid out.

Much less photovoltaic (ca. 0.9-3.2 TWh) than wind power is curtailed. This means that photovoltaic power tends to be used in the regions it is generated, mainly the south and west of Germany. For onshore wind power, the curtailment is also much lower than for offshore wind. It seems that there is a larger influence on curtailment if the generation is concentrated on only a few nodes. All offshore capacities are installed in the nodes Tennet-0 and 50hertz-0. Onshore wind capacities are more equally spread across the nodes, comparable with photovoltaic.

A clear result when comparing the different curtailments is that more curtailment takes place in all of the scenarios with six zones (about 84 TWh for 6Zones_0), compared to scenarios with less zones (e.g. 15 TWh in 3Zones_0). Because transmission capacity inside of zones is unconstrained (see section 4.3.1), more power can be distributed. Consequently, in 3Zones_0 more offshore energy can be transported to other parts of Germany and less of it has to be curtailed. Assuming that a country was represented by only node, there would be no transmission limits inside the country.
6Zones_3 is an exception to the above observation. In this aggregation consisting of six zones, node Tennet-0 was moved from zone 1 into zone 2. Since inside of zones there are no transmission limits, more energy can be transferred from the upper left node to zones 3, 4, 5 and 6. For scenario 6Zones_3, almost 7 times less energy must be curtailed compared to other scenarios with six nodes. In those scenarios, Tennet-0 is in a zone with lower transmission capacity to other zones and not all of the potential offshore power can be consumed by zone 1.

Furthermore, the influence of transmission capacities on curtailment is highly significant in these scenarios. Nodes Tennet-0 and 50hertz-0 both have high capacities[2] of offshore wind generation.

[1] Unless otherwise stated, unit of the y-axis is terawatt hours
[2] Here, the term "capacity" indicates the amount of energy that is (potentially) available

Both effects are also present in scenario 3Zones_0, where even less offshore wind is curtailed. In this case, all nodes containing offshore generation are part of the enlarged zone 1. All three zones are bigger and have more inter-zonal transmission capacities. As a result, more installed power is fed into the grid.
In scenario 3Zones_0, Run-of-the-river hydroelectricity (ROR, light blue) experiences more curtailment because energy produced by offshore wind plants is cheaper than ROR.

Another tendency for the examined scenarios can be observed: The less zones exist, the less curtailment occurs. Scenarios 4Zones_0 and 6Zones_3 do not adhere to this rule because of their specific aggregations. Besides, no statement can be made for more than 6 zones. The general tendency comes as no surprise. In the current implementation, overall transmission capacity increases with a decreasing number of zones because there is no capacity restriction inside of zones. While this seems obvious for a low number of zones, the effects can be equally significant for a large number of zones.

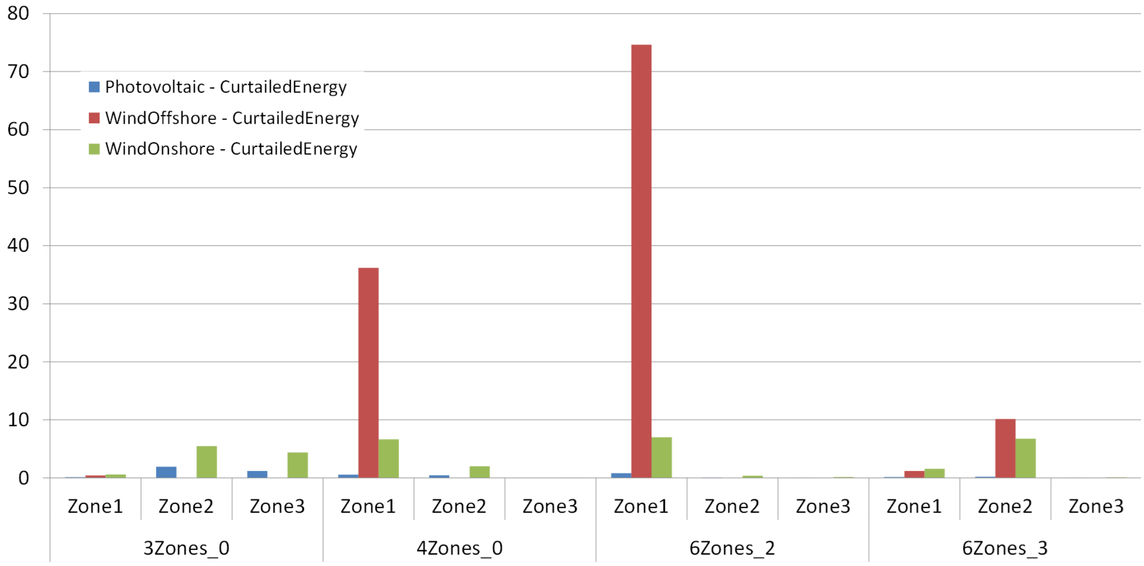


Figure 5.2.: Comparison of the curtailment of fluctuating renewable power generation in different zones

Providing more insight on which zones are affected by curtailment, figure 5.2 breaks down three technologies for zones 1 to 3 of four different scenarios. Caution has is needed when comparing the zones, as they are different in every scenario.
The figure some resemblance between 4Zones_0 and 6Zones_2. The highest magnitude of curtailment is in zone 1 where both nodes with offshore power generation are located.

The amount of dispatch in 4Zones_0 is more than three times the amount in 6Zones_3 but almost half of the value in 6Zones_2. The reason for lower dispatch than in 6Zones_2 is the strong interconnection of western and eastern Germany. Thus, zone 1 can inject more into the grid. A similar explanation can be given for the cause of low curtailment in scenarios 6Zones_3 and 3Zones_0. In both of the mentioned scenarios, node Tennet-0 has a limitless connection to more southern nodes where the injected power is used.

5.2. Annual energy production

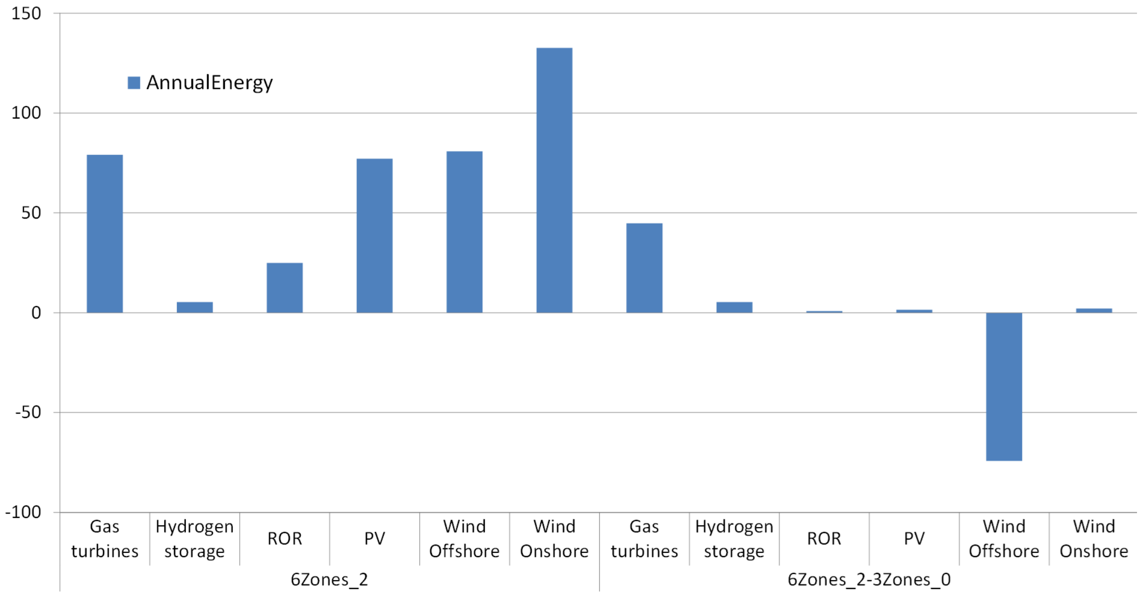


Figure 5.3.: Sum of annual energy supply per technology in Germany for 2050. Scenario 6Zones_2 compared to 3Zones_0

Figure 5.3 displays the annual energy production in two different ways. On the left, the amount of energy produced by some technologies in scenario 6Zones_2. The right side of the diagram shows the difference of these values in comparison with scenario 3Zones_0.

Photovoltaic (PV) seems not to be influenced by node aggregations in these scenarios. The ratio of curtailed to injected power ranges from 1% to 4%, in contrast to 92% of generated power that is curtailed in 6Zones_2. Aside from the local consumption effect explained above, a reason is that the injection of electricity produced by RES has priority over conventional sources. Another explanation could be that the installed photovoltaic does not reach the transmission limits, in contrast to the installed offshore capacities that exceed the limits.

A mix of various RES is supposed to be the optimization outcome because the scenarios are built with parameters that favor the dispatch of RES.

6. Conclusion & Outlook

As discussed in the introduction, aggregation of nodes in energy system models is a feasible way to reduce model complexity and therefore computing time, specifically REMix. In past scenarios, calculation time has been of the order of days while up to 300 and more model runs were started at a time. With the application of load flow models in energy system models, a compromise between both computational efficiency and accuracy needs to be found.

In the case of REMix, performing a reduction of a power network required much manual work. With the implementation of an node reduction algorithm and therefore with the automation of line aggregation, the creation of zonal mappings has become more convenient and faster. Using this advancement, different zonal layouts have been tested regarding their influence on a parameter of the optimization output.

Literature review showed that currently there seems to be no method based on Ward/REI that gives sufficiently accurate results for transmission capacities of a reduced network while at the same time being feasible to use in an energy system model. Therefore, a PTDF-based approach was used.

Oftentimes, node aggregations seem to be arbitrary and static, meaning they don't adjust to different power injection situations or scenarios. Nodes inside the same zone should have low electrical distance to each other so that the assumption of limitless transmission inside of zones is appropriate.

The quality of the power flow calculation was not validated in this thesis, whereas other publications indicate a reasonable compromise regarding the error of DC load flow simplification in high voltage networks. Although the implemented method from [Di Shi et al. 2012] claims to deliver precise results, detailed analysis for the present network needs to be done in further work. The REMix implementation to include power injection of every time step must be completed. Eventually, a compromise between load flow accuracy and run time needs to be determined, see also [Ortner et al. 2014].

Further research on the aggregation of capacity limits is necessary. In future works, different implementations of transmission limits need to be evaluated. One approach is shown in [Wonhyeok, S. Mohapatra, et al. 2013]

In the long term, higher spatial resolution of network data should be looked for. Using Kron's reduction, such data can be reduced without loss of accuracy. Combining this with an automatic cluster algorithm, node aggregations could be automated by aggregating nodes that are electrically close to each other. As a result, not even zone definitions need to be input manually.

All of this shows that more work should be put into the development of power flow models for market based scenarios and energy system analysis in general.
In a next step, the same scenario should be calculated with different aggregations in order to identify effects that are solely credited to network reduction. Looking at the findings in the context of different curtailments, the importance for a careful selection of node aggregations is seen.

References

[1] European Commission. *Energy roadmap 2050*. Energy. Luxembourg: Publications Office of the European Union, 2012.

[2] Yvonne Scholz. "Renewable energy based electricity supply at low costs: Development of the REMix model and application for Europe". PhD thesis. Stuttgart: Universitätsbibliothek der Universität Stuttgart, 2012. URL: `http://elib.uni-stuttgart.de/opus/volltexte/2012/7635`.

[3] Diego Luca de Tena. "Large scale renewable power integration with electric vehicles: Long term analysis for Germany with a renewable based power supply". PhD thesis. Stuttgart: Universitätsbibliothek der Universität Stuttgart, 2014. URL: `http://elib.uni-stuttgart.de/opus/volltexte/2014/9407`.

[4] Anke Eßer-Frey. "Analyzing the regional long-term development of the German power system using a nodal pricing approach". Dissertation. Karlsruhe: Karlsruhe Institute of Technology, 2012.

[5] Bundesnetzagentur. "Monitoring Report 2014". In: (2015).

[6] Di Shi and Daniel J. Tylavsky. "An Improved Bus Aggregation Technique for Generating Network Equivalents". In: *IEEE Power and Energy Society General Meeting* (2012), pp.1–8.

[7] Dietrich Oeding and Bernd Rüdiger Oswald. *Elektrische Kraftwerke und Netze*. Berlin, Heidelberg: Springer-Verlag Berlin Heidelberg, 2011. ISBN: 3642192467.

[8] entsoe. *Definitions of Transfer Capacities in liberalised Electricity Markets*. 2011. (Visited on 07/06/2015).

[9] Christof Duthaler et al. "Analysis of the use of PTDF in the UCTE transmission grid". In: (2008).

[10] Kenneth van den Berg, Erik Delarue, and D'haeseleer. "DC Power Flow in unit commitment models: TME Working Paper". In: (2014).

[11] Martin Kurzidem. “Analysis of Flow-based Market Coupling in Oligopolistic Power Markets”. Dissertation. Zürich: Eidgenössische Technische Hochschule Zürich, 2010.

[12] Antonis Papaemmanouil and Göran Andersson. “On the reduction of large power systemmodels for power market simulations”. In: (2011).

[13] J. B. Ward. “Equivalent circuits for power-flow studies”. In: *Electrical Engineering* 68.9 (1949), p. 794. DOI: 10.1109/EE.1949.6444973.

[14] Florian Dörfler and Francesco Bullo. “Kron Reduction of Graphs With Applications to Electrical Networks”. In: *IEEE Transactions on Circuits and Systems I: Regular Papers* 60.1 (2013), pp. 150–163. DOI: 10.1109/TCSI.2012.2215780.

[15] Gabriel Kron. *Tensors for Circuits*. New York: Dover, 1959.

[16] Homer E. Brown. *Solution of large networks by matrix methods*. 2nd ed. New York: Wiley, 1985. ISBN: 0471800740.

[17] A. E. Kennelly. “Equivalence of triangles and three-pointed stars in conducting networks”. In: *Electrical World and Engineer* 34 (1899), pp. 413–414.

[18] Ebrahim Shayesteh et al. “ATC-Based System Reduction for Planning Power Systems With Correlated Wind and Loads”. In: *IEEE Transactions on Power Systems* 30.1 (2015), pp. 429–438. DOI: 10.1109/TPWRS.2014.2326615.

[19] T. E. Liacco, S. C. Savulescu, and K. A. Ramaro. “An On-line Topological Equivalent of a Power System”. In: *IEEE Transactions on Power Systems* PAS-97.5 (1978), pp. 1550–1562.

[20] Paul Dimo. *Nodal analysis of power systems*. Eyrolles. Bucuresti and Tunbridge Wells, Rumania: Editura Academiei Republicii Socialisté România and Abacus Press, 1975. ISBN: 9780856260018.

[21] Savu Savulescu. “Equivalents for Security Analysis of Power Systems”. In: *IEEE Transactions on Power Apparatus and Systems* PAS-100.5 (1981), pp. 2672–2682. DOI: 10.1109/TPAS.1981.316783.

[22] L. Min and A. Abur. “Total Transfer Capability Computation for Multi-Area Power Systems”. In: *IEEE Transactions on Power Systems* 21.3 (2006), pp. 1141–1147. DOI: 10.1109/TPWRS.2006.876690.

[23] Liang Zhao and A. Abur. “Two-Layer Multi-Area Total Transfer Capability Computation”. In: *Bulk Power System Dynamics and Control* 6 (2004), pp. 499–503.

[24] Mauricio Granada Echeverri, Jesús María López Lezama, and José Roberto Sánchez Mantovani. "Decentralized AC power flow for multi-area power systems using a decomposition approach based on Lagrangian relaxation". In: *Revista Facultad de Ingeniería Universidad de Antioquia* 53 (2010), pp. 225–235.

[25] Xiaochen Zhang and Santiago Grijalva. "Multi-area ATC evaluation based on Kron reduction". In: *2013 IEEE International Conference on Smart Energy Grid Engineering (SEGE)*. DOI: 10.1109/SEGE.2013.6707921.

[26] Jang Wonhyeok, S. Mohapatra, et al. "Line limit preserving power system equivalent". In: *2013 IEEE Power and Energy Conference at Illinois (PECI 2013)*. 2013, pp. 206–212. DOI: 10.1109/PECI.2013.6506059.

[27] Jang Wonhyeok, Saurav Mohapatra, and T. J. Overbye, eds. *Towards a Transmission Line Limit Preserving Algorithm for Large-scale Power System Equivalents*. IEEE, 2015.

[28] S. Mohapatra, Jang Wonhyeok, and T. J. Overbye. "Equivalent Line Limit Calculation for Power System Equivalent Networks". In: *IEEE Transactions on Power Systems* 29.5 (2014), pp. 2338–2346.

[29] Yvonne Scholz et al. "Möglichkeiten und Grenzen des Lastausgleichs durch Energiespeicher, verschiebbare Lasten und stromgeführte KWK bei hohem Anteil fluktuierender erneuerbarer Stromerzeugung". In: (2014).

[30] Felix Cebulla, Tobias Naegler, and Markus Pohl, eds. *Future storage demand under high uncertainties*. 2015.

[31] Amprion. "Übersicht über die voraussichtliche Entwicklung der installierten Kraftwerksleistung und der Leistungsflüsse in den Netzgebieten der deutschen Übertragungsnetzbetreiber (Regionenmodell „Stromtransport 2013“)". In: (2011).

[32] dena German Energy Acency. "dena_Grid_Study_II_-_final_report". In: (2010).

[33] André Ortner and Tomas Kruijer, eds. *Transmission grid representations in power system models: The trade-off between model accuracy and computational time*. 2014.

List of Figures

A. Appendix

A.1. Node aggregations

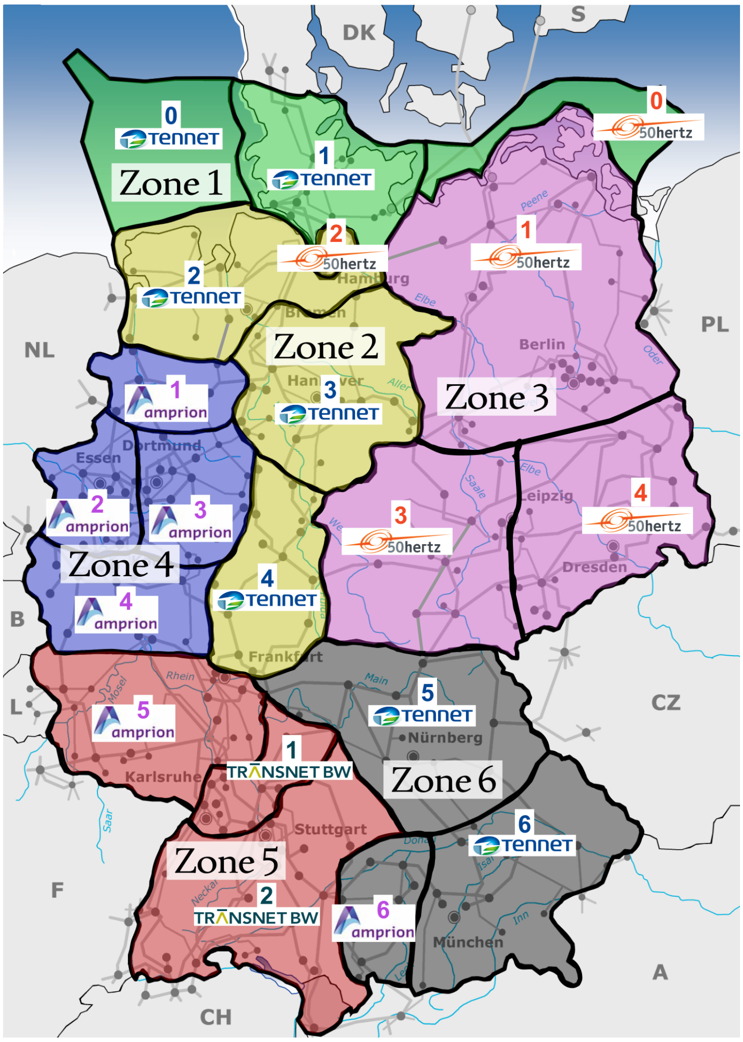


Figure A.1.: First aggregation: 6Zones_0

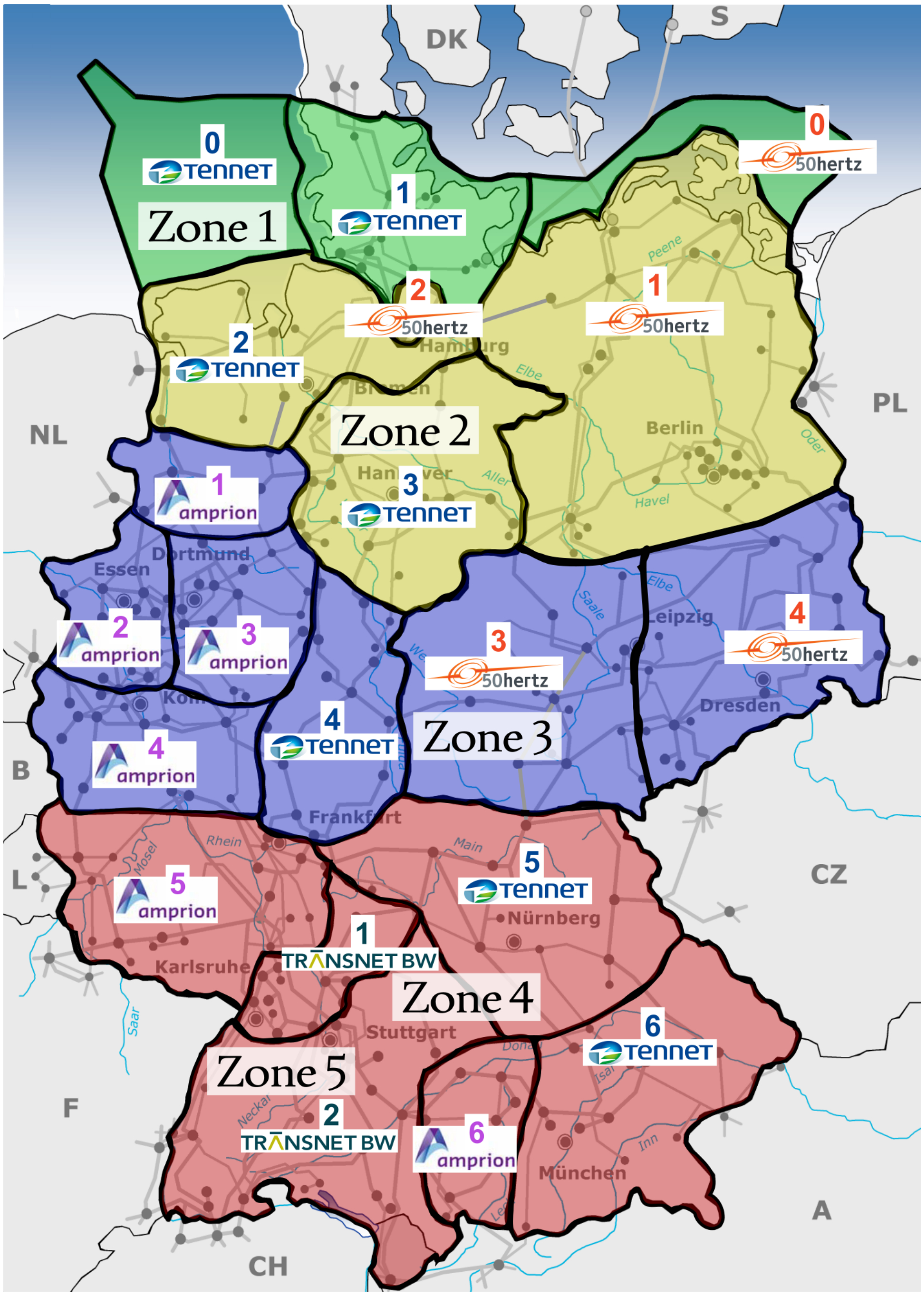


Figure A.2.: Second aggregation: 4Zones_0

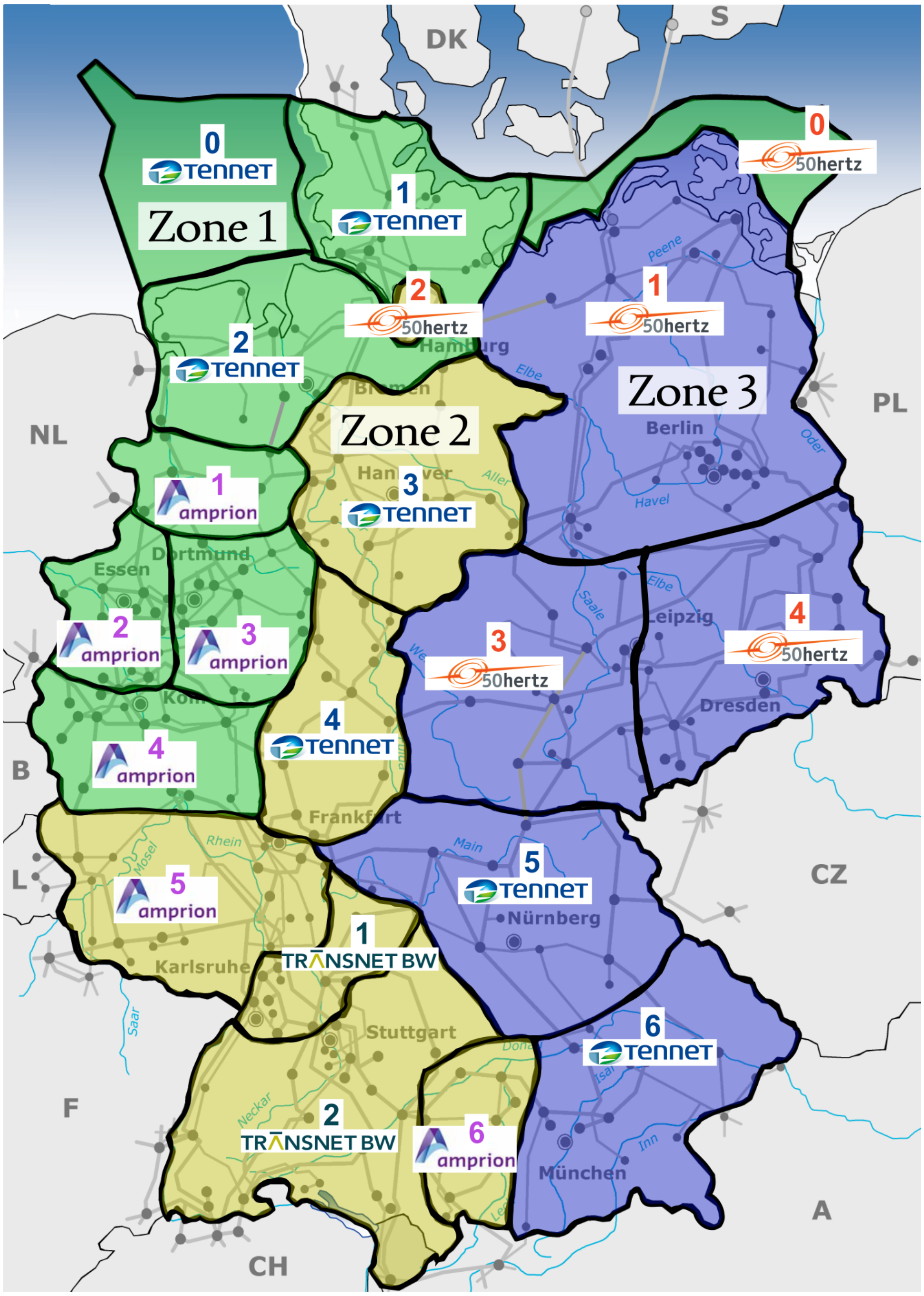


Figure A.3.: Third aggregation: 3Zones_0

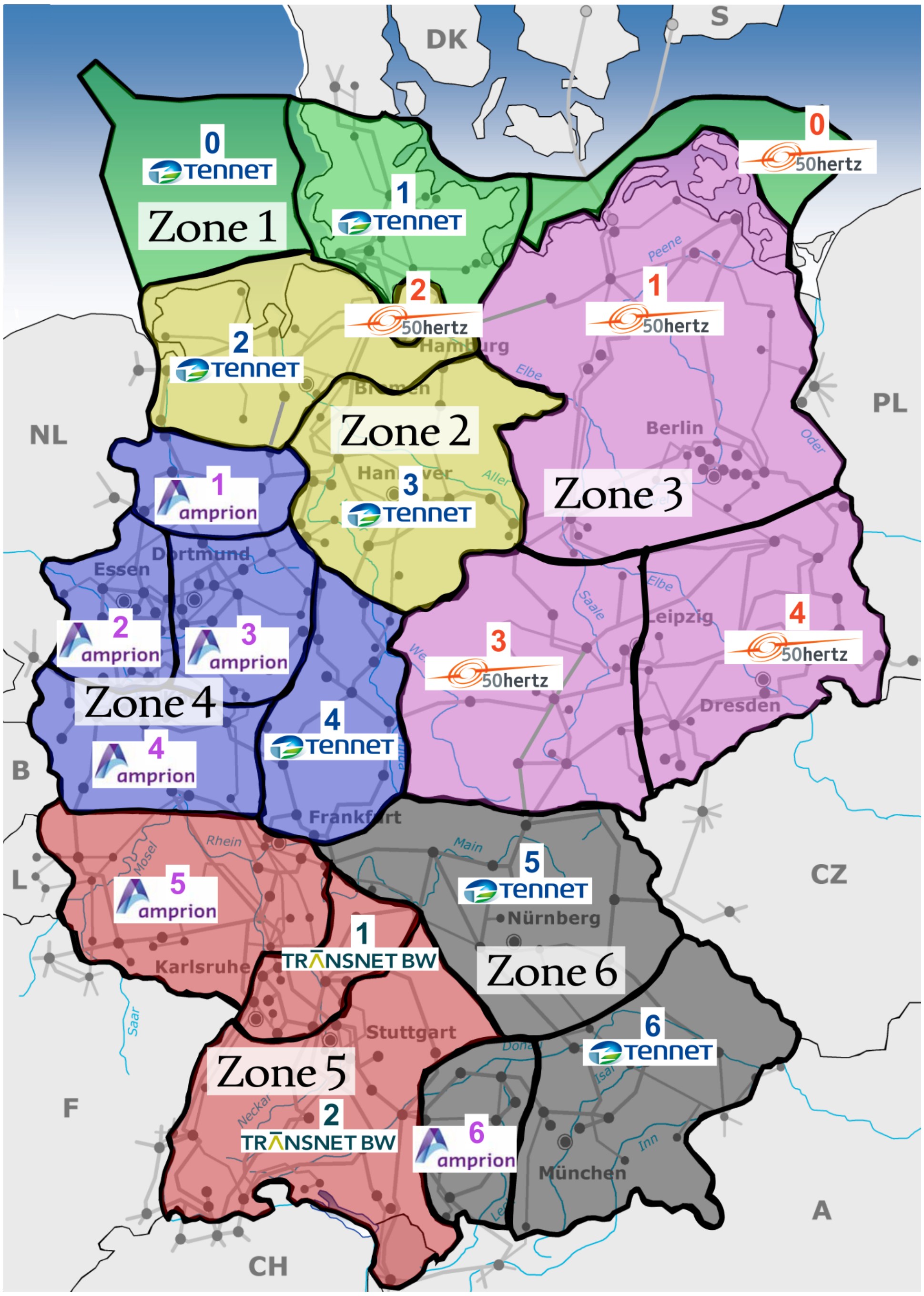


Figure A.4.: Fourth aggregation: 6Zones_1

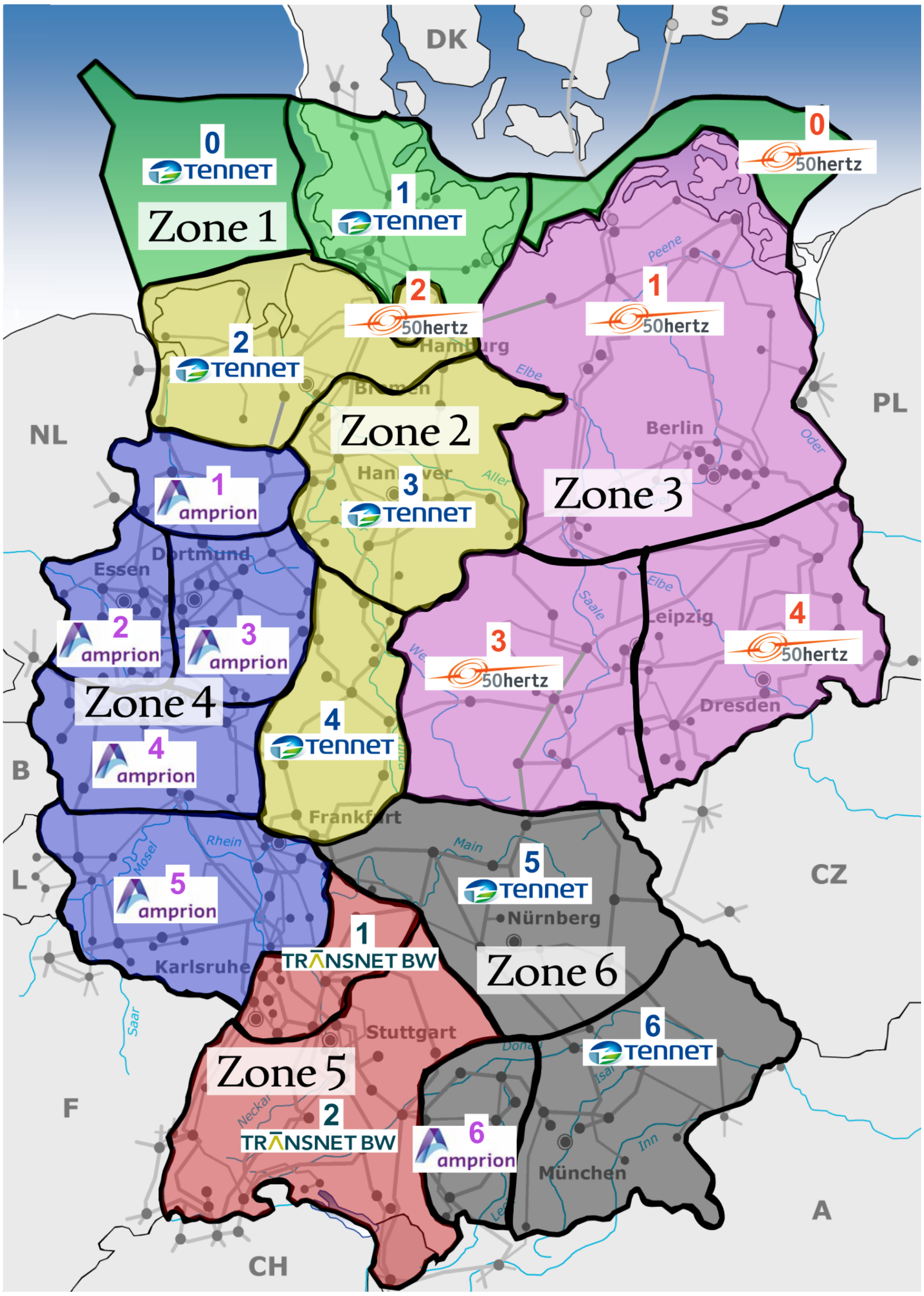


Figure A.5.: Fifth aggregation: 6Zones_2

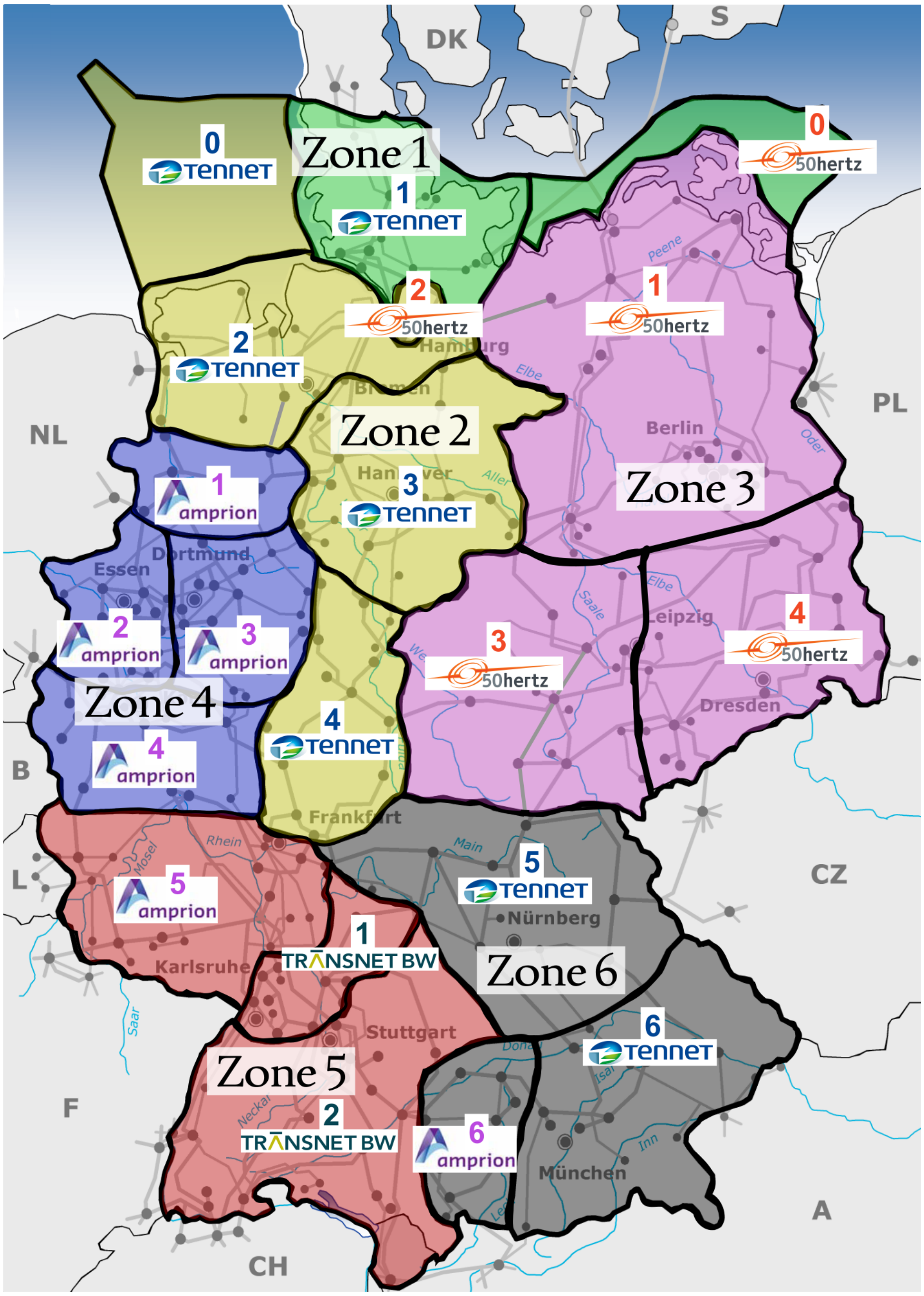


Figure A.6.: Sixth aggregation: 6Zones_3

A.2. JavaScript implementation to calculate Π_{flow}

This JavaScript code creates Π_{flow} after the zones have been defined and Pi_g has been formed.

The input parameter pi_g is a 2-dimensional array that matches Π_g. $incidenceMatrix$ is a 2-dimensional array representing the incidence matrix with $incidenceMatrix[i][j] \neq 0$ if node i is incident to transmission line j. $nodesModelList$ is an array containing the resulting zone names as unique strings.

The function returns an array containing Π_{flow} and a JavaScript object that maps all inter-zonal lines to the indices of data lines that are mapped to the zonal lines.

```
function calculatePi_flow(pi_g, incidenceMatrix, nodesModelList) {
  var nodeCount       = incidenceMatrix.length,
    nodalEdgesCount = incidenceMatrix[0].length,
    zoneCount       = pi_g.length,
    zonalEdgesCount = 0, // unknown until all zonal edges are found
    firstZoneFound,
    zoneOfNodeN,
    pi_flow         = createMatrix(nodalEdgesCount, nodalEdgesCount),
    e, n, z, key;

  /** first step: find all inter zonal edges **/

  // iterate edges
  for (e = 0; e < nodalEdgesCount; e++) {
    firstZoneFound = -1;

    // iterate nodes
    for (n = 0; n < nodeCount; n++) {
      // skip if node n isn't incident to edge e
      if (incidenceMatrix[n][e] == 0) continue;

      // look up which zone node n belongs to
      for (z = 0; z < zoneCount; z++) {
        if (pi_g[z][n] == 1) {
          zoneOfNodeN = z;
          break;
        }
      }
```

```
// is this the first incident node?
if (firstZoneFound == -1) {
  // first incident node found
  // remember its zone membership
  firstZoneFound = zoneOfNodeN;
}
// this is the second(==last) incident node
// what kind of edge do we have?
else if (firstZoneFound != zoneOfNodeN) {
  // zonal edge found
  // edge is incident to node in a different zone.

  // store zone pairs and line indices
  // the key must be unique
  if (firstZoneFound > zoneOfNodeN) {
    key = "modelEdge_" + nodesModelList[zoneOfNodeN] + "_" +
                        nodesModelList[firstZoneFound];
  }
  else {
    key = "modelEdge_" + nodesModelList[firstZoneFound] + "_" +
                        nodesModelList[zoneOfNodeN];
  }
  // either create new sub array or add to existing
  if (zonalEdges[key]) {
    // zonal edge was found before
    zonalEdges[key].push(e);
  }
  else {
    // new zonal edge
    ++zonalEdgesCount;
    zonalEdges[key] = [e];
  }

  // an edge can only be incident to
  // two nodes, stop iterating
  break;
}
else {
  // intra-zonal edge found
  // edge is incident to node in the same zone.
  // Ignore this edge and stop iterating.
  break;
```

```
      }
    }
  }

  /** second step: construct edge mapping a.k.a. pi_flow **/

  // now we know the dimensions of pi_flow
  pi_flow = createMatrix(zonalEdgesCount, nodalEdgesCount);

  e = 0;
  // create a matrix that maps all nodal/data edges
  // to zonal/model edges (summating mapping)
  for (key in zonalEdges) {
    zonalEdges[key].forEach(function(v) {
      pi_flow[e][v] = 1;
    });
    e++;
  }

  return [ pi_flow, zonalEdges ];
}
```