

# Single perturbation load approach: new definition for P1 and explaining the constancy of the buckling load

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## Abstract

The single perturbation load approach is a deterministic analysis method applied in the determination of design loads for imperfection sensitive structures. As verified by Hühne et al., imperfection sensitive cylindrical structures when subjected to axial compression will exhibit a progressively decreasing load carrying capacity for growing imperfection amplitudes up to a threshold, where the buckling load remains nearly constant. The method was named single perturbation load because a unitary lateral load is used to create the geometric imperfection, and the threshold perturbation load beyond which the buckling load is nearly constant is called P1. The aim of the present paper is to summarize what has been presented by Castro et al., explaining the constancy of the buckling load and giving a precise definition for the threshold perturbation load P1.

## 1 The single perturbation load approach

Winterstetter & Schmidt [1] classified the geometric imperfections as “realistic”, “worst” or “stimulating”. As discussed by Hühne et al, 2008 [2], single buckles are: “realistic”, because they can be verified in real test conditions; “worst case”, as explained by Deml & Wunderlich, 1997 [3], using a modified finite element model that was capable of finding the geometric imperfection that gave the minimum buckling load ; and “stimulating” since the buckling process is initiated by single buckles, as demonstrated by Esslinger, 1970 [4], using high-speed cameras.

Hühne observed that the buckling load remains nearly constant after a minimum perturbation load, called “P1”, as shown in Fig. 1. This figure shows many load-shortening curves at the left and the knock-down curve at the right, where the constancy of the buckling load can be observed. For this reason it was suggested to use as a design load the corresponding buckling load obtained when P1 is applied, called “N1”. It is important to emphasize that in the SPLA only geometric imperfections are taken into account, and other types of imperfection like load asymmetries are not considered.

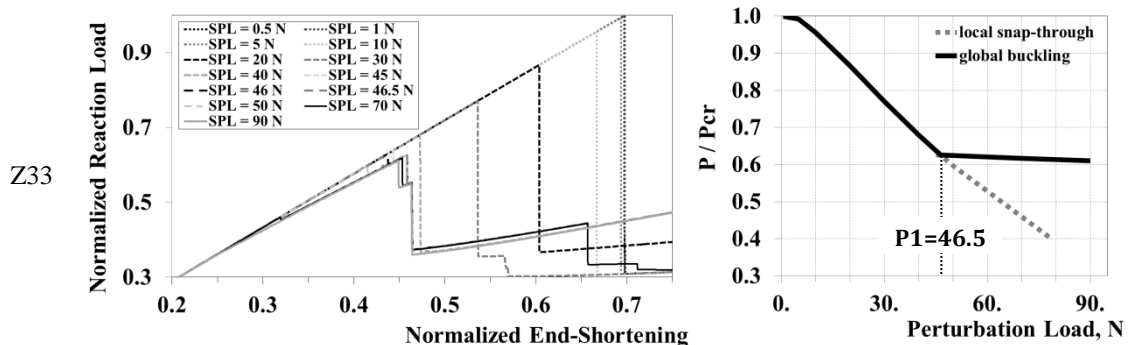


Fig. 1: Cone/cylinder coordinate system and geometric variables, modified from Castro et al. [5]

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## 2 New definition for the P1 value

Four load-shortening curves corresponding to perturbation loads close to  $P1$  will be closely evaluated in order to find a definition for  $P1$ . From Fig. 1, the curves corresponding to perturbation loads  $45\text{ N}$ ,  $46\text{ N}$ ,  $46.5\text{ N}$  and  $50\text{ N}$  were selected and are shown in Fig. 2, where the local snap-through and the global buckling are indicated, with an illustration of each one is given in Fig. 3. As schematically shown in Fig. 2, the threshold  $P1$  can be defined as the perturbation load that, when applied, will cause the local snap-through to appear at the same reaction load level as the global buckling load [5]. The threshold load indicated as  $PLST$  is the perturbation load that will induce a snap-through to appear before the global buckling, and in practice  $PLST$  is so close to  $P1$  that its determination is difficult, being acceptable to say that  $P1 \approx PLST$ .

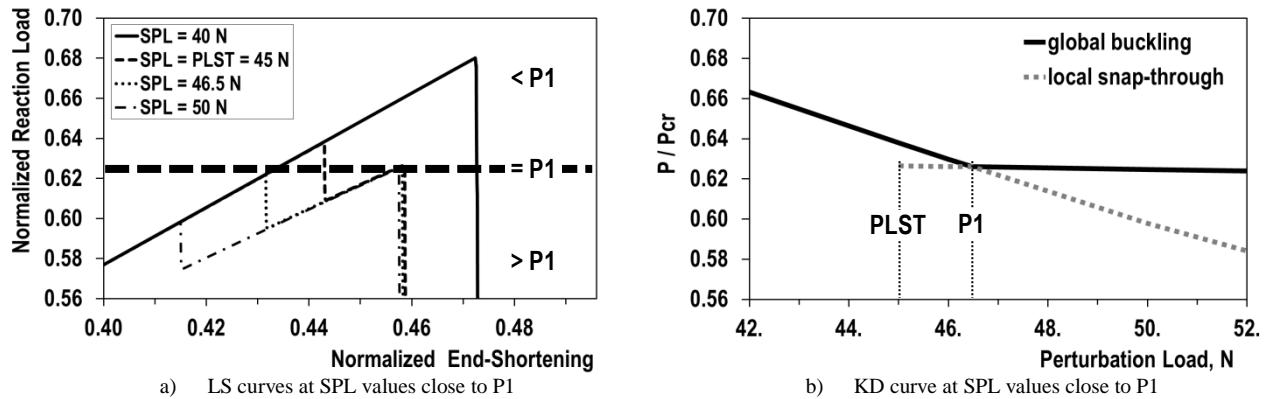


Fig. 2: Detailed evaluation of the load-shortening curves close to  $P1$ , copied from Castro et al. [5]

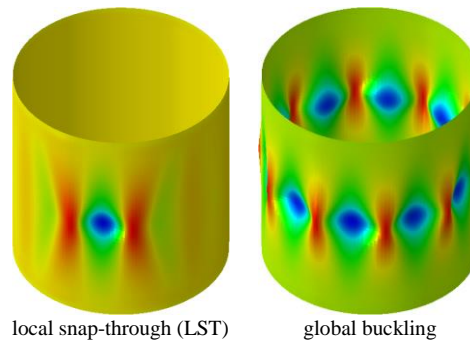


Fig. 3: Typical displacement patterns for the local snap-through and the global buckling when  $SPL > P1$ , copied from Castro et al. [5]

## 3 The constancy of the buckling load after $P1$

Based on many numerical experiments, Castro et al. [5] presented an explanation for the constancy of the buckling load after  $P1$ , customarily verified with the single perturbation load approach. In summary, when an initial perturbation load is applied to the completely unloaded structure it will create a defect that will grow with the axial compression. Keeping the axial compression constant, for small perturbation loads the defect disappears at the removal of the perturbation load, while for bigger perturbation loads ( $SPL > P1$ ) a snap-through is created and it can remain stable even with the withdrawal of the perturbation load. When such snap-through defects take place, they become the dominant geometric imperfection and further increase of the initial perturbation load will have a small influence on the size of the snap-through that is formed, which proved to be more dependent on the axial compression level [5]. Having a defect that is less dependent on the initial imperfection leads to the verified behaviour of nearly constant buckling loads after a threshold perturbation load  $P1$ .

#### 4 Final Remarks

The present study summarized the main outcomes obtained with the investigations presented by Castro et al. [5], giving an overview of the phenomena behind the single perturbation load approach.

#### 5 Acknowledgments

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