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Citation Notice

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Linear Parameter-Varying Feedforward Control: A Missile Autopilot Design

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The feedforward path of an autopilot is designed for the longitudinal dynamics of a tactical missile. A linear parameter-varying model is used to synthesize a self-scheduled control law based on a parameter-dependent Lyapunov function. The controller is evaluated on a nonlinear model of industrial complexity both under nominal conditions and parametric uncertainty. Tracking performance is significantly enhanced while leaving robustness properties of an existing feedback controller unaltered.

I. Introduction

The benefits of a two-degree-of-freedom control system, i. e., a controller consisting of a distinct feedforward and feedback path are well known.[1] With feedback controllers being able to cause instability, sufficiently large robustness margins are mandatory, which leads to the well-known trade-off between performance and robustness inherent to all single-degree-of-freedom designs. A two-degree-of-freedom configuration relieves this conflict and separates tracking requirements from other objectives such as disturbance attenuation and robustness.

From a designer's point of view, a two-step design with two independent controllers has several appealing aspects compared to a combined design: A feedback controller can be designed with respect to robustness and disturbance attenuation and a feedforward controller can be designed to achieve fast tracking. An important implication is, that with such a design strategy, it is possible to add a feedforward controller to an already existing closed-loop system without the need to redesign the whole control system.

The prevalent approach to design the feedforward path is to perform a nonlinear dynamic inversion of the plant. This technique is however limited to minimum-phase systems and thus excludes a large class of challenging control applications. Furthermore, it is difficult to effectively limit the feedforward control authority in the design. Recently, a design procedure has been proposed that recasts the feedforward control problem for linear systems as a state feedback problem.[2,3] Consequently, it is applicable also to non-minimum phase systems and allows to incorporate loop shaping strategies into the design.

The purpose of this paper is to present a successful autopilot design for a nonlinear missile model of industrial complexity, using this methodology and the linear parameter-varying (LPV) systems framework.

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The LPV formulation has received a lot of attention in the last two decades for its ability to produce self-scheduled controllers. LPV gain scheduling is thus an area of interest for academia and industry alike, as it allows well-developed linear design tools to be applied to a number of nonlinear systems.

LPV control has already proved useful for missile autopilot applications and several results are published which consider feedback controllers.[4–8] The only published LPV feedforward control application examples up to this date are also considered with a missile autopilot but use a highly simplified two-state model.[2,3] The present work's major contribution is thus proving applicability to an industrial high-fidelity model where highly nonlinear aerodynamic data necessitates the use of a gridding approximation as introduced in Ref. 9 in contrast to the polytopic and linear fractional representations used in Refs. 2, 3.

The paper is structured as follows. In Section II, a brief introduction to the class of LPV systems and the LPV state feedback problem is presented. Section III relates the mixed sensitivity concept to feedforward control. Section IV introduces the LPV model of the missile. Details of the design procedure are given in Section V and finally Section VI discusses the simulation results.

II. Preliminaries

A. Induced $\mathcal{L}_2$-Norm Control of LPV Systems

A time-varying scheduling parameter vector is defined as a function $\rho\colon \mathbb{R} \rightarrow \mathcal{P}$ with a compact set $\mathcal{P} \subset \mathbb{R}^{n_\rho}$, where n_ρ denotes the number of scheduling signals. The variation rates of the parameters $\dot{\rho}\colon \mathbb{R} \rightarrow \dot{\mathcal{P}}$ are defined to lie in the polyhedron $\dot{\mathcal{P}} = \{q \in \mathbb{R}^{n_\rho} \mid |q_i| \leq \nu_i \ , \quad i = 1, \dots, n_\rho\}$. A state space representation of an LPV dynamical system in terms of the continuous matrix functions $A\colon \mathcal{P} \rightarrow \mathbb{R}^{n_x \times n_x}$, $B\colon \mathcal{P} \rightarrow \mathbb{R}^{n_x \times n_u}$, $C\colon \mathcal{P} \rightarrow \mathbb{R}^{n_y \times n_x}$, $D\colon \mathcal{P} \rightarrow \mathbb{R}^{n_y \times n_u}$ and the state $x\colon \mathbb{R} \rightarrow \mathbb{R}^{n_x}$, input $u\colon \mathbb{R} \rightarrow \mathbb{R}^{n_u}$ and output $y\colon \mathbb{R} \rightarrow \mathbb{R}^{n_y}$ is then given by

$$\begin{aligned} \dot{x}(t) &= A(\rho(t))\ x(t) + B(\rho(t))\ u(t) \\ y(t) &= C(\rho(t))\ x(t) + D(\rho(t))\ u(t)\ . \end{aligned} \tag{1}$$

Since the parameter trajectories are not known a priori, the parameter space $\mathcal{P} \times \dot{\mathcal{P}}$ represented by the auxiliary variables $p \in \mathcal{P}$ and $q \in \dot{\mathcal{P}}$ is considered. From here on, explicit dependence on time and parameters is dropped for the sake of readability.

It is common practice in LPV control to specify performance in terms of the induced $\mathcal{L}_2$-norm of a generalized plant.[10] A generalized LPV plant P is said to have an induced $\mathcal{L}_2$-norm $\|P\|$ if there exists a constant $\gamma > 0$ such that for all $z = P\,w$, $w \in \mathcal{L}_2$ the following relation holds:

$$\|z\|_2 \leq \gamma \|w\|_2\ , \quad \|z\|_2 = \sqrt{\int_0^\infty z^T(\tau)\ z(\tau)\ \mathrm{d}\tau}\ . \tag{2}$$

If all states are available for feedback, an induced $\mathcal{L}_2$-norm and stability of the closed-loop system can be achieved with

Theorem 1 (Ref. 9) *Consider a compact set $\mathcal{P} \times \dot{\mathcal{P}}$, a performance specification $\gamma > 0$ and a generalized LPV plant of the form*

$$\begin{bmatrix} \dot{x} \\ \hline z_1 \\ z_2 \end{bmatrix} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_{11} & 0 & 0 \\ C_{12} & 0 & I \end{array}\right] \begin{bmatrix} x \\ \hline w \\ u \end{bmatrix}. \tag{3}$$

Let $R\colon \mathcal{P} \rightarrow \mathbb{R}^{n_x \times n_x}$ be a continuously differentiable symmetric matrix function such that $\forall (p, q) \in \mathcal{P} \times \dot{\mathcal{P}}$

$$R \succ 0\ , \tag{4a}$$

$$\begin{bmatrix} R\ (A - B_2\, C_{12})^T + (A - B_2\, C_{12})\ R - B_2\, B_2^T - \sum_{i=1}^{n_\rho} \frac{\partial R}{\partial p_i}\, q_i & R\, C_{11}^T & \gamma^{-1}\, B_1 \\ C_{11}\, R & -I & 0 \\ \gamma^{-1}\, B_1^T & 0 & -I \end{bmatrix} \prec 0\ , \tag{4b}$$

Then, the state feedback control law

$$u = -\left(B_2^T\, R^{-1} + C_{12}\right)\, x \tag{5}$$

renders the closed-loop system asymptotically stable with an induced $\mathcal{L}_2$-norm less than γ.

Since the linear matrix inequality (LMI) constraints (4) are infinite dimensional due to the functional dependence on the scheduling parameter p, the practical approach proposed in Ref. 9 is to grid the parameter space and to enforce the constraints only on a finite dimensional set $\{p_k\}_1^{n_{\text{grid}}} \in \mathcal{P}$. Further, the arbitrary dependence of the Lyapunov matrix R on the parameters needs to be restricted to arrive at a tractable formulation. Choosing basis functions g_i for

$$R(p) = \sum_{i=1}^{n_R} R_i \, g_i(p) \tag{6}$$

leads to the semidefinite program

$$\min_{\gamma, \{R_i\}_1^{n_R}} \gamma \text{ such that} \qquad \text{LMI (4) holds } \forall (p,q) \in \{p_k\} \times \text{vert}\left(\dot{\mathcal{P}}\right), \tag{7}$$

where $\text{vert}(\dot{\mathcal{P}})$ denotes the vertices of the polyhedron $\dot{\mathcal{P}}$. Problem (7) is readily solved with convex optimization software and a parameter-dependent controller is explicitly given by Eq. (5).

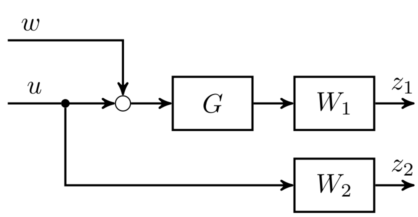
B. Mixed Sensitivity Design

In order to formulate design specifications in terms of the induced $\mathcal{L}_2$-norm, a generalized plant formulation and mixed sensitivity minimization 11 are used. Mixed sensitivity design has proved a useful design paradigm, allowing to enforce certain specifications such as integral action and roll-off on the controller.

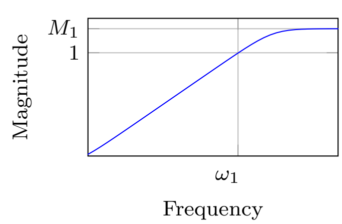
The generalized plant depicted in figure 1(a) is used and a controller is synthesized to minimize the induced $\mathcal{L}_2$-norm from w to z. The weighting scheme exploits the roll-off of the plant to ensure that there is no feedthrough from w to z_1 in the generalized plant, i. e., the sensitivity is weighted with the plant model G and W_1. As a byproduct, the inherent limitations of the plant due to non-minimum phase behavior are included in the design: if w is interpreted as a reference signal, the closed-loop system is required only to follow a reference filtered by the system dynamics. The integral filter W_1 is used to set the closed-loop bandwidth as well as to guarantee steady-state accuracy. The high-pass filter W_2 limits control action. They are depicted in figure 1 and parameterized as

$$W_1(s) = \frac{\frac{s}{M_1} + \omega_1}{s}, \quad W_2(s) = \frac{s + \frac{\omega_2}{M_2}}{\epsilon\, s + \omega_2}. \tag{8}$$

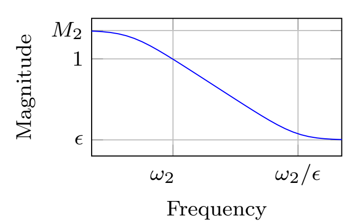
Remark: For LPV systems, the notion of a transfer function does not exist, still a "frozen parameter" interpretation is useful, especially in conjunction with the gridding approach that treats the LPV system as a collection of LTI systems.



(a) Generalized plant interconnection



(b) Frequency response of W_1^{-1}



(c) Frequency response of W_2^{-1}

Figure 1. Generalized plant and frequency response of the weighting filters for mixed sensitivity design.

III. Feedforward Control Design

The interconnection of a feedforward controller with a closed-loop system is shown in figure 2. The dashed box represents the feedforward controller that takes the reference signal r and provides a control signal $u_r = F\, r$ and a reference output $y_r = G\, F\, r$.[12] Note that in contrast to Ref. 12, different variables are used to distinguish between the plant model G and the actual plant P. The feedback controller is denoted K and for further analysis, an output disturbance d and measurement noise n are injected into the loop at the plant output and the controller input, respectively. The total control signal u is formed as the sum of the feedforward control signal u_r and the feedback control signal $\tilde{u}$.

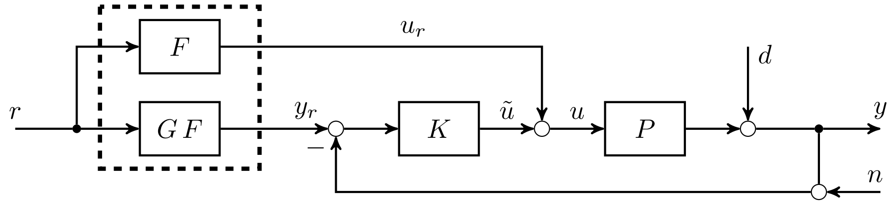


Figure 2. Feedforward controller interconnection with closed-loop system.

A. Separation of Feedforward and Feedback Path

The closed-loop transfer function corresponding to figure 2 is

$$y = \underbrace{G\,F\,r}_{\text{reference tracking}} + \overbrace{S\;(P-G)\;F\,r}^{\text{model mismatch}} + \underbrace{S\,d}_{\text{disturbances}} - \overbrace{T\,n}^{\text{measurement noise}}\;, \tag{9}$$

where $S = (I + P\,K)^{-1}$ and $T = I - S$ are the sensitivity and complementary sensitivity functions of the closed-loop system that do not depend on the feedforward controller. The only interaction of feedforward and feedback controller is due to model mismatch, which is filtered through the sensitivity function S. An important consequence is that if $S(0) = 0$, i. e., the feedback controller includes integral action, there is no steady-state error regardless of model mismatch.

Considering only nominal performance, i. e., $G = P$ and the absence of disturbances and noise, Eq. (9) simply becomes $y = G\,F\,r$ or equivalently in terms of the error

$$r - y = (I - G\,F)\;r\;. \tag{10}$$

The transfer function $I - G\,F$ in Eq. (10) is precisely the feedforward sensitivity introduced in Ref. 11 to describe the effect of feedforward control on a closed-loop system. Since the feedforward sensitivity governs the nominal error dynamics, it is desirable to shape it similar to the classical sensitivity function. In a mixed sensitivity formulation, this leads to the problem

$$\min_F \left\| \begin{bmatrix} W_1\;(I - G\,F) \\ W_2\,F \end{bmatrix} \right\|, \tag{11}$$

where the filters W_1 and W_2 are those given in Eq. (8).

B. Synthesis as a State Feedback Problem

There are several ways to motivate the synthesis of the feedforward controller as a state feedback problem. A formal proof based on Youla parameterization is given in Ref. 2. An alternative approach is presented here, based on the substitution[12]

$$F = K_r\;(I + G\,K_r)^{-1}\;. \tag{12}$$

With this, the feedforward sensitivity can be written as

$$(I - G\,F) = (I + G\,K_r)^{-1}\;, \tag{13}$$

which is the sensitivity function for a closed-loop system composed of the plant model G and the feedback controller K_r. Consequently, expression (11) becomes a standard S/KS minimization problem for the fictitious closed-loop system. However, there are two important differences: First, the problem can be solved irrespectively of any robustness requirements since only the model G is to be controlled, which is known and neither subject to disturbances nor noise. Second, all states of G (and also of the filters W_1 and W_2) are available, even if not all states are measurable on the real plant. Therefore, K_r can always be designed as an (augmented) state feedback controller and synthesized by means of program (7). Note that since K_r internally stabilizes G, both F and $G\,F$ are stable, even if G is not.

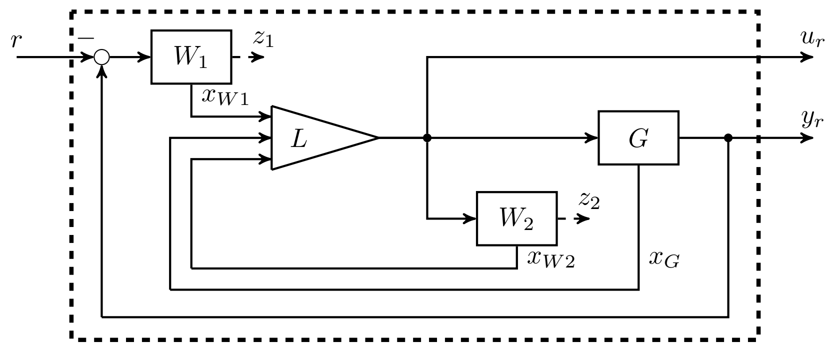


Figure 3. Feedforward controller realization.

With the generalized plant (3) and Wu's feedback law (5) written as $u = L\,x$, a state space realization of F is given by

$$\left[\begin{array}{c} \dot{x}_F \\ \hline u_r \end{array}\right] = \left[\begin{array}{c|c} A + B_2\,L & B_1 \\ \hline L & 0 \end{array}\right] \left[\begin{array}{c} x_F \\ \hline r \end{array}\right] \quad \text{with } L = -B_2^T\,R^{-1} - C_{12}\,. \tag{14}$$

A generalized plant corresponding to expression (11) with F realized as the state space model (14) is then

$$\left[\begin{array}{c} \dot{x} \\ \dot{x}_F \\ \hline z_1 \\ z_2 \end{array}\right] = \left[\begin{array}{cc|c} A & B_2\,L & B_1 \\ 0 & A + B_2\,L & B_1 \\ \hline C_{11} & 0 & 0 \\ C_{12} & D_{W2}\,L & 0 \end{array}\right] \left[\begin{array}{c} x \\ x_F \\ \hline r \end{array}\right], \tag{15}$$

where D_{W2} is the feedthrough matrix of the filter W_2. From this representation, it is now possible to proof the sufficiency of the solution as a state feedback problem by applying the state transformation

$$\begin{bmatrix} \xi \\ x_F \end{bmatrix} = \begin{bmatrix} I & -I \\ 0 & I \end{bmatrix} \begin{bmatrix} x \\ x_F \end{bmatrix}. \tag{16}$$

This decouples the system into

$$\left[\begin{array}{c} \dot{\xi} \\ \dot{x}_F \\ \hline z_1 \\ z_2 \end{array}\right] = \left[\begin{array}{cc|c} A & 0 & 0 \\ 0 & A + B_2\,L & B_1 \\ \hline C_{11} & C_{11} & 0 \\ C_{12} & C_{12} + D_{W2}\,L & 0 \end{array}\right] \left[\begin{array}{c} \xi \\ x_F \\ \hline r \end{array}\right] \tag{17}$$

and shows that the states ξ are not controllable from the reference input r. They can thus be disregarded for input-output behavior. Since the feedforward controller is completely parameterized by the state feedback gain L and the model matrices, the minimization problem (11) can be solved as

$$\min_L \left\| \left[\begin{array}{c|c} A + B_2\,L & B_1 \\ \hline C_{11} & 0 \\ C_{12} + D_{W2}\,L & 0 \end{array}\right] \right\|. \tag{18}$$

For implementation of the feedforward controller, the structure shown in figure 3 is used. Complexity of the feedforward controller is moderate. It has the same order as the generalized plant, i. e., the number of states of the plant and the weighting filters. Apart from the online evaluation of the LPV plant model G, only one inversion of a positive definite matrix, that is efficiently performed using LDL^T factorization, is necessary.

IV. LPV Model of the Missile

A generic missile model is illustrated in figure 4. The longitudinal motion short period dynamics LPV model used here for synthesis is derived in Ref. 13 using a function substitution technique on the nonlinear equations of motion. It is given as

$$\left[\begin{array}{c} \dot{w} \\ \dot{q} \\ \hline a_z \\ q \end{array}\right] = \left[\begin{array}{cc|c} K_1\,C_{Z0} & V\,\cos(\alpha)\,\cos(\beta) & K_1\,V\,\frac{C_{Z\delta}}{C_{M\delta}} \\ \frac{K_2}{I_{yy}}\,C_{M0} & \frac{K_3}{I_{yy}}\,C_{Mq} & \frac{K_2\,V}{I_{yy}} \\ \hline K_1\,C_{Z0} & 0 & K_1\,V\,\frac{C_{Z\delta}}{C_{M\delta}} \\ 0 & 1 & 0 \end{array}\right] \left[\begin{array}{c} w \\ q \\ \hline C_{M\delta} \end{array}\right] . \tag{19}$$

The two states are the pitch rate q and the velocity w in z-direction. The output to be controlled is the plunge acceleration, denoted a_z.

The parameter vector $p = [Ma \quad \alpha \quad \beta]^T$ is used as a scheduling signal, where Ma denotes the Mach number, α the angle of attack and β the sideslip angle. The terms K_1, K_2 and K_3 depend on Ma and geometric data, I_{yy} is a scalar moment of inertia and $V(p)$ is the true airspeed. Further, $C_{i0}(p)$ are dimensionless coefficients, $C_{Mq}(p)$ is the pitch-rate pitching-moment effectiveness and $C_{i\,\delta}(p, \delta)$ are coefficients depending also on the deflection of the control surface. All these coefficients are highly nonlinear functions of the scheduling parameters and available only as lookup tables. Further, a third order linear time invariant actuator model is included. The overall system is thus strictly proper and of fifth order.

The missile is in a cruciform configuration and controlled by deflecting the four fins at the tail. Doing so does not only change the aerodynamic moments but also produces aerodynamic forces acting in the opposite of the desired direction. Therefore, the system exhibits non-minimum phase characteristics.

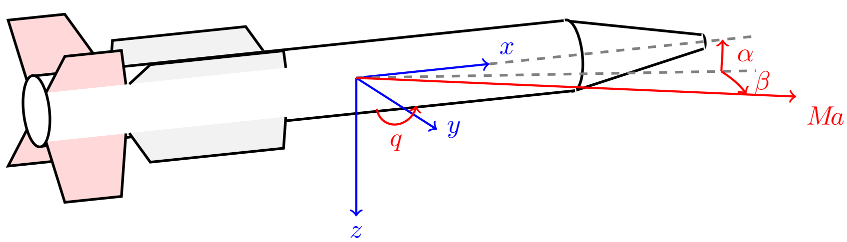


Figure 4. Model of a generic tactical missile.

V. Design

A. Technicalities

The admissible parameter space is gridded as $\{p_k\} = Ma_{\text{grid}} \times \alpha_{\text{grid}} \times \beta_{\text{grid}}$ with $Ma_{\text{grid}} = \{1.2, 1.5, 1.8, 2.1, 2.4\}$, $\alpha_{\text{grid}} = \{0.1°, 1°, 4°, 7°, \ldots, 19°, 22°\}$ and $\beta_{\text{grid}} = \{0.1°, 10°\}$. The grid covers a wide range of the flight envelope and consists of $n_{\text{grid}} = 90$ points. The controller is designed only for positive values of α and β. For implementation, symmetry with respect to α and β is exploited. As β has a relatively small influence on the system dynamics, two grid points are considered sufficient to represent the minimum and maximum occurring values. The parameter rate bounds are selected as $\dot{Ma} \in [-0.5, 0.5]\ \frac{1}{\text{s}}$, $\dot{\alpha} \in [-100°, 100°]\ \frac{1}{\text{s}}$, and $\dot{\beta} \in [-100°, 100°]\ \frac{1}{\text{s}}$ based on simulation results.

Inevitably, some trial and error is required to select the basis functions for the Lyapunov matrix R. The use of a relative performance index, where γ is locally normalized at each grid point with respect to the achievable $\mathcal{H}_\infty$ norm, however, considerably simplifies the procedure since it provides a lower bound on the achievable performance.[13] Starting with a few basis functions and consecutively adding terms until the relative improvement in the performance index is small thus appears to be a good strategy. For the present case, it leads to

$$R(p) = R_0 + R_{Ma}\,Ma + R_\alpha\,\alpha + R_\beta\,\beta + R_{Ma,\alpha}\,Ma\,\alpha + R_{Ma^2}\,Ma^2 + R_{\alpha^2}\,\alpha^2 . \tag{20}$$

B. Tuning the Feedforward Controller

Tuning within the mixed sensitivity framework translates to selecting appropriate weighting filters. The choice of these filters such that time-domain criteria are met, is a highly nontrivial task despite the interpretability of each parameter. For this reason, the multi-objective optimization environment MOPS[14] is employed to set the weighting filters. Tuning therefore consists of the following four steps:

1. parameterization of the weighting filters
2. formulation of optimization criteria in terms of time domain performance specifications
3. execution of the optimization routine
4. evaluation in nonlinear simulation

Step 1 determines the number of optimization variables and hence the computational complexity of the task. Since the missile has strongly varying dynamics over different operating points, the coefficients M and ω in Eq. (8) are chosen to depend affinely on the scheduling parameters, leading to 16 tuner variables.

Step 2 is the essential part of the tuning procedure. Since Eq. (9) shows that nominal performance of the overall system can be inferred from the response $r \to y_r$, the LPV feedforward controller's reference output is evaluated at all grid points, i. e., for a collection of LTI systems. As robustness is not required and the control action is already limited by the mixed sensitivity design, it is deemed sufficient to use only the speed-of-response specifications and penalty terms for undershoot and overshoot as objectives. The cost function to be minimized is formed as a sum-of-squares

$$J = \sum_{k=1}^{n_{\text{grid}}} \sum_{i=1}^{n_{\text{obj}}} \left(\frac{O_{i,k}}{D_{i,k}} \right)^2 , \tag{21}$$

where $O_{i,k}$ denotes the value of the ith objective at grid point k and $D_{i,k}$ denotes the corresponding demand.

Consisting of time domain criteria, the objective function is highly nonlinear, nonconvex and not available in analytical form. Therefore, in Step 3, direct search methods are preferred over gradient-based ones and a pattern search algorithm is employed. Good initial values for the tuners are easily obtained from engineering insight. Consequently, the use of local optimization strategies is justified. The controller synthesis problem (7) is solved using a Matlab toolbox developed at the DLR Institute of System Dynamics and Control by the second and fourth author. Since the Lyapunov matrix R needs to be inverted in Eq. (5) to reconstruct the controller, a suboptimal synthesis algorithm that indirectly minimizes the condition number of R is used. It minimizes an upper bound on the largest and maximizes a lower bound on the smallest eigenvalue of R.[15] This avoids numerical problems often associated with LMI design.

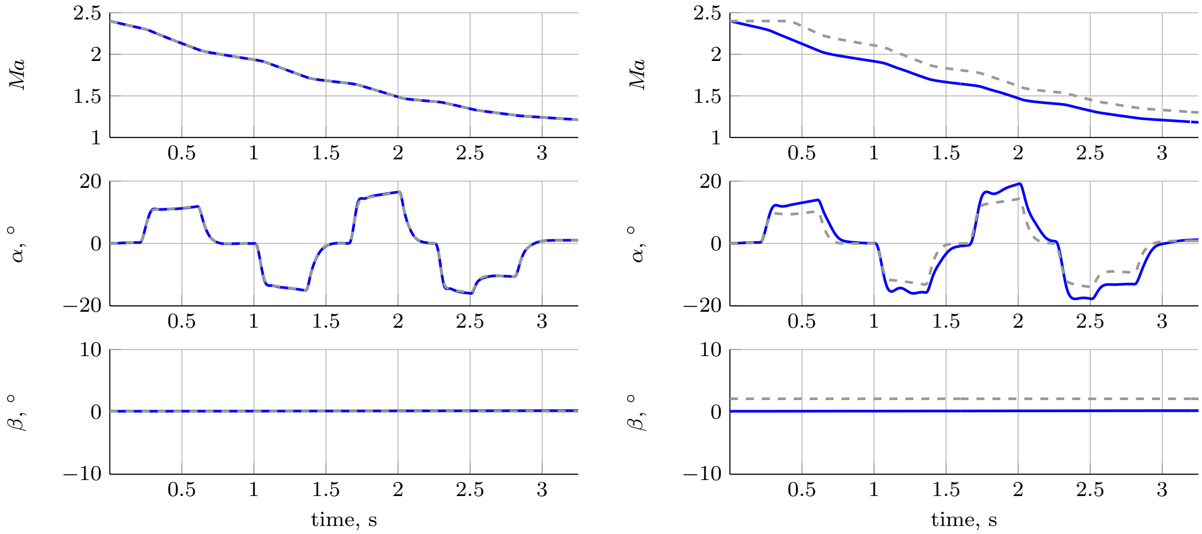
In Step 4, the controller synthesized with the optimal parameters is tested in nonlinear simulation. In the present work, the linear system responses at the grid points appear to cover the nonlinear system behavior very well. The largest discrepancy in nonlinear simulation is observed for the overshoot criterion. In order to satisfy a 5% demand in nonlinear simulation, Step 2–4 had to be repeated with the demand for the linear responses adjusted to 1% in the cost function.

VI. Assessment

A. Simulation Results

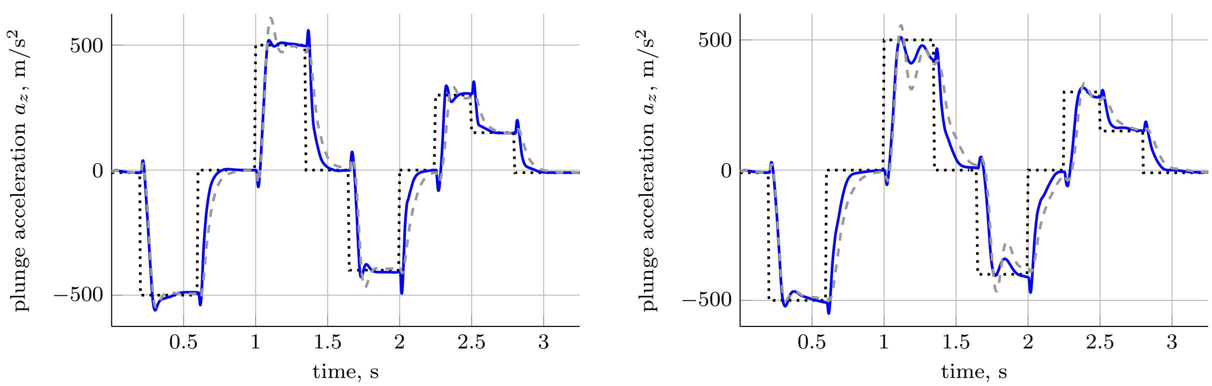
Simulation is performed on a full nonlinear 6-degree-of-freedom model, incorporating both fifth order sensor dynamics and third order actuator dynamics. Tracking performance is compared between an ad hoc gain-scheduled PI feedback-only controller and a configuration with additional feedforward controller. The considered scenario is representative of the homing phase, i. e., there is no thrust and the Mach number steadily decreases (see figure 5).

For feedback, the vertical acceleration a_z and pitch rate q are used with scheduling via Ma, α and β. The feedforward controller receives only the Mach number Ma as an exogenous scheduling signal and further uses the internal model states for self-scheduling via estimates of α and β. Figure 5 confirms that the parameter trajectories during simulation cover a large area of the flight envelope. The predictions used to schedule the feedforward controller can be seen to precisely resemble the true flight conditions of the nominal case. In



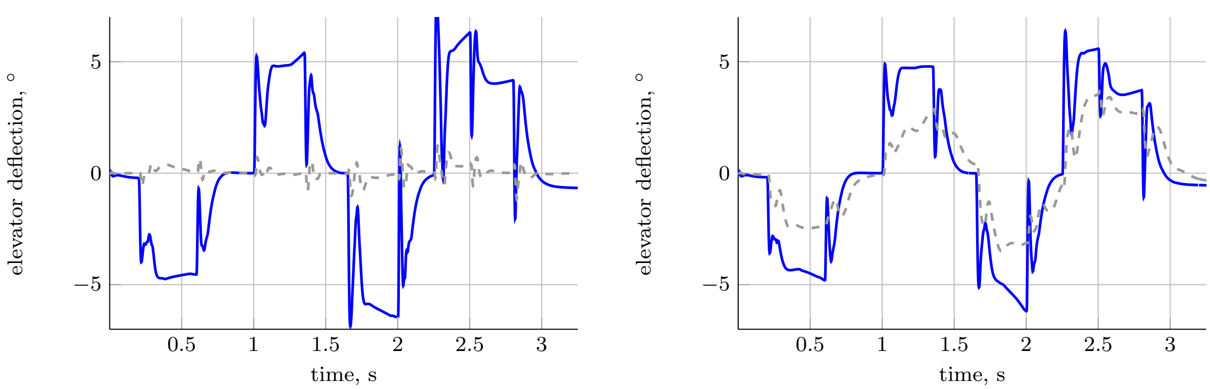
(a) Nominal (b) Uncertain

Figure 5. True (——) and predicted (- - -) scheduling parameter trajectories.



(a) Nominal (b) Uncertain

Figure 6. Reference (······), response with (——) and without (- - -) additional feedforward controller.



(a) Nominal (b) Uncertain

Figure 7. Control inputs from feedforward controller (——) and feedback controller (- - -) when used together.

addition, simulation results are also shown for parametric uncertainty in the aerodynamic coefficients C_{Z0} and C_{M0}, the scheduling parameters Ma, α, β, and the position of the center of gravity.

Figure 6 shows the reference trajectory for the plunge acceleration a_z and the system responses with and without additional feedforward control. The feedforward controller consistently improves the speed of all responses: Settling time is reduced by up to 40%. Noticeably, the largest improvement is achieved for changes from high to low accelerations. Overshoot is reduced in two cases and remains of comparable magnitude to the feedback controller for the others. Undershoot inevitably increases with the faster response, but remains inside the allowed range. The increase in performance under parametric uncertainty is very similar to the nominal case.

Figure 7 further shows that for the nominal case, the system is indeed driven almost entirely through the feedforward control signal and thus confirms the separation of feedforward and feedback path. The latter only accounts for the mismatch between the LPV model and the fully nonlinear model, which is largest at high frequencies and thus excited most at the very instant of the steps. With uncertainty in the parameters, the feedback controller is much more involved, reflecting the need to compensate for the model mismatch.

B. Discussion

With the additional LPV feedforward controller, performance over the whole parameter space is considerably improved. While for some operating conditions, the improvement is very significant, for others, it is just marginal. At these points, the existing feedback controller is expected to already operate near the physical limits of the system. Its bandwidth limitation is thus due to the non-minimum phase characteristics rather than robustness requirements. As the feedforward controller is also subject to these inherent limitations, performance cannot be further increased. It is notable that the overall benefit in performance is also preserved for the system with parametric uncertainty. From the separation of control paths, it can further be expected that an additional increase in robustness may be achieved by retuning the feedback controller without impairing tracking performance.

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