Electromagnetic Scattering from Impedance-Matched Bodies

Andrey V. Osipov
Microwaves and Radar Institute, German Aerospace Center (DLR), Oberpfaffenhofen, 82234 Wessling, Germany
Andrey.Osipov@dlr.de

***Abstract*—Electromagnetic scattering from impedance bodies with the surface impedance equal to the wave impedance of the surrounding medium shows a number of extremal features, including vanishing axial backscattering cross section in the case of arbitrarily sized bodies of revolution. This paper examines various scattering cross sections (mono- and bistatic, total scattering, absorption and extinction) of electrically large arbitrarily shaped impedance-matched bodies, and it is shown that the backscattering cross section of such bodies vanishes in the optical limit. Furthermore, it is conjectured that considered as functions of the surface impedance, the mean total scattering cross section has a global minimum and the mean absorption cross section a global maximum when the body is illuminated by plane waves with random incidence direction and polarization.**

***Index Terms*—electromagnetic scattering by absorbing media, electromagnetic theory, microwave absorbers, radar scattering, remote sensing, scattering cross sections, surface impedance.**

I. Introduction

The impedance (Leontovich) boundary condition relates the total electric and magnetic fields $\mathbf{E}$ and $\mathbf{H}$ on a scattering surface,

$$\hat{n}\times\mathbf{E} = Z_s \hat{n}\times\hat{n}\times\mathbf{H} \tag{1}$$

where Z_s is the surface impedance and $\hat{n}$ is the outward unit vector normal to the surface, e.g. [1]. The boundary condition applies to a wide variety of material interfaces, including metamaterial absorbers, thus enabling numerical simulations of electromagnetic scattering from electrically large bodies coated with modern absorbing materials [2-5].

If Z_s is equal to the wave impedance Z_0 of the surrounding medium and the scatterer is invariant under a 90° rotation about some axis, then the backscattering in the axial direction is exactly zero [6]. A subtle feature of scattering from impedance-matched bodies is, however, that for a generally shaped body, general incidence and scattering directions and a general polarization of the incident wave the level of the scattered field is not necessarily lower than for a body of the same shape with a mismatched value of Z_s. In the case of an impedance plane it is the reflectance averaged over all polarization and incidence directions that has a global minimum when $Z_s = Z_0$ [7,8].

The recent progress in the design of thin metamaterial flat absorbers and metasurfaces with engineered values of the surface impedance, e.g. [9-11], has motivated this work. The paper describes the dependence of electromagnetic fields scattered by electrically large arbitrarily shaped impedance bodies on the surface impedance and presents the following results which, to the best of author's knowledge, are new.

II. Main Results

Let k be the wavenumber, $\eta = Z_s / Z_0$ the relative surface impedance of the scattering boundary (a complex-valued parameter with $\mathrm{Re}\,\eta > 0$ for every passive boundary; $\eta = 1$ for an impedance-matched boundary), χ an angle defining the orientation of the polarization plane in an incident linearly polarized plane wave, σ the bistatic scattering cross section (monostatic when the incidence and scattering directions coincide), σ_{T} the total scattering cross section, σ_{abs} the absorption cross section and σ_{ext} the extinction cross section. The main results of this work can be summarized as follows.

A. Vanishing Backscattering Cross Section as $k \to \infty$

When the shape of the impedance-matched body does not satisfy the symmetry required by Weston's theorem or the incidence direction deviates from the symmetry axis, the backscatter does not vanish. It can be shown, however, that the backscattering cross section vanishes in the limit $k \to \infty$, provided that the scatterer is convex. The proof is based on the stationary point estimation of the scattered field. Figure 1 illustrates this property by showing the backscattering width of a circular cylinder illuminated at a right angle to its axis.

B. Invariance of Polarization-Averaged Cross Sections when $\eta \to \eta^{-1}$, and Their Local Extrema at $\eta = 1$

If polarization of the incident wave is random with χ uniformly distributed over the interval $0 \le \chi < 2\pi$, then the mathematical expectation of a scattering cross section can be determined by the averaging operation

$$\langle \dots \rangle_\chi = \frac{1}{2\pi}\int_0^{2\pi} \dots \,\mathrm{d}\chi. \tag{2}$$

Using a symmetry of Maxwell's equations and boundary conditions (1) under the transformation: $\mathbf{E} \to \mathbf{H}$, $\mathbf{H} \to -\mathbf{E}$, $Z_0 \to 1/Z_0$ and $Z_s \to 1/Z_s$ [12], it can be shown that the polarization-averaged (mean) cross sections $\langle \sigma \rangle_\chi$, $\langle \sigma_{\mathrm{T}} \rangle_\chi$, $\langle \sigma_{\mathrm{abs}} \rangle_\chi$ and $\langle \sigma_{\mathrm{ext}} \rangle_\chi$ are invariant under the transformation $\eta \to 1/\eta$.

A consequence of this invariance is an extremum of the averaged cross sections at $\eta = 1$. Indeed, every function compliant with the symmetry can be represented as a function of the argument $t = \eta + \eta^{-1}$, implying that the derivative

$$\frac{\partial}{\partial \eta} f(t) = f'(t)\left(1 - \frac{1}{\eta^2}\right) \qquad (3)$$

vanishes at $\eta = 1$.

A closer look at the behavior of the cross sections in a vicinity of the point $\eta = 1$ on the complex η-plane reveals, however, that the type of the extremum (minimum, maximum, saddle point) depends on the incidence direction and, in the case of $\langle \sigma \rangle_\chi$, on the scattering direction. The situation is similar to the behavior of the reflectance of planar impedance boundaries [8].

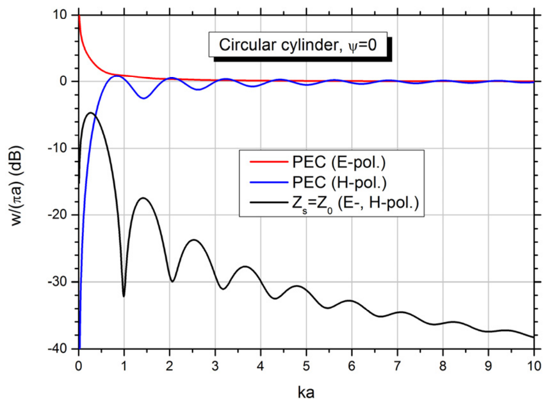

Fig. 1. Backscattering width of a circular cylinder as a function of ka where a is the radius of the cylinder; the width is normalized to its geometrical optics limit.

C. *Global Minimum of $\langle \sigma_{\mathrm{T}} \rangle$ and Maximum of $\langle \sigma_{\mathrm{abs}} \rangle$ at $\eta = 1$*

Referring to the case of a planar boundary, it is reasonable to expect that averaging $\langle \sigma_{\mathrm{T}} \rangle_\chi$ and $\langle \sigma_{\mathrm{abs}} \rangle_\chi$ over all possible incidence directions should lead to functions, denoted by $\langle \sigma_{\mathrm{T}} \rangle$ and $\langle \sigma_{\mathrm{abs}} \rangle$, with, respectively, a minimum and a maximum at $\eta = 1$ (a minimum of $\langle \sigma_{\mathrm{T}} \rangle$ means a maximum of $\langle \sigma_{\mathrm{abs}} \rangle$ since for electrically large scatterers, $\sigma_{\mathrm{abs}} + \sigma_{\mathrm{T}} = \sigma_{\mathrm{ext}} \approx 2S$ where S is the area of the geometric cross section of the shadow region behind the scatterer [13]).

Furthermore, physical reasoning suggests that the minimum of $\langle \sigma_{\mathrm{T}} \rangle$ and the maximum of $\langle \sigma_{\mathrm{abs}} \rangle$ must be global on the complex half-plane $\mathrm{Re}\,\eta > 0$. The conjecture is supported by the curves in Figs. 2 and 3, plotted by using the exact analytical solution for an impedance sphere.

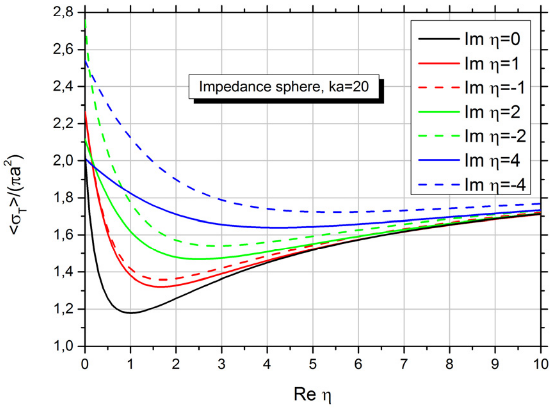

Fig. 2. Normalized mean total scattering cross section of an impedance sphere with $ka = 20$ as a function of $\mathrm{Re}\,\eta$ for several values of $\mathrm{Im}\,\eta$; a is the radius of the sphere.

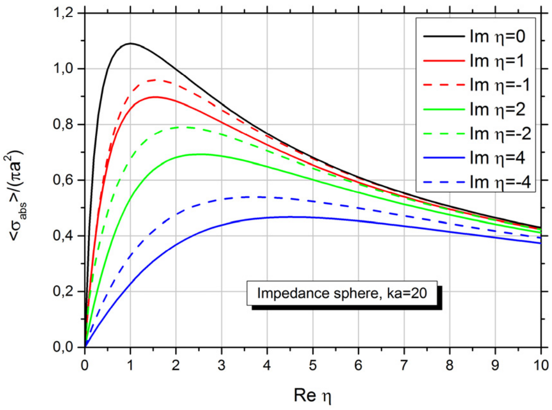

Fig. 3. The same as in Fig. 2 but for the mean absorption cross section.

Since averaging corresponds to the evaluation of a mathematical expectation, the obtained results can be understood as extremal properties of the mean cross sections of impedance bodies illuminated by plane waves with random polarization and incidence directions. The latter scenario is typical for moving radar targets.

III. Concluding Remarks

For cylindrical bodies, similar results can be formulated in terms of scattering widths, which are the cross sections per unit length of the scatterer. For example, it is the total scattering width $\langle w_{\mathrm{T}} \rangle$ and the absorption width $\langle w_{\mathrm{abs}} \rangle$, averaged over polarization and incidence directions, that are at minimum and maximum at $\eta = 1$, respectively.

A rigorous proof of the property C is unavailable to the author at the moment of writing this paper. So far this physically plausible conjecture has been checked numerically by using exact analytical solutions for impedance spheres and circular cylinders. Numerical and experimental tests of more general geometries are desirable but difficult because of the need to collect scattering data for electrically large bodies over the whole range of incidence and scattering directions and polarization cases.

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