SVD-based RF interference detection and mitigation for GNSS

Matteo Sgammini, Felix Antreich and Michael Meurer,
Institute of Communications and Navigation, German Aerospace Center (DLR)

BIOGRAPHY

Matteo Sgammini received the MEng degree in electrical engineering in 2005 from the University of Perugia. He joined the Institute of Communications and Navigation of the German Aerospace Center (DLR), in 2008. His field of research is in interference detection and mitigation in navigation systems.

Felix Antreich received the diploma and the Ph.D. degree in Electrical Engineering and Information Technology from the Munich University of Technology (TUM), Munich, Germany, in 2003 and 2011, respectively. Since July 2003, he has been with the Institute of Communications and Navigation of DLR. His research interests include sensor array signal processing for global navigation satellite systems (GNSS) and wireless communications, estimation theory and signal design for synchronization, and GNSS.

Michael Meurer received the diploma in electrical engineering and the Ph.D. degree from the University of Kaiserslautern, Germany. After graduation, he joined the Research Group for Radio Communications at the Technical University of Kaiserslautern, Germany, as a senior key researcher, where he was involved in various international and national projects in the field of communications and navigation both as project coordinator and as technical contributor. From 2003 till 2013, Dr. Meurer was active as a senior lecturer and Associate Professor (PD) at the same university. Since 2006 Dr. Meurer is with the German Aerospace Centre (DLR), Institute of Communications and Navigation, where he is the director of the Department of Navigation and of the center of excellence for satellite navigation. In addition, since 2013 he is a professor of electrical engineering and director of the Institute of Navigation at the RWTH Aachen University. His current research interests include GNSS signals, GNSS receivers, interference and spoofing mitigation and navigation for safety-critical applications.

ABSTRACT

In this work an eigenspace method for interference detection and mitigation is presented. Several problems arise when using the SVD-based algorithm in single antenna GNSS receivers, causing the impairment of the correlation matrix estimate. Solutions to the problems are proposed. The derived approaches are assessed by computer simulations. It is shown that detection and mitigation of wide-sense stationary narrow-band interference can be achieved with low complexity, while wide-band and/or non-stationary interferences require more degrees of freedom and thus more complexity.

INTRODUCTION

The rapid growth of the wireless telecommunication sector and consequently the high demand of spectrum assigned to the new services caused the frequency spectrum to be very crowded and quite saturated. Due to the very weak received signal power on ground of the Global Navigation Satellite Systems (GNSS) signals spurious harmonics from other systems can cause unintentional interference and, therefore, a serious problem to the reliable estimation of user position, velocity and time (PVT). Besides unintentional interference, more virulent intentionally radiated signals, called jammer, may knock out the GNSS receiver; this is especially the case when a jammer with high time-frequency dynamic (e.g. Chirp-like jammer) affects the GNSS signal spectrum [1].

To enhance robustness and reliability of the GNSS receiver, several interference detection and suppression methods, successfully integrated and fully operational in GNSS receiver, can be found in literature. They differentiate by the domain in which they operate and in which way they try to separate useful signals from interfering signals. We can distinguish between: time-domain techniques, mostly based on ‘pulse blanking’ [2], frequency-domain techniques [3], time-frequency domain techniques, i.e. FDAF [4], space-domain techniques [5], space-time domain and space-frequency domain techniques [4],[6] and [7] and other orthogonal signal domain based techniques like Wavelet [3] or Karhunen-Loève Transform (KLT) [8]. Up to now, space-time domain techniques are the most promising technique for mitigation of wide-band and high dynamic interferences. However, they make use of antenna arrays, which implies higher costs both for the antenna and the receiver hardware and, in general, does not always allow to fulfill SWaP (Space, Weight and Power) and form factor requirements coming from the relevant applications.

Therefore, in this paper a novel method for interference detection and suppression is proposed which is free of aforementioned disadvantages. The method is based on singular value decomposition (SVD) and operates in digital domain at signal processing level before signal correlation. We demonstrate that this method can be easily adapted to be used in GNSS systems for radio frequency interference (RFI) detection and mitigation. The computational complexity of the whole system is scalable and can be designed to fit the hardware requirements. It is shown that a large number of degrees of freedom are needed to properly mitigate wideband RFI, while the algorithm works well even with a low number K (e.g. $\mathrm{K}=4$) when the RFI is narrowband and stationary.

SIGNAL MODEL

Let $\mathbf{x}[\mathrm{k}] \in \mathbb{C}^{\mathrm{N}\times 1}$ denote the base-band signal collecting N time instances starting from time k, with elements $\mathrm{x}[\mathrm{k}], \mathrm{x}[\mathrm{k}+1], \ldots, \mathrm{x}[\mathrm{k}+\mathrm{N}-1]$.

The received signal $\mathbf{x}[\mathrm{k}]$ is assumed to be

$$\mathbf{x}[\mathrm{k}] = \sum_{\mathrm{l}=1}^{\mathrm{L}} \mathbf{s}_{\mathrm{l}}[\mathrm{k}] + \mathbf{i}[\mathrm{k}] + \mathbf{n}[\mathrm{k}] \tag{1}$$

where L is the number of satellite signals impinging on the receiver antenna, $\mathbf{s}_{\mathrm{l}}[\mathrm{k}] \in \mathbb{C}^{\mathrm{N}\times 1}$ is the l-th satellite signal, $\mathbf{i}[\mathrm{k}] \in \mathbb{C}^{\mathrm{N}\times 1}$ is a wide-sense stationary (WSS) interference and $\mathbf{n}[\mathrm{k}] \in \mathbb{C}^{\mathrm{N}\times 1}$ is the additive zero-mean white Gaussian noise with variance σ^2.

The corresponding covariance matrix $\mathbf{R} \in \mathbb{C}^{\mathrm{N}\times\mathrm{N}}$ of the zero mean wide-sense stationary signal $\mathbf{x}[\mathrm{k}]$ is

$$\mathbf{R}[k] = \mathbf{R} = \mathrm{E}\left[\mathbf{x}[\mathrm{k}]\mathbf{x}^{\mathrm{H}}[\mathrm{k}]\right] \tag{2}$$

We now consider a collection of K realizations of the signal vector $\mathbf{x}[\mathrm{k}]$, with starting time $\mathrm{k} \in [0,1,\mathrm{K}-1]$, and we form a data observation matrix $\mathbf{X} \in \mathbb{C}^{\mathrm{K}\times\mathrm{N}}$, in canonical form, expressed in matrix notation as

$$\mathbf{X} = \begin{bmatrix} \mathrm{x}[0] & \mathrm{x}[1] & \cdots & \mathrm{x}[\mathrm{N}-1] \\ \mathrm{x}[1] & \mathrm{x}[2] & \cdots & \mathrm{x}[\mathrm{N}] \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{x}[\mathrm{K}-1] & \mathrm{x}[\mathrm{K}] & \cdots & \mathrm{x}[\mathrm{K}+\mathrm{N}-2] \end{bmatrix} = \begin{bmatrix} \mathbf{x}^{\mathrm{T}}[0] \\ \mathbf{x}^{\mathrm{T}}[1] \\ \vdots \\ \mathbf{x}^{\mathrm{T}}[\mathrm{K}-1] \end{bmatrix} \tag{3}$$

We can approximate the correlation matrix using the sample correlation matrix $\widehat{\mathbf{R}} \in \mathbb{C}^{\mathrm{K}\times\mathrm{K}}$ as

$$\widehat{\mathbf{R}} = \mathbf{X}\mathbf{X}^{\mathrm{H}} \tag{4}$$

The longer the observation time of the data matrix $\mathbf{X}$ (larger N) the better the estimation of the sample correlation matrix, and the higher the ratio between the principal eigenvalue of the interference space and those of the noise space.

Singular Value Decomposition

The sample correlation matrix $\widehat{\mathbf{R}}$ can be factorized using the Eigen Value Decomposition (EVD) as follows

$$\widehat{\mathbf{R}} = \mathbf{V}_{\mathrm{x}}\mathbf{\Lambda}_{\mathrm{x}}\mathbf{V}_{\mathrm{x}}{}^{\mathrm{T}} \tag{5}$$

given

$$\mathbf{V}_{\mathrm{x}}{}^{\mathrm{T}}\mathbf{V}_{\mathrm{x}} = \mathbf{V}_{\mathrm{x}}\,\mathbf{V}_{\mathrm{x}}{}^{\mathrm{T}} = \mathbf{I}_{\mathrm{N}} \tag{6}$$

where $\mathbf{V}_{\mathrm{x}} = [\mathbf{v}_1^{\mathrm{x}}, \mathbf{v}_2^{\mathrm{x}}, \ldots, \mathbf{v}_{\mathrm{K}}^{\mathrm{x}}]$ is the $\mathrm{K}\times\mathrm{K}$ orthonormal matrix of eigenvectors, and $\mathbf{\Lambda}_{\mathrm{x}} = \mathrm{diag}(\lambda_{\mathrm{k}}^{\mathrm{x}})$ is the $\mathrm{K}\times\mathrm{K}$ diagonal matrix of eigenvalues $[\lambda_1^{\mathrm{x}}, \lambda_2^{\mathrm{x}}, \ldots, \lambda_{\mathrm{K}}^{\mathrm{x}}]$.

There is a strong relationship between the EVD and the SVD, applying the SVD to the data matrix $\mathbf{X}$, we can factorize $\mathbf{X}$ as

$$\mathbf{X} = \mathbf{V}_{\mathrm{x}}\,\mathbf{\Phi}_{\mathrm{x}}\,\mathbf{U}_{\mathrm{x}}{}^{\mathrm{T}} \tag{7}$$

where $\mathbf{V}_{\mathrm{x}}$ is the $\mathrm{K}\times\mathrm{K}$ orthonormal matrix of left singular vectors equal to the matrix of eigenvectors, $\mathbf{\Phi}_{\mathrm{x}} = [\mathrm{diag}(\varphi_{\mathrm{k}}^{\mathrm{x}}) \quad | \quad \mathbf{0}]$ where $\mathrm{diag}(\varphi_{\mathrm{k}}^{\mathrm{x}})$ is the $\mathrm{K}\times\mathrm{K}$ diagonal matrix of singular values $[\varphi_1^{\mathrm{x}}, \varphi_2^{\mathrm{x}}, \ldots, \varphi_{\mathrm{K}}^{\mathrm{x}}]$, having the property $\varphi_{\mathrm{k}} = \sqrt{\lambda_{\mathrm{k}}}$, and $\mathbf{U}_{\mathrm{x}}$ is the $\mathrm{N}\times\mathrm{N}$ orthonormal matrix of right singular vectors. Hereafter we indicate with $\varphi_{\mathrm{k}}^{\mathrm{s}}$, $\varphi_{\mathrm{k}}^{\mathrm{i}}$ and $\varphi_{\mathrm{k}}^{\mathrm{n}}$ the singular values belonging to signal, interference, and noise subspace respectively, descendingly ordered from the biggest (φ_1) to the smallest one (φ_{K}).

We assume each replica $\mathbf{s}_{\mathrm{l}}[\mathrm{k}]$ having a signal power much smaller than the noise power, in general between 20 dB and 40 dB weaker. For the purpose of this work, we also assume that, in case an interfering signal is present, it applies that $\varphi_{\mathrm{k}}^{\mathrm{n}} < \varphi_1^{\mathrm{i}}\ \forall\, \mathrm{k} \in [1,\mathrm{K}]$. We can summarize these assumptions as

$$\varphi_1^{\mathrm{s}} \ll \varphi_{\mathrm{k}}^{\mathrm{n}} < \varphi_1^{\mathrm{i}} \qquad \forall\, \mathrm{k} \in [1,\mathrm{K}] \tag{8}$$

Let us consider the interference subspace to be of rank P, with $P < \mathrm{K}$. In order to separate the interference subspace from the noise plus signal subspace, it is convenient to rewrite the SVD as in (9).

$$\mathbf{X} = \left[\mathbf{V}_{\mathrm{x}}{}^{(K\times P)} | \mathbf{V}_{\mathrm{x}}{}^{(K\times K-P)}\right] \left[\begin{matrix} \mathbf{\Phi}_{\mathrm{x}}{}^{(P)} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_{\mathrm{x}}{}^{(K-P)} \end{matrix} \quad \middle| \quad \mathbf{0}^{(K\times N-K)}\right] \left[\begin{matrix} \mathbf{U}_{\mathrm{x}}{}^{\mathrm{T}\,(P\times N)} \\ \mathbf{U}_{\mathrm{x}}{}^{\mathrm{T}\,(N-P\times N)} \end{matrix}\right] \tag{9}$$

$$\begin{aligned} \mathbf{X} &\approx \left[\mathbf{V}_{\mathrm{i}}{}^{(K\times P)} | \mathbf{V}_{\mathrm{i}}{}^{(K\times K-P)}\right] \left[\begin{matrix} \mathbf{\Phi}_{\mathrm{i}}^{(P)} + \sigma\,\mathbf{I}^{(P)} & \mathbf{0} \\ \mathbf{0} & \sigma\,\mathbf{I}^{(K-P)} \end{matrix} \quad \middle| \quad \mathbf{0}^{(K\times N-K)}\right] \left[\begin{matrix} \mathbf{U}_{\mathrm{i}}{}^{\mathrm{T}\,(P\times N)} \\ \mathbf{U}_{\mathrm{i}}{}^{\mathrm{T}\,(N-P\times N)} \end{matrix}\right] \\ &= \mathbf{V}_{\mathrm{i}}{}^{(K\times P)}\left(\mathbf{\Phi}_{\mathrm{i}}^{(P)} + \sigma\,\mathbf{I}^{(P)}\right)\mathbf{U}_{\mathrm{i}}{}^{\mathrm{T}\,(P\times N)} + \mathbf{V}_{\mathrm{i}}{}^{(K\times K-P)}\left(\sigma\,\mathbf{I}^{(K-P)}\right)\mathbf{U}_{\mathrm{i}}{}^{\mathrm{T}\,(K-P\times N)} \end{aligned} \tag{10}$$

Since the interference subspace has rank P and (8) holds, we can approximate the equation in (9) substituting the left singular vector matrix of the signal $\mathbf{V}_{\mathrm{x}}$, with the left singular vector matrix of the interference $\mathbf{V}_{\mathrm{i}}$ obtaining the decomposition in (10). Equation (10) reveals that

$$\begin{cases} \mathbf{V}_{\mathrm{i}} \approx \mathbf{V}_{\mathrm{x}} \\ \mathbf{\Phi}_{\mathrm{x}}{}^{(P)} \approx \mathbf{\Phi}_{\mathrm{i}}^{(P)} + \sigma\,\mathbf{I}^{(P)} \\ \mathbf{\Phi}_{\mathrm{x}}{}^{(\mathrm{K-P})} \approx \sigma\,\mathbf{I}^{(K-P)} \end{cases} \tag{11}$$

Thus we can approximate singular vectors of the interference subspace with singular vectors resulting from the SVD of the input signal.

INTERFERENCE DETECTION

In this section we describe the interference detection problem and present the SVD-based detector. We address two issues arising when the algorithm is implemented in a GNSS receiver and propose different solutions to solve them.

Detection problem

Signal detection is based on the Neyman-Pearson lemma [11]. The Neyman-Pearson criterion maximizes the probability of detection (P_D) by a given maximal value of probability of false alarm (P_{FA}). Hypothesis H_0 as given in (12) is referred to as the simple (not composite) *null hypothesis*, and represent the case when the discrete time signal $\mathbf{x}[\mathrm{k}]$ consists of only noise plus the "weak" satellite signals. The alternative hypothesis H_1 represents the case when an additive interference $\mathbf{i}[\mathrm{k}]$ affects $\mathbf{x}[\mathrm{k}]$. The two mutually exclusive hypotheses can be written as

$$\begin{aligned} H_0 &: \mathbf{x}[\mathrm{k}] = \sum_{\mathrm{l}=1}^{\mathrm{L}} \mathbf{s}_{\mathrm{l}}[\mathrm{k}] + \mathbf{n}[\mathrm{k}] \\ H_1 &: \mathbf{x}[\mathrm{k}] = \sum_{\mathrm{l}=1}^{\mathrm{L}} \mathbf{s}_{\mathrm{l}}[\mathrm{k}] + \mathbf{i}[\mathrm{k}] + \mathbf{n}[\mathrm{k}] \end{aligned} \tag{12}$$

Depending on interference signal characteristics, different optimal or suboptimal estimators can be used. The generalized likelihood ratio test (GLRT) has been used here to determine the estimator functionality. It produces in general good detection performances and small loss comparing with an optimal estimator. While determining the probability density function (*PDF*) under H_0, indicated as $p(\mathbf{x}; H_0)$, can be an easy task, since the signal mainly consists of white Gaussian noise, determining $p(\mathbf{x}; H_1)$, the *PDF* under H_1, can be very challenging and sometimes prohibitive. After fixing the P_{FA}, we can determine the detection threshold γ from the probability density function $p(\mathbf{x}; H_0)$, as follows

$$P_{FA} = \int_{\gamma}^{\infty} p(\mathbf{x}; H_0)\,dx \tag{13}$$

SVD-based detection

SVD-based signal detection is largely used in cognitive radio networks to detect the presence of wireless signals [9].

The test statistics is the principal singular value obtained after SVD factorization of the data matrix $\mathbf{X}$.

Since the singular values are descendingly ordered, the principal singular value corresponds to the largest singular value φ_1^{x}.

$$\mathrm{T}(\mathrm{x}) = \varphi_1^{\mathrm{x}}$$

It is shown in [12] that if the K-variate sample correlation matrix $\widehat{\mathbf{R}}$, under H_0 (*null case*), follows the standard Wishart distribution $W_p(\mathbf{I}, \mathrm{N})$, φ_1^{x} follows a Tracy-Widom distribution

$$\varphi_1^{\mathrm{x}} \sim TW_{\beta}\left(\mu_{\varphi}, \sigma_{\varphi}\right) \tag{14}$$

where μ_{φ} and σ_{φ} are the centering parameter and the scaling parameter, respectively, defined as

$$\begin{aligned} \mu_{\varphi} &= \left(\sqrt{K - 1/2} + \sqrt{N - 1/2}\right)^2 \\ \sigma_{\varphi} &= \left(\sqrt{K - \frac{1}{2}} + \sqrt{N - \frac{1}{2}}\right)\left(\frac{1}{\sqrt{K - \frac{1}{2}}} + \frac{1}{\sqrt{N - \frac{1}{2}}}\right)^{\frac{1}{3}} \end{aligned} \tag{15}$$

We can now determine the detection threshold γ_{SVD} inverting (13) and making use of the cumulative distribution function.

Noise sample correlation matrix

The data observation matrix in (3) collects data from one single sensor, the receiving single antenna. The K different realizations of the signal vector $\mathbf{x}[\mathrm{k}]$ are in fact a time shift of the same realization. One row of the data observation matrix differs from the previous one by just one sample. This means the rows of $\mathbf{X}$ are not independent. In the case that only the additive white Gaussian noise (AWGN) is present, we can express the sample correlation matrix $\widehat{\mathbf{R}} = \widehat{\mathbf{R}}_{\boldsymbol{n}}$ in the following form

$$\widehat{\mathbf{R}}_{\boldsymbol{n}} = \begin{bmatrix} \sigma^2 & R_{n_0,n_1} & R_{n_0,n_2} & \cdots & R_{n_0,n_{K-1}} \\ & \sigma^2 & R_{n_1,n_2} & \cdots & R_{n_1,n_{K-1}} \\ & & \sigma^2 & \ddots & \vdots \\ & \cdots & & \ddots & R_{n_{K-2},n_{K-1}} \\ & & & & \sigma^2 \end{bmatrix} \quad (16)$$

where R_{n_i,n_j} is the cross-correlation of the noise realization i with the noise realization j. The diagonal elements of $\widehat{\mathbf{R}}_{\boldsymbol{n}}$ have the property

$$R_{n_0,n_\Delta} \approx R_{n_1,n_{\Delta+1}} \approx \cdots \approx R_{nn}(\Delta)$$

The result is a sample correlation matrix $\widehat{\mathbf{R}}_{\boldsymbol{n}}$ which is not Wishart. This affects the statistical properties of the principal singular value defined in (14) and results in a signal detector with degraded performance.

There are mainly two possibilities to avoid this problem. The first is to use K different sensors to collect the different realizations, the second is to collect the realizations spaced by a delay equal or greater than N, the length of the input signal vector. The first option is the less practicable as it proposes to extend the receiver by K receiving channels. The second option can be afforded without any additional cost, in this case the starting times k the realizations $\mathbf{x}[\mathrm{k}]$ will be collected would become

$$\mathrm{k} \in [0, \mathrm{N}+\delta, \dots, (\mathrm{N}+\delta)(\mathrm{K}-1)]$$

Hereafter we call this type of data observation matrix the block-wise data observation matrix, indicated by $\mathbf{X}_{\mathrm{b}}$.

Satellite sample correlation matrix

In (8) we have assumed the satellite signal power is far below the noise floor and consequently its eigenvalues to be much smaller than the eigenvalues of the interference subspace. This assumption is reasonable if the interference power is strong enough to enable the interference to be the principal, or dominant source in the eigenspace, or if we are using the block-wise data observation matrix $\mathbf{X}_{\mathrm{b}}$. However, in case the interference power is not strong enough and/or there are several satellite signals in view, this assumption becomes weak.

If we observe the sample correlation matrix under the assumption that the signals are uncorrelated among each other, we can write

$$\widehat{\mathbf{R}} = \widehat{\mathbf{R}}_s + \widehat{\mathbf{R}}_i + \widehat{\mathbf{R}}_n \quad (17)$$

where $\widehat{\mathbf{R}}_s$ is the satellite sample correlation matrix, $\widehat{\mathbf{R}}_i$ the interference sample correlation matrix and $\widehat{\mathbf{R}}_n$ the noise sample correlation matrix.

Neglecting the cross-correlation between the pseudorandom noise codes of the different satellite signals and in case the canonical data observation matrix in (3) has been used, we can approximate the satellite sample correlation matrix as

$$\widehat{\mathbf{R}}_s = \begin{bmatrix} \sum_{s=1}^{S} R_{ss}(0) & \cdots & \sum_{s=1}^{S} R_{ss}(K-1) \\ & \ddots & \vdots \\ \cdots & & \sum_{s=1}^{S} R_{ss}(0) \end{bmatrix}$$

where $R_{ss}(\Delta)$ is the autocorrelation of the satellite signal s at time Δ. Assuming that the satellite signals have all the same modulation and power, we can write

$$\widehat{\mathbf{R}}_s = \mathrm{S} \cdot \begin{bmatrix} R_{ss}(0) & \cdots & R_{ss}(K-1) \\ & \ddots & \vdots \\ \cdots & & R_{ss}(0) \end{bmatrix} \quad (18)$$

The resulting satellite correlation matrix has off-diagonal elements different than zero and are not negligible. The impairment of the signal correlation matrix $\widehat{\mathbf{R}}$ due to the presence of the satellite signal results in a degradation of the interference detection and mitigation performance. In particular interference mitigation shall take into account the satellite sample correlation to properly perform mitigation. This will be discussed in more detail in the Interference Mitigation section.

Estimation of noise power

In order to separate the interference subspace from the noise plus signal subspace, a correct estimation of the noise power is required. Independently from the detection method used to determine the presence of an additive interfering signal in white Gaussian noise, the detection threshold has to be scaled by the noise power. We will consider the estimation of the noise power as a deterministic signal scaling the observation matrix.

Recalling the power of a generic signal $\mathbf{x}[\mathrm{k}]$ can be calculated as

$$W_x = \frac{1}{K}\sum_{n=1}^{K}\lambda_n^x = tr(\mathbf{\Lambda}_n) \quad (19)$$

While assuming that the interference subspace has rank P and (8) and (11) hold, we can calculate the interference power W_i and the signal power W_x as

$$\begin{aligned} W_i &= \frac{1}{K}(tr(\mathbf{\Lambda}_i^P) - P\sigma^2) \\ W_x &= \frac{1}{K}\sum_{k=1}^{K}\lambda_x^k = \frac{1}{K}E[tr(\boldsymbol{x}[k]\boldsymbol{x}[k]^T)] \end{aligned} \quad (20)$$

The noise power $W_n = \sigma^2$can be expressed as

$$\sigma^2 = W_x - W_i = W_x - \frac{1}{K}tr(\mathbf{\Lambda}_i^P) + \frac{P}{K}\sigma^2 \quad (21)$$

Solving for σ^2 gives

$$\sigma^2 = \frac{K}{K-P}W_x - \frac{1}{K-P}tr(\mathbf{\Lambda}_i^P) \quad (22)$$

Additionally an estimation of the rank of the interference subspace is needed. A simple way to estimate P can be to count the number of signal eigenvalues exceeding the detection threshold γ.

INTERFERENCE MITIGATION

The algorithm can be further enhanced to adaptively mitigate the detected interference. Mitigation is also based on eigendecomposition of the signal space.

Knowledge about the signal eigenspace can be efficiently applied for interference mitigation in two ways. The first makes use of the knowledge about the signal's principal components to obtain the coefficients of a transversal filter. The second makes use of the principal components to construct a projection matrix spanning the orthogonal complement domain with respect to the interference subspace. The first approach is more suitable for stationary interference, the filter's coefficients can be calculated once and updated over a time period longer than the observation time. The second approach requires constant update of the projection matrix coefficients, this can be done block-wise or in a recursive way. The second approach is more computationally expensive, but has the advantage to be effective against both stationary and non-stationary interference.

In this work we focus on mitigation based on the projection matrix.

Rank reduction filter

Applying the Karhunen-Loève expansion the input data vector $\mathbf{x}[k]$ can be expressed as a linear combination of eigenvectors in the following form

$$\boldsymbol{x}[k] = \sum_{n=1}^{N}\boldsymbol{v}_n^x[k]c_n[k] \quad (23)$$

where $c_n[k]$ are zero-mean, uncorrelated random variables defined by the inner product

$$c_n[k] = \boldsymbol{v}_n^{x\,T}[k]\boldsymbol{x}[k]$$

We assume the principal components mainly containing the interference energy, in other words they represent the "eigensignals" or "natural modes" of the interference. We can then perform a rank reduction of the data observation matrix in order to mitigate the interference. We define the output data matrix $\mathbf{Y}[k]$ as

$$\begin{aligned} \mathbf{Y} &= \sum_{k=P+1}^{K}\mathbf{v}_k^x[k]\mathbf{c}_k[k] \\ &= \mathbf{X} - \sum_{k=1}^{P}\mathbf{v}_{\boldsymbol{k}}^x[k]\mathbf{c}_k[k] \\ &= \mathbf{X} - \left(\sum_{k=1}^{P}\mathbf{v}_k^x[k]\mathbf{v}_k^{x\,T}[k]\right)\mathbf{X} \end{aligned} \quad (24)$$

and

$$\mathbf{y}[k] = \mathbf{\Pi}^T\mathbf{Y} \quad (25)$$

where $\Pi^T = [1,0,\dots,0]$ is the $1 \times N$ top "pinning" vector. The new vector $\mathbf{y}[k]$ has rank $N - P$. The rank reduction in (19) can be also obtained via multiplication of the data observation matrix with a projector $\mathbf{P}_\mathbf{x}^\perp$ spanning the orthogonal complement of the interference subspace. The projector is defined as

$$\mathbf{P}_{\boldsymbol{x}}^\perp = \mathbf{I} - \sum_{k=1}^{P}\mathbf{v}_k^x[k]\mathbf{v}_k^{x\,T}[k] = \mathbf{I} - \mathbf{V}_x{}^{(P)}\mathbf{V}_x{}^{(P)\,T} \quad (26)$$

and the output signal vector can be obtained as

$$\mathbf{y}[k] = \mathbf{\Pi}^T\mathbf{P}_{\boldsymbol{x}}^\perp\mathbf{X} = \mathbf{w}^T[k]\,\mathbf{X} \quad (27)$$

where $\mathbf{w}^T[k]$ is coefficient vector of the transversal filter of length K taps, as shown in Figure 1.

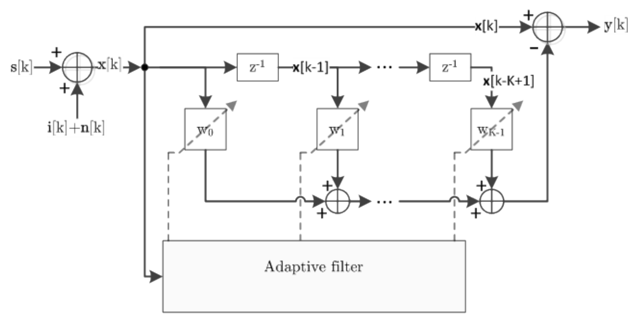

Figure 1 Adaptive transversal filter

Sample correlation matrix compensation

As already discussed before, the satellite sample correlation matrix can affect the signal's space and impair the interference subspace estimate. To weaken the impact of the satellite signals on the signal's subspace, we can use an estimate $\widetilde{\mathbf{R}}_s$ of the satellite sample correlation matrix $\widehat{\mathbf{R}}_s$ and compensate the signal correlation matrix $\widehat{\mathbf{R}}$ as following

$$\widehat{\mathbf{R}}_{\mathrm{C}} = \widehat{\mathbf{R}}_s + \widehat{\mathbf{R}}_i + \widehat{\mathbf{R}}_n - \widetilde{\mathbf{R}}_s \quad (28)$$

An estimation of $\widehat{\mathbf{R}}_s$ can be obtained by computing $\widehat{\mathbf{R}}$ under H_0, for $N \to \infty$ the matrix $\widehat{\mathbf{R}}$ becomes

$$\widehat{\mathbf{R}} \approx \begin{bmatrix} \sigma^2 + S \cdot R_{ss}(0) & \cdots & S \cdot R_{ss}(K-1) \\ & \ddots & \vdots \\ \cdots & & \sigma^2 + S \cdot R_{ss}(0) \end{bmatrix} \quad (29)$$

Assuming all the satellite signals in view have the same modulation and the modulation is known, we can determine $R_{ss}(0)$ based on the values of the off-diagonal elements, obtaining an estimate for the satellite correlation matrix in (18).

NUMERICAL RESULTS

Numerical results have been obtained by numerical simulations; all simulations were performed in Matlab.
If not otherwise specified numerical simulations report results using a data observation matrix with size $K = 4$ and $N = 2000$. Simulations were performed using the parameters summarized in Table 1.

Interference detection

The interference detector's performance has been evaluated using as test metrics the probability of detection P_D. The probability of false alarm $P_{FA}$was set to 10^{-3}. The SVD-based detector's performance has been compared with the GLRT for the detection of a sinusoid [13], based on the FFT of the data observation matrix.

Simulation parameters	
Sampling frequency	20 MHz
Discriminator type	E-L coherent
Correlator space	0.5 chip
Integration time	1 ms
Satellite signal	GPS-L1 C/A PRN1
Average CN0	45 dB-Hz
Chirp RFI	Sweep 0-20 MHz within 20µs
Sine-wave RFI	Carrier frequency 1.5 MHz
DME RFI	Single double pulse within 1ms, carrier frequency 1.5 MHz
Band-limited RFI	Bandwidth 2MHz cent. at 1.5MHz

Table 1 Simulation parameters

Figure 2 shows the performance degradation in case the noise correlation matrix's distribution is not Wishart and has the form described in (16). Figure 3 shows the degradation caused by the presence of the satellite signals. In this case satellite signal's CN0s were, on average, 45dB-Hz. The figure shows also the improvement obtained using $\widetilde{\mathbf{R}}_s$ compensation as indicated in (28). Figure 4 and Figure 5 show the performance in case of a single DME pulse interference, and a chirp-like interference, respectively. The graphs depict the different behavior of the two data observation matrix types, the canonical and the block-wise. In general the canonical provides better results for non-stationary, low duty cycle and intermittent interferences, while the block-wise approach provides better results for stationary interferences. The block-wise type has the advantage that the resulting $\widehat{\mathbf{R}}$ under H_0 has a Wishart distribution and is also unaffected by the presence of satellite signals. Figure 6 and Figure 7 depict a comparison between the SVD-based and the FFT-based detectors for a wideband interference (2MHz band-limited noise) and a sinusoid interference, respectively. As expected SVD-based provides better results in detecting wideband interferences, like band-limited noise, DME and chirp-like RFIs, while the FFT-based approach is more indicated for narrow-band RFIs.

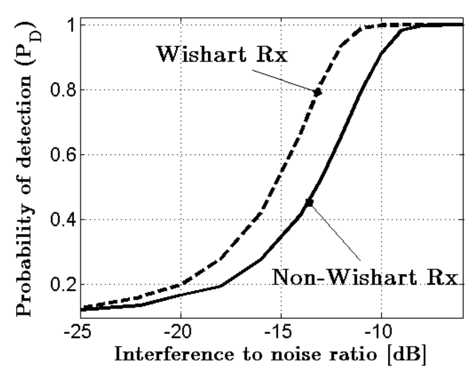

Figure 2 P_D of Chirp interference – performance degradation for non-Wishart $\widehat{\mathbf{R}}_n$

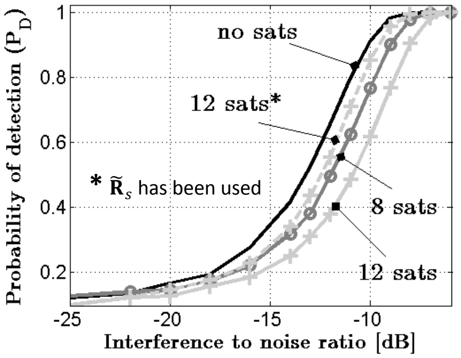

Figure 3 P_D of Chirp interference – Effect of satellite signals

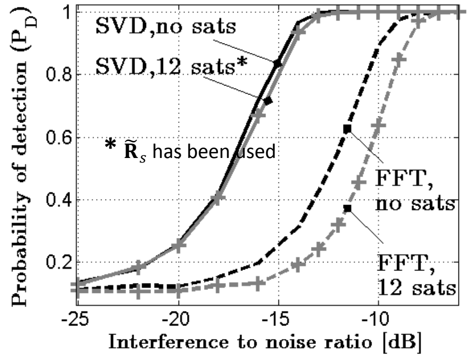

Figure 6 P_D of Band-limited interference (2Mhz bandwidth) – Comparison between SVD-based and FFT-based detection

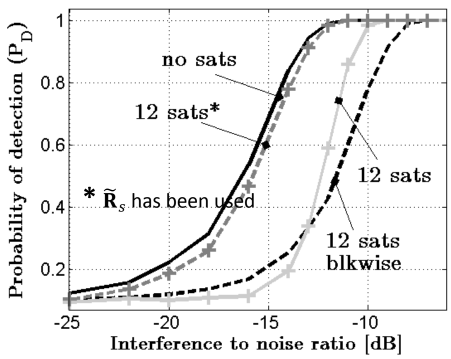

Figure 4 P_D of DME interference – Comparison between canonical and block-wise observation matrices

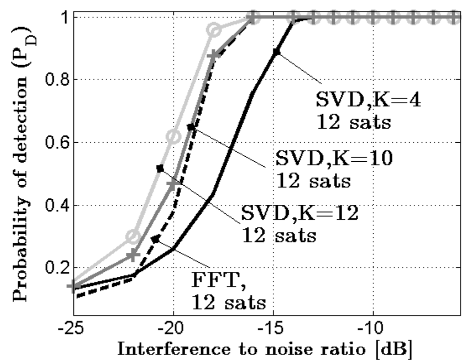

Figure 7 P_D of Sine-wave interference – Comparison between SVD-based and FFT-based detection

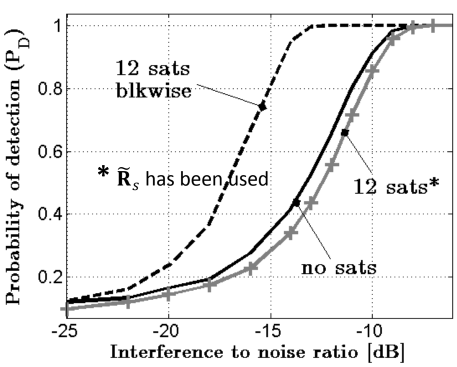

Figure 5 P_D of Chirp interference – Comparison between canonical and block-wise observation matrices

Interference mitigation

Interference mitigation performance has been evaluated using two types of test metrics: the early-late discriminator jitter and the interference mitigation capability, the latter expressed as the amount of rejected interference power. The SVD-based mitigation has been compared with the mitigation using the frequency domain adaptive filter (FDAF) described in [4]. Simulation results have been obtained using an ideal receiver, processing floating point data and having an input signal dynamic range large enough to avoid clipping and overflow.

Figure 8 and Figure 9 show the results after mitigation of a sine-wave. The sine-wave frequency has been selected to fall in between two adjacent frequency bins; this represents a worst-case, since it causes the maximum amount of spectral leakage. Using FDAF the receiver robustness could be increased by about 20dB, in this case the FFT-based technique's performance was limited by the leakage effect. On the contrary, SVD-based algorithm does not suffer from the leakage problem and is able to optimally separate the interference from the signal subspace. In other words a perfect projector onto the orthogonal complement of the sine wave can be obtained.

Figure 8 also reveals the improvement in tracking jitter and RFI mitigation when a canonical data observation matrix is used, compared with the block-wise approach. Figure 8 and Figure 9 show the results in case a DME interference is affecting the receiver. In this case the carrier frequency of the interference was besides the main lobe of the satellite signal spectrum. SVD-based mitigation was able to reject the DME interference and ensure tracking. Anyway it was necessary to compensate the sample correlation matrix as indicated in (28). This operation is crucial, and its effect can be seen in Figure 12 and Figure 13. Figure 12 shows the transfer function $H(f)$ of the equivalent transversal filter with coefficients $\mathbf{w}^T[k]$ as given in (27). The filter transfer function obtained without matrix compensation would filter out part of the satellite signal, since the algorithm would treat the signal subspace as part of the interference subspace. In contrast, using matrix compensation prior to the eigendecomposition would separate the satellite from the interference subspace and preserve the satellite signal. Figure 13 shows the resulting satellite signal after mitigation in time domain. Successful mitigation of the DME wide-band non-stationary signal could be achieved as long as the interference spectrum was not fully overlapping the main lobe of the satellite spectrum. Further simulations were conducted for different wide-band interference sources, like chirp-like and band-limited noise RFIs. In general mitigation of wide-band and non-stationary interferences requires additional degrees of freedom. In other words, for this kind of RFIs it is necessary to increase the number of realizations K in order to enable the SVD-based algorithm to properly separate the signal from the interference subspace.

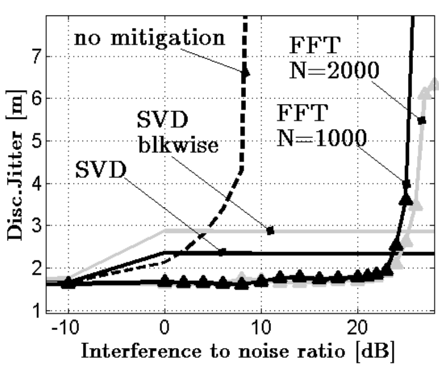

Figure 8 E-L discriminator jitter after Sine-wave interference mitigation

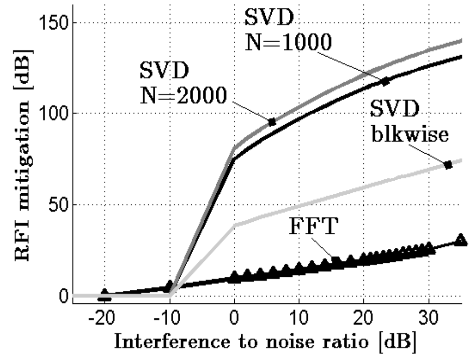

Figure 9 Rejected Sine-wave interference power after mitigation

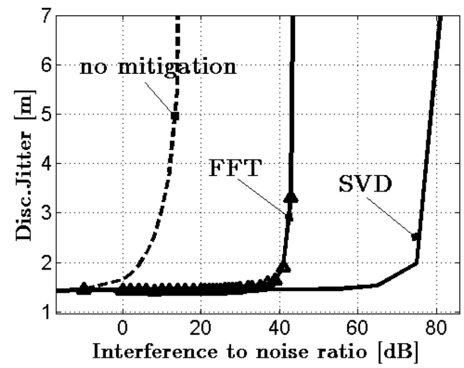

Figure 10 E-L discriminator jitter after DME interference mitigation

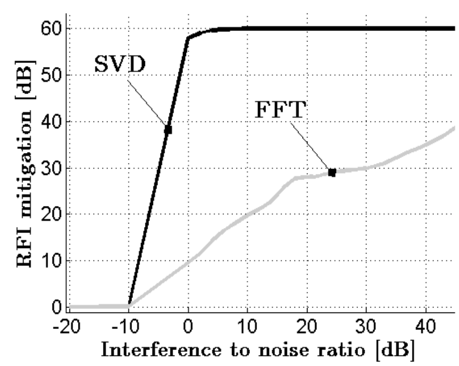

Figure 11 Rejected DME interference power after mitigation

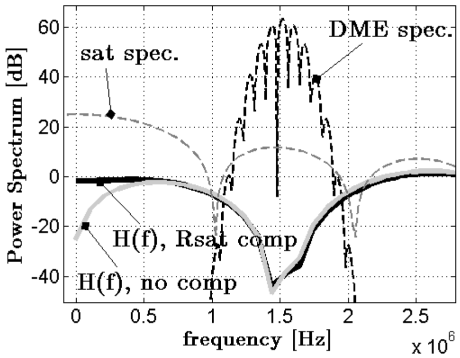

Figure 12 Equivalent transversal filter response – improvement using the satellite correlation matrix compensation

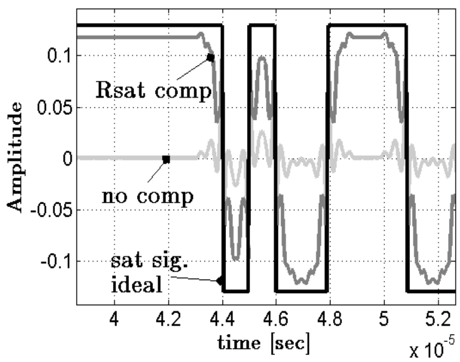

Figure 13 Satellite time signal after DME mitigation – improvement using the satellite correlation matrix compensation

CONCLUSIONS

In this work we have presented two methods, one for interference detection and one for interference mitigation, both making use of an eigendecomposition of the data space. Particular attention was paid to the form of the data observation matrix and how it can affect the algorithms. We have shown that interference mitigation suffers when the data observation matrix has overlapping rows resulting in a noise correlation matrix not following the usual Wishart distribution. In order to better separate the interference from the signal plus noise subspace we have proposed an approach making use of the estimate of the satellite sample correlation matrix.

Using a software representation of a GNSS receiver it has been demonstrated that the usage of the SVD-based algorithms in single antenna GNSS systems are a powerful tool to detect interfering signals and increase the robustness of the receiver. The computational complexity is scalable and can fit the desired requirement. Increasing the degrees of freedom of the algorithm and the complexity of the hardware leads to a sensible improvement of the RFI mitigation performance against wideband and non-stationary RFI. The mathematical principle and its practical implementation have similarities with the ones proposed in [6] and [10]. Hence the derivation of the computational requirement made in [10] is also applicable for in this work proposed schemes.

ACKNOWLEDGMENTS

The study which results are reported in this paper was performed in the frame of BaSE-II (Bavarian security receiver) project co-funded by the Bavarian Ministry of Economic Affairs. This support is greatly acknowledged.

REFERENCES

[1] T. Kraus et al., “Survey of In-Car Jammers - Analysis and Modeling of the RF Signals and IF Samples (Suitable for Active Signal Cancelation)”, ION GNSS 2011.

[2] G.X. Gao, "DME/TACAN Interference and its Mitigation in L5/E5 Bands", ION GNSS 2007.

[3] Paonni et al., "Innovative interference mitigation approaches: Analytical analysis, implementation and validation", NAVITEC 2010.

[4] M. Cuntz et al., “Field Test: Jamming the DLR Adaptive Antenna Receiver”, ION GNSS 2011.

[5] M. Cuntz et al., "GALANT - Architecture of a Real-Time Safety of Life Receiver". ION GNSS 2009.

[6] M. Sgammini et al., “Blind Adaptive Beamformer Based on Orthogonal Projections for GNSS”, ION GNSS 2012.

[7] Castañeda et al., "Joint Space-Time Interference Mitigation for Embedded Multi-Antenna GNSS Receivers" ION GNSS 2013.

[8] Musumeci et al.,"A comparison of transformed-domain techniques for pulsed interference removal on GNSS signals," ICL-GNSS 2012.

[9] X. Shaoyi et al., "SVD based sensing of a wireless microphone signal in cognitive radio networks", ICCS 2008.

[10] Kurz et al., "An architecture for an embedded antenna-array digital GNSS receiver using subspace-based methods for spatial filtering," NAVITEC 2012.

[11] J. Neyman, E.S. Pearson, “On the Problem of the Most Efficient Tests of Statistical Hypotheses,” Philosophical Transactions of the Royal Society of London. Series A, Containing Papers of a Mathematical or Physical Character 231, 1933, pp. 289-337.

[12] I.M. Johnstone, "On the distribution of the largest eigenvalue in principle components analysis," The Annals of Statistics, vol. 29, no. 2, 2001, pp. 295-327.

[13] S. M. Kay, “Fundamentals of statistical signal processing - Detection Theory”, Vol.II, 1998, pag.279-283.