Imaging ocean surface statistics using Geosynchronous Correlating SAR (CoSAR)

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Abstract

This paper discusses a novel radar imaging concept called a Correlating SAR (CoSAR) and its application to the observation of the second order statistics of the radar echoes corresponding to the ocean surface. It is shown that these statistics can be retrieved by jointly processing the radar echoes received by two radars that have a relative motion.

1 Introduction

A general assumption allowing Synthetic Aperture Radar (SAR) imaging is that the observed scene does not change during the aperture time. Even for Ground Moving Target Indication (GMTI) techniques or Along-Track Interferometry (ATI) it must be assumed that the target remains coherent during some time. This limitation is generally acceptable for the observation of land. In contrast, it is clear that this assumption does not hold for the fast decorrelating ocean surface. One consequence is that the azimuth resolution becomes limited by the coherence time of the surface. However, for most applications the reduced resolution is not a problem, since the desired products are usually relatively low resolution products (resolutions in the order of $0.1\,\mathrm{km}$ to $1\,\mathrm{km}$), which are usually achievable, and the much finer nominal resolution can still be exploited to generate large numbers of independent looks.

A stronger limitation becomes apparent when SAR observations are compared to radar observations made by fixed coastal radars. Indeed, those allow the observation of the spatio-temporal statistics of the radar echoes, for example in the form of the spatially varying Doppler spectra. In this sense, aside from the NRCS, both single channel and ATI-capable SAR systems are limited to the estimation of the first moment of these Doppler spectra, which are derived from the Doppler Centroid anomaly [1] or the along-track interferometric phase [2], respectively.

This paper discusses a novel radar imaging concept which we call a Correlating SAR (CoSAR) [3]. The basic idea is to operate two physically separated radars (which may share a common transmitter) with a relative motion so that their azimuth (cross-range) separation varies with time. Pairs of echoes acquired at each instant of time and relative position can be combined to produce estimates of the spatial autocorrelation function of the received signal. Estimates of this autocorrelation function for different positions can then be combined to high resolution images of some statistical properties of the scene, including estimates of the space-varying Doppler spectrum. This approach to imaging, which follows from the Van Cittert-Zernike Theorem, is generally used for imaging radiometers [4], radio-telescope arrays [5], and imaging Mesoscale Stratosphere Troposphere (MST) radars [6].

In particular, we are interested in a CoSAR system consisting of two geosynchronous spacecraft with a relative motion around a nominal geostationary position. Such a configuration would allow the observation of the ocean surface at moderate resolution, with regional coverage, and high temporal sampling (two observations per day). Besides allowing the estimation of the Doppler spectrum of the surface, the configuration discussed will also yield a a cross-track interferometric phase, from which ocean topography could be derived.

2 Signal Model

The backscattering of a monochromatic single polarization radar signal on a surface can be described by a complex scattering coefficient,

$$s(x, y, t_s),$$

which is, generally speaking, spatially and slow time varying, where with slow time, t_s, we refer to time at scales large compared to the pulse repetition interval.

Since we are interested in radar observations of the ocean surface, we may assume that the scattering coefficient decorrelates quickly, so that $s(\cdot)$ should be treated as a multi-dimensional random process. We will assume that it is a complex zero-mean locally homogeneous and temporally ergodic process, which is usually valid over some temporal scales, so that it can be described (at least partially) by its second order statistics. In particular, we assume

$$\begin{aligned} &E[s(x+\Delta x, y+\Delta y, t_\mathrm{s}+\tau)\cdot s^*(x,y,t_\mathrm{s})] \\ &\quad = R_\tau(x,y,\tau)\cdot\delta(\Delta x,\Delta y) \end{aligned} \tag{1}$$

where $E[\cdot]$ is the expected value operator, $\delta(\cdot)$ the Dirac-delta function, and $R_\tau(\cdot)$ a space-varying temporal autocorrelation function that can be expressed as

$$R_\tau(x, y, \tau) = \sigma_0(x, y) \cdot \gamma_\tau(x, y, \tau) \tag{2}$$

with $\sigma_0(\cdot)$ the space-varying real-valued NRCS, $\gamma_\tau(\cdot)$ a complex-valued temporal coherence function (with $\gamma_\tau(x, y, 0) = 1$). Note that the Doppler centroid would be given in this model by the derivative of the phase of $\gamma_\tau(x, y, 0) = 1$ with respect to τ at $\tau = 0$.
A complete derivation of the radar signal model was given in [3] and will be omitted in this paper due to space limitations. The key result is the spatio-temporal cross-correlation function of the echoes ($v_1(\cdot)$ and $v_2(\cdot)$) received by a pair of radar systems as illustrated in Fig. 1:

$$\begin{aligned}\Gamma(t_\mathrm{s}, t_{\mathrm{f},1}, t_{\mathrm{f},2}, \tau) \\ &= E[v_2(t_\mathrm{s} + \tau, t_{\mathrm{f},2}) \cdot v_1^*(t_\mathrm{s}, t_{\mathrm{f},1})] \\ &= \int \widetilde{W}(x, y, t_\mathrm{s}, t_{\mathrm{f},1}, t_{\mathrm{f},2}) \cdot R_\tau(x, y, \tau) \\ &\quad \cdot e^{-2\mathrm{j} \cdot k_0 \Delta r(x,y,t_\mathrm{s})} dx dy,\end{aligned} \tag{3}$$

where t_s represents slow-time, $t_{\mathrm{f},i}$ fast time (range delay) for the i-th radar, and $\widetilde{W}(\cdot)$ is a combined weighting function that depends on the two range time positions and the beam patterns. The term $\Delta r(x, y, t_\mathrm{s})$ is the CoSAR equivalent to the range history in a regular SAR system. The CoSAR differential range history is similar to the general case of a bistatic SAR range history. Processing approaches and issues studied for bistatic SAR may, therefore, be applicable to CoSAR.

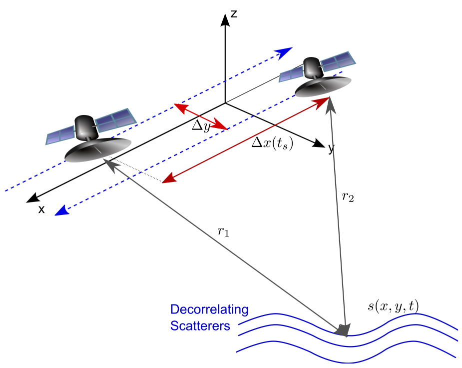


Figure 1: Illustration of CoSAR acquisition geometry, with two radar systems moving in opposite directions in the azimuth (cross-range) direction.

2.1 Resolution

Range and azimuth resolution can be derived in a way analogous to that of a bistatic system, for example following [7]. For practical CoSAR configurations, with quasi-parallel slant-range vectors, the range resolution will be that of a single radar and, therefore, determined by the pulse bandwidth. The *azimuth* (synthetic aperture) resolution is given by

$$\delta \vec{r}_\mathrm{az}(x, y) = \frac{\nabla f_{\Delta D}(x, y, t_\mathrm{s})}{||\nabla f_{\Delta \mathrm{D}}(x, y, t_\mathrm{s})||^2} \frac{1}{T_\mathrm{int}(x, y)}, \tag{4}$$

where the operator ∇ represents the gradient in the x-y plane, T_int is the integration time, and

$$f_{\Delta \mathrm{D}}(x, y, t_s) = -\frac{2}{\lambda} \frac{\partial}{\partial t_\mathrm{s}} \Delta r(x, y, t_\mathrm{s}) \tag{5}$$

can be defined as a delta-Doppler frequency. Once the azimuth direction has been identified, the corresponding resolution can be approximated as

$$\delta r_\mathrm{az} \approx \frac{\lambda_0 \cdot R_0}{2 \cdot \Delta v_\mathrm{az} \cdot T_\mathrm{int}}, \tag{6}$$

with Δv_az the relative velocity of the radars in the azimuth direction, and R_0 the slant-range distance between the radars and the imaged area. Here it we can recognize the product of the relative velocity and the integration time, $\Delta v_\mathrm{az} \cdot T_\mathrm{int}$, as the CoSAR aperture length.

3 CoSAR imaging

The simplified signal model in (3) quickly suggests and approach to CoSAR imaging. The idea is to obtain estimates of $\Gamma(\cdot)$ for a range of relative positions. The phase term resulting from the delta-range history will be a fast varying term that allows the application of the Principle of Stationary Phase, so that estimates of the signal of interest can be obtained by multiplying the estimates of $\Gamma(\cdot)$ by a complex conjugated reference function, and integrating over slow time:

$$\begin{aligned}\hat{R}_\tau(x, y, \tau) = \int \hat{\Gamma}(t_\mathrm{s}, t_{\mathrm{f},1}(x, y), t_{\mathrm{f},2}(x, y), \tau) \\ \cdot e^{2\mathrm{j} \cdot k_0 \Delta \tilde{r}(x,y)} dt_\mathrm{s},\end{aligned} \tag{7}$$

where the first term inside the integral is the estimate of $\Gamma(\cdot)$ for the two *back-projected* range-delay positions corresponding to a decorrelating scatterer at the reference position (x,y). Particularizing to $\tau = 0$ will give us an estimate of the NRCS, $\hat{\sigma}_0(x, y)$. It is interesting to highlight some properties of the CoSAR focused images:

- The estimated NRCS ($\hat{R}_\tau(x, y, 0)$) should be a positive real number. A non-zero phase would be the result of noise, an insufficient number of independent samples for the estimation of $\Gamma(\cdot)$ and, most notably, trajectory knowledge errors and an erroneous assumed height. Reversing this latest statement, the surface height can be inverted by finding the value that results in a real valued NRCS estimate.
- Multilooking is done before CoSAR imaging, during the estimation of $\hat{\Gamma}(\cdot)$, so that CoSAR image products are, in principle, speckle free.

The required estimate of $\Gamma(\cdot)$ is a second-order moment of an ergodic process. It can, therefore, be estimated by averaging independent realizations of the product of the two signals. These independent samples can be obtained in different ways:

- Averaging independent range-looks. In this case, the range resolution of the individual radar must be higher than the desired CoSAR resolution.

- By exploiting the temporal decorrelation of the surface. This requires that the relative positions vary slowly in terms of the coherence time surface.
- If the two radars have, in addition to their relative motion, a common-mode motion, this will provide independent looks in the common Doppler domain.

4 Interpretation

It is worth spending a few words interpreting the meaning of (3) and (7). Both expressions represent, basically, a Fourier transform in the CoSAR azimuth direction (which is given by $\delta\vec{r}_{\mathrm{az}}$ in (4)). Thus, for each relative position of the two radars the function $\Gamma(\cdot)$ represents a Fourier component of the signal of interest, $R_\tau(x, y, \tau)$, in the azimuth direction. By varying the relative positions of the two radar we collect different Fourier components of this signal which are used later to reconstruct the space-domain signal.
With this understanding, and since $R_\tau(x, y, \tau)$ is generally a low-pass signal in the spatial domain, it should be clear that the CoSAR acquisition geometry should allow sampling of the small wavenumber Fourier components. As derived in [3], this is achieved if at some point during the CoSAR aperture the ground-projections of the slant-range vectors of the two radars to the target of interest are aligned. This implies that the *azimuth* separation of the two radars must vanish at some point during the aperture, but allows a cross-track baseline to exist at all points.
It has all ready been pointed out that the existence of such a cross-track baseline renders the system sensitive to topography. This sensitivity does not come for free: a cross-track baseline will cause range spectral shift [8]. This implies a loss of common range bandwidth, with the associated imperative of performing common-band filtering, and requires accommodating this spectral shift in transmit pulse bandwidths.

5 Radiometric Quality

The derivation of the radiometric error budget is lengthy and, therefore, beyond the scope of this paper. Here we present the key results. Since CoSAR imaging is closely related to spectral analysis, we borrow the quality measure used for spectral power estimators, the ratio between the square of the estimate and the variance of the estimation error:

$$Q = \frac{\left|E\left[\hat{R}_\tau(\cdot)\right]\right|^2}{E\left[\left|\delta R_\tau[\cdot]\right|.^2\right]} \tag{8}$$

Manipulating the previous results, the quality of CoSAR image can be expressed as

$$Q_W = \frac{N_{\mathrm{r}} \cdot T_{\mathrm{i}}}{F_{\mathrm{n}} \cdot \tau_{\mathrm{ca}}} \cdot \frac{\left|\delta\vec{r}_{\mathrm{az}}(x, y)\right|^2 \left|R_\tau(x, y, \tau)\right|^2}{\left|\int \widetilde{W}(x, y, t_{\mathrm{s}}, t_{\mathrm{f},1}, t_{\mathrm{f},2}) \cdot \sigma_0(x, y)\right|^2}, \tag{9}$$

where N_{r} is the number of independent range samples used in the estimation of $\Gamma(\cdot)$, T_{i} the CoSAR integration time, τ_{ca} the coherence time of the radar echoes, and

$$F_{\mathrm{n}} = \left(\frac{N_{\mathrm{ca}} \cdot \mathrm{SNR} + 1}{N_{\mathrm{ca}} \cdot \mathrm{SNR}}\right)^2 \tag{10}$$

a noise degradation factor that depends on the single-pulse SNR and the number of coherently integrated pulses, $N_{\mathrm{ca}} = \tau_{\mathrm{ca}} \cdot \mathrm{PRF}$.

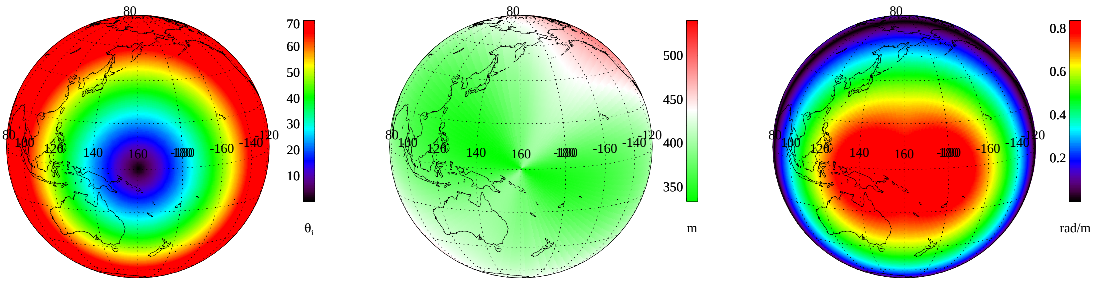
Orbit inclination	0.05°
Orbit eccentricity	0.005
Argument of perigee	90°
Mean longitude	160E
Center frequency	9.65 GHz
Integration time	300 s

Table 1: Some parameters of an example geosynchronous CoSAR mission.

The numerator in the last term of (9)represents the square of the intensity of the signal of interest integrated over the resolution cell. With everything else left equal, improving the azimuth resolution leads to a degraded quality. The denominator is, normalization factors aside, the square of the power of the range compressed signal. For the estimation of the NRCS ($\tau = 0$) and for an homogeneous scene, the entire fraction is the square of the ratio of the azimuth resolution and the width of the antenna footprint in the azimuth direction. Clearly, this ratio will tend to be much smaller than one.
The noise degradation factor will play a small role as long as the product $N_{\mathrm{ca}} \cdot \mathrm{SNR}$ is at least in the order of 5 to 10. The quality scales with the number of independent range samples, which can be easily increased by either degrading the product resolution or by improving the range resolution. The ratio $T_{\mathrm{i}}/\tau_{\mathrm{ca}}$ is the number of independent azimuth samples processed. Increasing the integration time will only improve the quality if the azimuth resolution is kept constant.

6 Mission concept

This section briefly discusses a geosynchronous CoSAR mission concept to monitor ocean winds and currents. The proposed concept (see Tab. 1), consists of two radar satellites in geosynchronous orbits centered at 160E, to cover the oceans in South-East Asia and the Australian East coast. Both spacecraft would have a small eccentricity of 0.005, with an argument of perigee of 90 degree, and an orbital inclination of 0.5 degree. The spacecraft would fly in identical orbits but with a 180 degree relative phase, corresponding to a 12 hour delay. With these parameters, the mean velocity of the spacecraft, in an Earth-Centered Earth-Fixed (ECEF) coordinate system, is of only 3.05 m/s, while the mean distance between the two spacecraft is approximately 85 km. The ECEF orbit is a vertically tilted ellipse, with an almost circular ground projection.



(a) Incident Angle **(b)** Cross-range resolution **(c)** Range spectral shift

Figure 2: Incident angle, CoSAR azimuth resolution and magnitude of range specral shift after a 12 hour observation period and a 300s integration time.

The use of X-band provides a large allocated bandwidth and reasonable expected NRCS values over the oceans for large incident angles. A 300 s CoSAR integration time has been arbitrarily set for illustration purposes.
Figure 2 illustrates the CoSAR performance. First, the incident angle is shown, as this would limit the observable region. The second panel shows the cross-range (*azimuth*) CoSAR resolution obtained after 12 hours observation, with the assumed 300 s integration time for each geographical location. The last panel shows the corresponding spectral shift in range. The values, in the range of 0.5 to 0.8 rad/m are significant and require, for adequate performance, either pulse bandwidths in the order of 100 MHz, or a range adaptive frequency offset of tens of MHz. These values correspond to heights of ambiguity between 6 and 8 m, thus providing very high sensitivity to surface topography.

7 Outlook

CoSAR provides a new approach to radar imaging of the ocean surface. In this paper we have provided the fundamental relations governing the imaging performance in terms of resolution and radiometric errors. The detailed derivation and in-depth discussion of these relations are the subject of a journal paper currently in review process. Future work will include the analysis of the mission performance in terms of geophysical products and experimental demonstration and validation of the concept.

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