Analysis of Position and Timing Solutions for an APNT-System – A Look on Convergence, Accuracy and Integrity

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BIOGRAPHY

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ABSTRACT

Alternative Position Navigation and Timing (APNT) considers different possible ranging sources like DME, UAT, LDACS, etc for which multipath is a major threat. Therefore code smoothing with Doppler measurement techniques have been considered in this paper.
But ranging errors are not the only issues encountered in APNT; in fact the positioning algorithm needs to be revisited as the linearization assumption for GNSS does not apply for the short range case. Indeed, two problems have been observed while applying the Newton-Raphson approach for the LDACS data collected during the 2012 flight trials: the initial point when taken too far away from the true position prevents the iterative positioning algorithm to converge; and when the initial point is close to the solution but the ranging errors are too large we also observed a convergence problem of the Newton-Raphson algorithm.
From this observation, we have decided to investigate the performance of a direct method. A hybrid method taking advantage of both approaches (initial point provided by a direct method followed by the Newton-Raphson method) has been suggested and compared with the stand alone direct method. This paper describes these methods and their performances are assessed based on simulations over Germany using DME stations as ranging source locations.
We show that the direct and hybrid methods provide a solution also when the Newton-Raphson algorithm does not converge.

INTRODUCTION

GNSS is foreseen by both air traffic management research programs NextGen in the US and SESAR in Europe to be the primary mean of navigation for all phases of flight. GNSS offers flexibility, global availability and outstanding performance to support performance based navigation (PBN) and precision approach under low visibility conditions. Unfortunately, due to the low level of its signal power, this system is vulnerable to radio frequency interference and the user may potentially loose the navigation service in a wide area during a critical phase of flight. In order to ensure continuity of the navigation service, a backup solution must be provided, that provides a PBN service, i.e. navigation including integrity monitoring. There are different concepts for APNT-Systems under investigation, most of them rely on ranging or pseudo ranging with ground based stations. The ground based ranging is much more resilient to interference than space based ranging sources (such as GNSS) thanks to a higher emitted signal power and a shorter distance between transmitter and receiver. But the change in geometry and the length of ranges has an implication on the positioning algorithms. The traditional GNSS positioning algorithm does not apply anymore. There are different ways to approach this problem, e.g. compute only the horizontal position and take the altitude from a barometric altimeter or apply algorithms that are less sensitive to high Dilution of Precision (DOP). But the DOP will have a significant impact no matter which algorithm is used, we can only avoid divergence of iterative algorithms and the failure to determine a position solution. We will show that depending on the density of the network of stations and the accuracy of the range measurement we need to combine independent altitude measurements with improved algorithms to get a reliable position solution.
We will start by analyzing the number of visible stations and the dilution of precision for ground stations located in Germany. Then we will shortly describe the error model that is used for the ranging errors. After this we will discuss the advantages and disadvantages of several algorithms and analyze their performance with simulations and flight trial data.

DENSITY OF STATIONS

In this section we will investigate how the density of ground stations impacts the availability of the positioning service. Three or four (for a three dimension problem) stations have to be in view to make a position solution possible. The availability largely depends on the altitude of the airborne receiver, as the visibility is mainly limited by the curvature of the earth, not the distance from the station. But having a line of sight connection to sufficiently many (more than three or four) stations does not guarantee an accurate position solution. This mainly depends on the Dilution of Precision (DOP) of the ranging sources and the accuracy of the range measurements. We have investigated the situation at the example of Germany's DME stations and for a subset of these stations limited to one station per airport. In the Figures below you can see the number of visible stations as well as the horizontal Dilution of Precision (HDOP) and the global DOP at 10000ft, 20000ft and 30000ft above mean sea level (AMSL). We did not include detailed terrain information to compute the solutions, as we did not consider aircraft altitudes close to the ground.

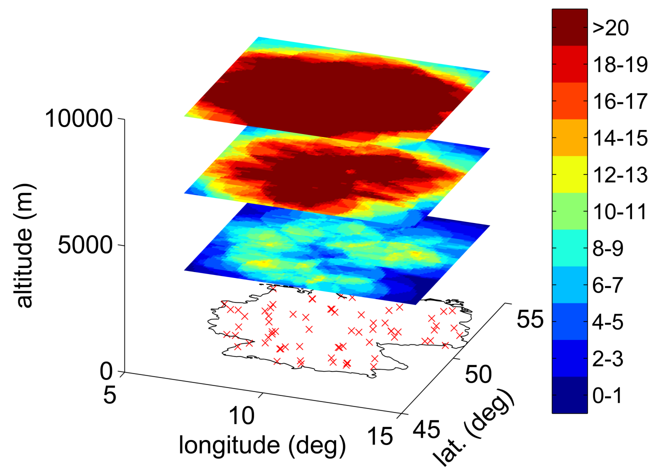

Fig. 1 Number of visible stations with currently used DME stations in Germany

In Figure 1 we can see that the German DME stations (not including the DMEs used as beacons for ILS) cover most of the German airspace at and above 10000ft. When close to the borders availability gaps may occur, but this situation can be avoided when considering the contribution of other DME stations of neighboring countries.

The horizontal dilution of precision (shown in Figure 2)

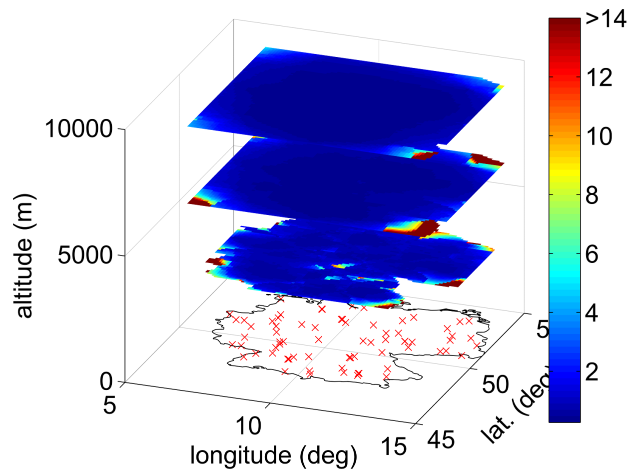

Fig. 2 HDOP with currently used DME stations in Germany

is below 2 in all relevant areas. So we have a good configuration for horizontal position determination. But the geometric dilution of precision (GDOP) (see Figure 3) reaches extremely high values (even exceeding a few hundreds of kilometers especially at 10000ft). So a vertical position determination will be quite hard to achieve with the geometrical constellation in this situation.

But there are 86 DME stations in Germany, without the

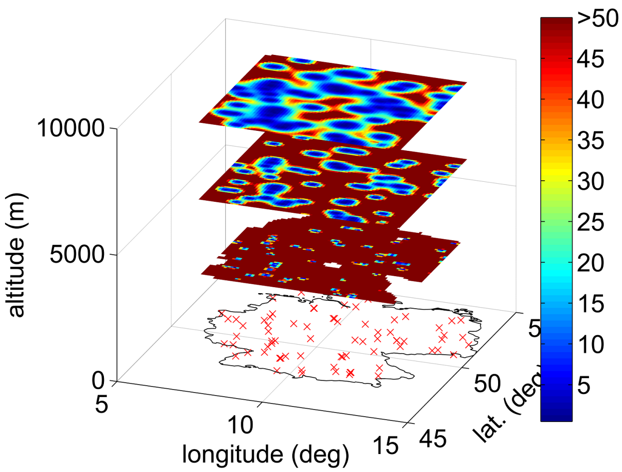

Fig. 3 GDOP with currently used DME stations in Germany

ones used to substitute the beacons for ILS. So we investigated the situation with a thinned out station network and used just 33 stations close to airports (including small regional airports). This could be a sufficient set of stations for a communication system, such as LDACS.

At an elevation of 10000ft AMSL we observe that the cov-

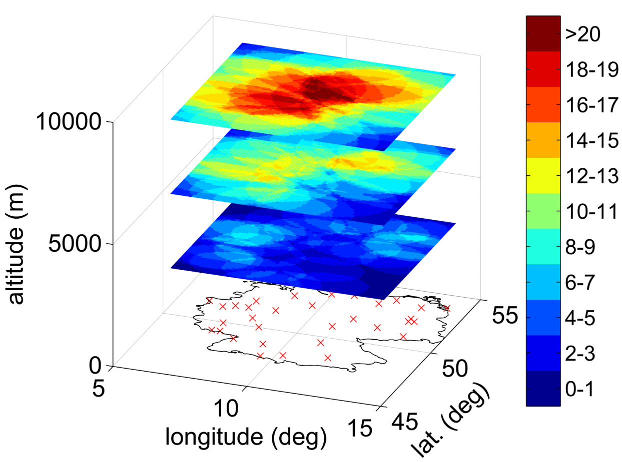

Fig. 4 Number of visible stations with one ground station per airport in Germany

erage is seriously degrading, and availability problems may accur in large areas. But thou there are much less stations, we still see a horizontal DOP below 2 at 20000ft and above (see Figure 5).

The global DOP degrades also further (see Figure 6),

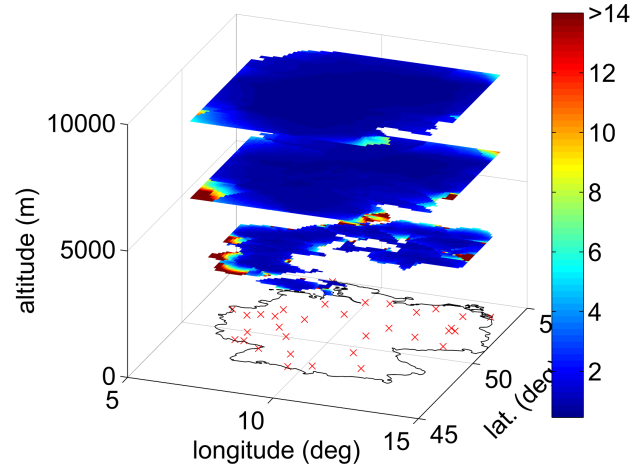

Fig. 5 HDOP with one ground station per airport in Germany

showing that there would not be any availability for 3-dimensional positioning at FL100 in wide areas.

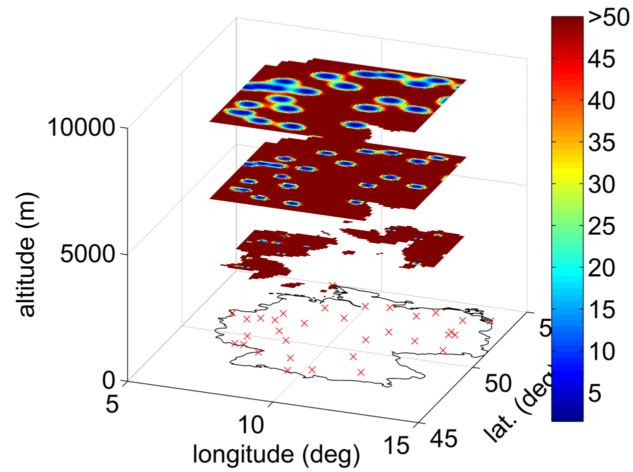

Fig. 6 GDOP with one ground station per airport in Germany

RANGE ERROR MODELS

In 2012 a flight trial with the LDACS communication signal was conducted to prove that this signal can also be used for ranging [7]. From the measured data we took a several samples at different altitudes showing nominal behavior. Then we derived a model for the ranging error by overbounding the estimated error in these samples using a normal distribution. We could improve our range measurements by applying a carrier phase smoothing filter (also called Hatch filter [3]) with different smoothing times. After the smoothing the distribution of the range error is not similar to a normal distribution anymore, but a Gaussian overbound can be constructed to model an upper bound for the ranging error. With this method we get the standard deviations for different altitudes considered during the measurement campaign and different smoothing constants shown in Table 1 were evaluated. Figure 7 shows the ranging error measured to one of the stations during a sample recorded at about 8.5km (AMSL) which is approximately flight level 280.

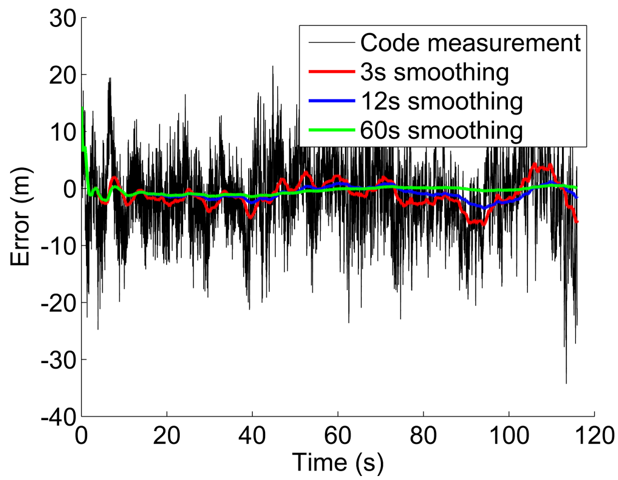

Fig. 7 Ranging Errors measured at altitude 8.5km AMSL

altitude	Code	3s	12s	60s
3.0 km	370.9m	11.7m	3.6m	1.1m
8.5 km	17.4m	4.3m	2.1m	1.1m
11.5 km	10.0m	3.1m	2.1m	1.5m

Table 1 Modeled standard deviation of ranging errors

CONVERGENCE OF ITERATIVE ALGORITHMS

The iterative Gauss-Newton algorithm, often also called Newton-Raphson algorithm, has shown to converge to wrong solutions as well as to diverge in our simulations and with the recorded data from our flight trial. The existence and the size of an area of convergence around the optimal solution of the pseudorange equations depends on the geometry of the visible stations, how fast the geometry changes in the area around the position solution and how good the solution fits the measured pseudoranges. With the Gauss-Newton method we want to minimize the following function

$$F : \mathbb{R}^4 \to \mathbb{R}^N : \begin{pmatrix} \mathbf{x} \\ b \end{pmatrix} \mapsto (\rho_i - \|\mathbf{s}_i - \mathbf{x}\| - b)_{1 \le i \le N}$$

where N denotes the number of stations and $\mathbf{s}_i$ the coordinates of each station and ρ_i the measured pseudorange from the receiver to the station at $\mathbf{s}_i$. We will now write $\mathbf{z} = (\mathbf{x}^T, b)^T$. We usually minimize F by finding a fixed point of

$$\Phi(\mathbf{z}) = \mathbf{z} - \underbrace{(\mathbf{G}(\mathbf{z})^T \mathbf{G}(\mathbf{z}))^{-1}}_{=:\mathbf{H}(\mathbf{z})} \mathbf{G}(\mathbf{z})^T F(\mathbf{z})$$

where $\mathbf{G}(\mathbf{z})$ is the Jacobian matrix of F and also the so called geometry matrix for the location $\mathbf{x}$. To guarantee the existence of a positive radius of convergence to a fixed point $\mathbf{z}^*$ of Φ we need to show that [6, p. 220]

$$\|\Phi'(\mathbf{z}^*)\| = \|\mathbf{H}(\mathbf{z}^*)\|_2 \cdot \| \sum_{i=1}^{N} F_i(\mathbf{z}^*) F_i''(\mathbf{z}^*) \|_2 < 1$$

where $F_i(\mathbf{z})$ is the i-th coordinate of $F(\mathbf{z})$, and F_i'' is the Jacobian matrix of this coordinate. For this it is sufficient to show that

$$\mathrm{DOP}^2 \sum_{i=1}^{N} \frac{\rho_i - (\|\mathbf{s}_i - \mathbf{x}\| + b)}{\|\mathbf{s}_i - \mathbf{x}\|} < 1.$$

DIRECT ALGORITHM

APNT is not the only application for which the geometry of the constellation makes a position determination unusually hard to achieve. Deep space navigation is an other such application, for which different direct positioning algorithms were developed e.g. the algorithm by Krause [4] or by Bancroft [1]. In this paper we select the second algorithm as it is capable of handling more than four pseudo ranges, but it is not possible to directly handle an altitude sensor information like the barometric altimeter. For the horizontal only solution we project the ranges into a plane and transform the 3 D problem to a 2 D problem. We will briefly describe the direct algorithm we used in our simulations, for a detailed description of the algorithm see [1]. With the notation used in the previous section we have

$$\mathbf{A} := \begin{pmatrix} \mathbf{s}_1 & \rho_1 \\ \vdots & \vdots \\ \mathbf{s}_n & \rho_n \end{pmatrix}.$$

With

$$\mathbf{B} := \mathbf{A}^+ \text{ the pseudo inverse of } \mathbf{A}$$

and

$$\mathbf{J} := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

and

$$r_i := \frac{1}{2} \mathbf{s}_i^T \mathbf{J} \mathbf{s}_i$$

we define

$$E := \begin{pmatrix}1 & \dots & 1\end{pmatrix} \mathbf{B}^T \mathbf{J} \mathbf{B} \begin{pmatrix}1 \\ \vdots \\ 1\end{pmatrix}$$

$$F := \begin{pmatrix}1 & \dots & 1\end{pmatrix} \mathbf{B}^T \mathbf{J} \mathbf{B} \begin{pmatrix}r_1 \\ \vdots \\ r_n\end{pmatrix} - 1$$

$$G := \begin{pmatrix}r_1 & \dots & r_n\end{pmatrix} \mathbf{B}^T \mathbf{J} \mathbf{B} \begin{pmatrix}r_1 \\ \vdots \\ r_n\end{pmatrix}.$$

Let λ_1 and λ_2 be the solutions of the quadratic equation

$$E\lambda^2 + 2F\lambda + G = 0$$

then either $\mathbf{z}_1$ or $\mathbf{z}_2$ with

$$\mathbf{z}_i = \begin{pmatrix}\mathbf{x}_i \\ b\end{pmatrix} = \lambda_i \mathbf{B} \begin{pmatrix}1 \\ \vdots \\ 1\end{pmatrix} + \mathbf{B} \begin{pmatrix}r_1 \\ \vdots \\ r_n\end{pmatrix}$$

is the position and timing solution. The right solution $\mathbf{y}_1$ or $\mathbf{y}_2$ has to be selected by verifying that it is sufficiently satisfying the pseudorange equations. The performance of the algorithm mainly depends on the accuracy of the pseudo range measurements, because it solves an other minimization problem than the classical Newton-Raphson algorithm:

$$\begin{pmatrix}\mathbf{x} \\ b\end{pmatrix} = \arg\min_{z \in \mathbb{R}^4} \sum_{i=1}^{N} \left((\rho_i - b)^2 - \|\mathbf{x} - \mathbf{s}_i\|^2\right)^2 \qquad (1)$$

and not

$$\begin{pmatrix}\mathbf{x} \\ b\end{pmatrix} = \arg\min_{z \in \mathbb{R}^4} \sum_{i=1}^{N} \left(\rho_i - b - \|\mathbf{x} - \mathbf{s}_i\|\right)^2. \qquad (2)$$

Therefore the solution is not optimal in the same sense as the solution computed by the Newton-Raphson algorithm.

HYBRID ALGORITHM

The algorithm presented in this section combines the two algorithms above, by computing the position using the direct method by Bancroft [1] and then taking it as an initial position vector for the Newton-Raphson method. So we (usually) get a sufficiently good initial vector for the Gauss-Newton algorithm to converge to the right position solution. One advantage of this method is that the position solution is optimal in the sense of Equation 2. Furthermore the propagation of the distribution of the ranging error to the position error and the horizontal protection level can be computed in the same way as for the Newton-Raphson method.

PERFORMANCE ANALYSIS

To analyse the performance of the algorithms in different situations we look at the results of simulations as well as theoretical computations. We describe the quality of the position solution derived for simulations by computing an overbounding normal distribution. For the horizontal positioning error we use the principal component analysis to estimate the dominant error direction. Then we compute a one-dimensional Gaussian overbound to the distribution of the error in this direction. The standard deviation σ of this distribution is then taken as σ_{pos} for the horizontal positioning error. For all our simulations we used 100 sample range errors for each station.

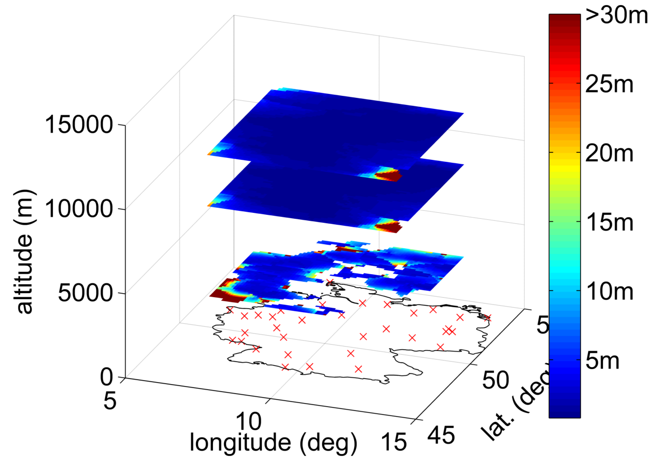

Fig. 8 σ_{bound} for σ_{range} deduced smoothing with 12s time constant see Table 1

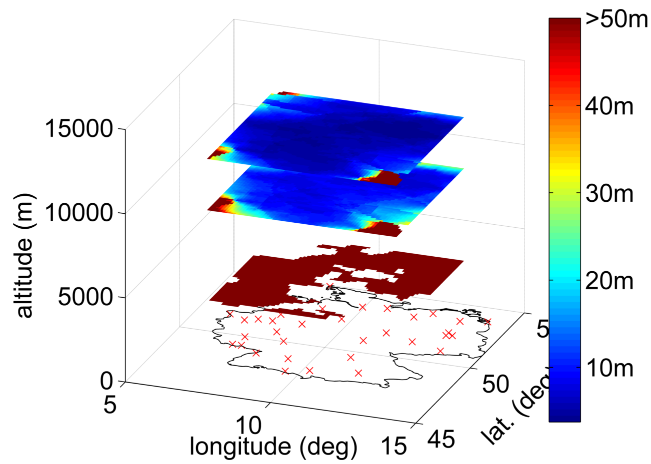

Fig. 9 σ_{bound} for σ_{range} deduced from ranging with code measurements see Table 1

For the Newton-Raphson algorithm we can compute a horizontal standard deviation σ_{bound} overbounding the horizontal error, depending only on the geometry and the standard deviation of the ranges σ_{range}.

$$\sigma_{bound} = \sqrt{\frac{H_{1,1} + H_{2,2}}{2} + \sqrt{\left(\frac{H_{1,1} - H_{2,2}}{2}\right)^2 + H_{1,2}^2}}$$

This leads to a similar result as σ_{pos} for the hybrid algorithm, as the final Position computation is done with the Newton-Raphson method and is shown in Figures 8 and 9.

High Ranging Accuracy

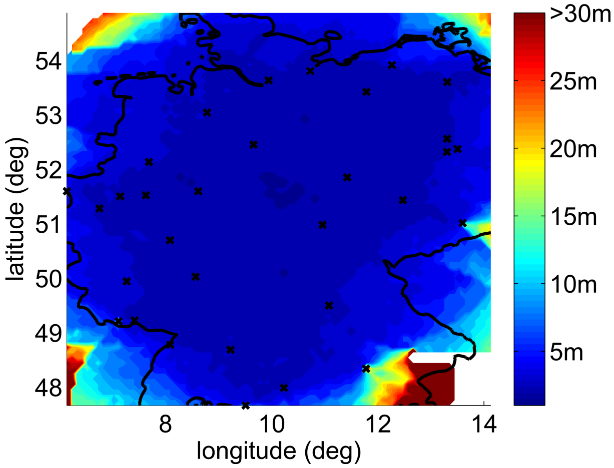

Fig. 10 σ_{pos} for high ranging accuracy ($\sigma_{range} = 2.1$m), with one ground station per airport at altitude 8.5km (AMSL) using the hybrid algorithm.

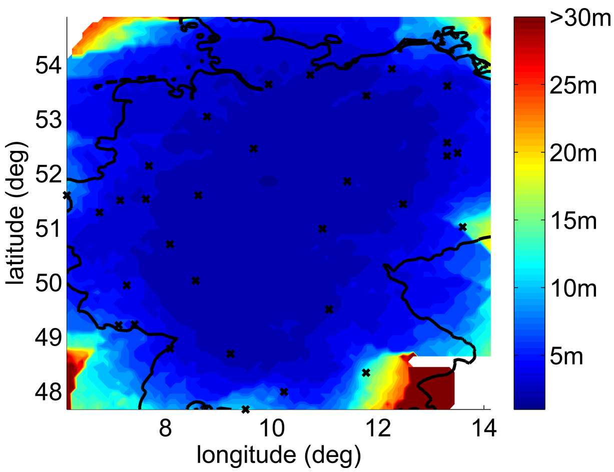

Fig. 11 σ_{pos} for high ranging accuracy ($\sigma_{range} = 2.1$m), with one ground station per airport at altitude 8.5km (AMSL) using the direct method.

First we will compare the performance of the hybrid and the direct positioning methods for a high ranging accuracy and known altitude. We will do so by simulation at an altitude of 28000ft (flight level 280) with one station per airport in Germany. As standard deviation for the ranging error we use 2.1m (overbound of the 12s smoothed ranges at this altitude). The resulting σ_{pos} is shown in Figures 10 and 11.

Low Ranging Accuracy

Secondly we will compare the performance of the hybrid and the direct positioning method for a lower ranging accuracy at known altitude. We will do so by simulation at an altitude of 28000ft (flight level 280) with one station per airport in Germany. As standard deviation for the ranging error we use 17.4m (overbound of the ranges from code measurements at this altitude).The resulting σ_{pos} is shown in Figures 12 and 13.

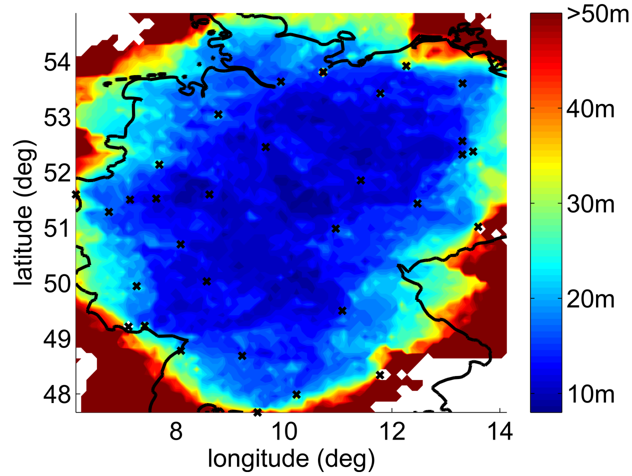

Fig. 12 σ_{pos} for low ranging accuracy ($\sigma_{range} = 17.4$m), with one ground station per airport at altitude 8.5km (AMSL) using the hybrid algorithm.

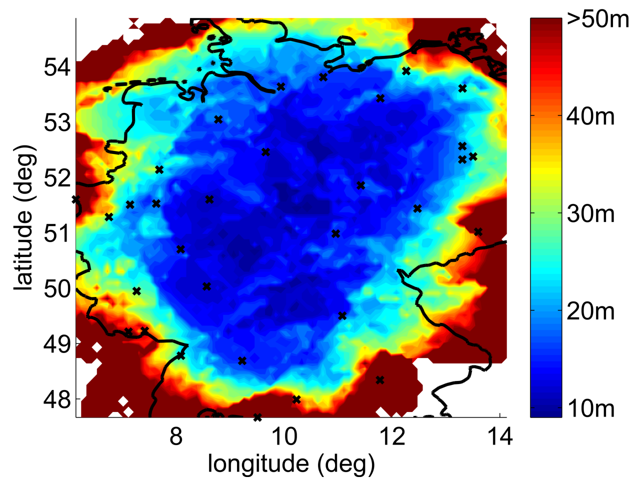

Fig. 13 σ_{pos} for low ranging accuracy ($\sigma_{range} = 17.4$m), with one ground station per airport at altitude 8.5km (AMSL) using the direct algorithm.

At Low Altitudes

At about 10000ft (flight level 100) less stations are visible and the ranging performance is worse than at higher altitudes. So positioning with the ranges determined through the code measurement is hardly possible, if at all. So we included all DME stations in this simulation and chose to use the standard deviation of the overbounding distribution of the 12s smoothed ranges and we consider the altitude as known. As the results for the direct and the hybrid algorithms are very similar we show only the results of the hybrid algorithm in Figure 14.

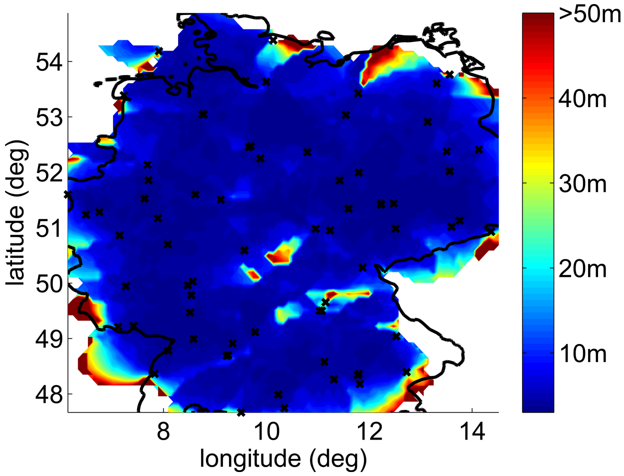

Fig. 14 σ_{pos} for high ranging accuracy ($\sigma_{range} = 3.6$m), with all DME stations at altitude 3.0km (AMSL) using the hybrid algorithm.

3D Positioning

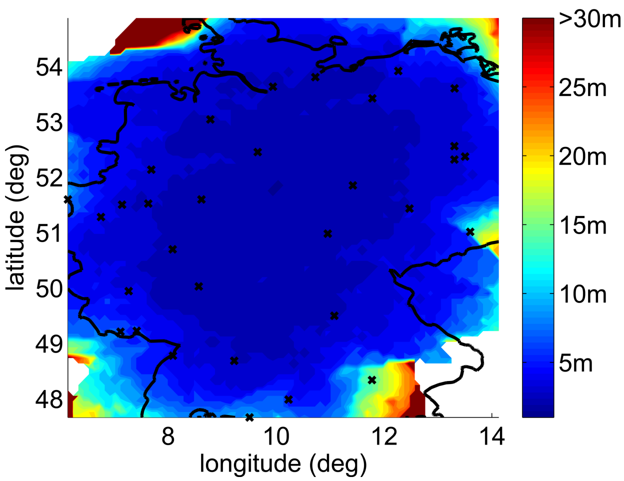

Fig. 15 σ_{pos} for high ranging accuracy ($\sigma_{range} = 2.1$m), with one ground station per airport at altitude 8.5km (AMSL) using the direct algorithm for 3D Positioning.

Finally we analyze the horizontal positioning error of a 3-dimensional positioning solution. Which can be computed if four or more stations are in view, but without the use of a barometric altimeter. In Figures 15 and 16 we show the overbounding σ_{pos} for the horizontal position error for a 3-dimensional position solution. First for the direct method with high ranging accuracy, second for the hybrid algorithm with lower ranging accuracy.

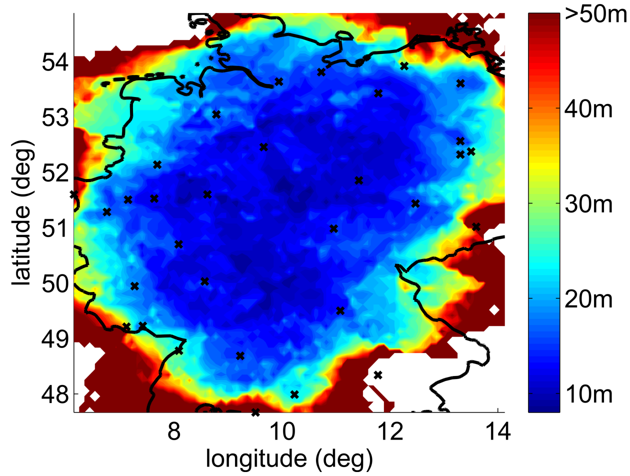

Fig. 16 σ_{pos} for low ranging accuracy ($\sigma_{range} = 17.4$m), with one ground station per airport at altitude 8.5km (AMSL) using the hybrid algorithm for 3D Positioning.

EVALUATION WITH FLIGHT TRIAL DATA

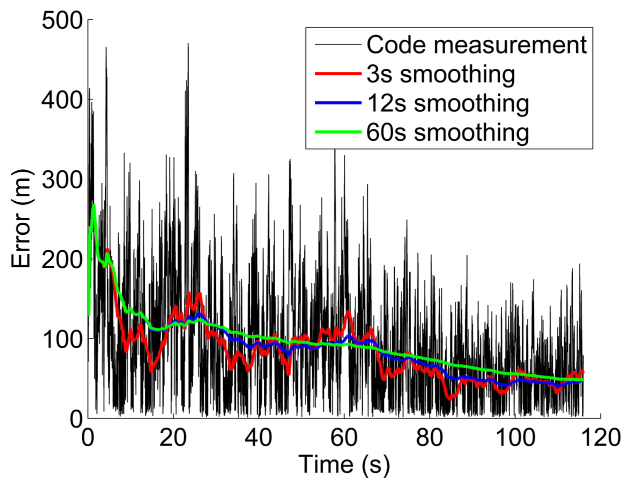

Fig. 17 Position error at 8.5km (AMSL)

We also tested these algorithms with the ranges measured during our flight trials in 2012 [7] and compared the computed position with the reference position determined with GPS. In Figure 17 and 18 we see the total and the horizontal position error when using the hybrid algorithm for 3-dimensional positioning with the range data recorded at approximately 8500m above MSL. We used different smoothing constants for the filtering of the ranges to show the impact of the different errors in the range measurements (see Table 1) on the positioning.

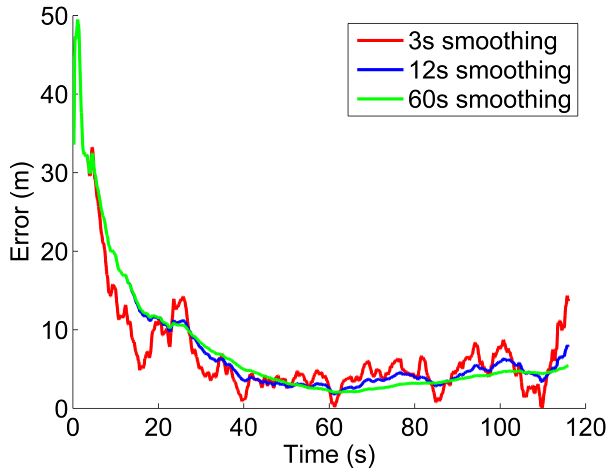

Fig. 18 Horizontal position error at 8.5km (AMSL)

CONCLUSIONS

In this paper we did an analysis of the positioning algorithm performance for a ground based APNT system. The difficult geometric conditions can be overcome by direct and hybrid algorithms. With these algorithms we prevent convergence problems and get high performance and accuracy for horizontal positioning. The coverage can be extended by using an additional baro-altimeter, whereby the number of necessary stations can be reduced to three. But even despite the high geometric dilution of precision a 3-dimensional position solution can be computed whenever enough stations are visible and a good ranging performance is assumed.

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