Introduction

- Developments in space technology have paved the way for more challenging missions which require advanced guidance and control algorithms for safely and autonomously landing on celestial bodies.
- Instant determination of hazards, automatic guidance during landing maneuvers and likelihood maximization of a safe landing are of paramount importance.
- Reachability analysis is used to obtain attainable landing areas for the final phase of interplanetary space missions given initial conditions, admissible control inputs and landing constraints.

Problem Statement

Equations of motion of the moon lander are taken from [1]. The vector of states and control inputs is defined as follows:

\[ \mathbf{x}(t) = (d, h, c, d, h, c, \beta, x, m)^T \quad \mathbf{u}(t) = (T_u, T_v, T_q, \omega_p, \omega_x)^T \]

Initial condition and terminal conditions:

\[ \mathbf{x}(0) = (d_0, h_0, c_0, 0, h_0, 0, \beta_0, x_0, m_0)^T \]
\[ \mathbf{x}(T_f) = (0, 0, 0, 0, 0, 0, 0, 0, 0)^T \]

Condition for successful landing:

\[ \Delta x_{\text{max}} = \sqrt{(1 m/s, 1 m/s, 0, free, 1 m/s, free, 10^6, 180^\circ, free)^T} \]

Reference Frame: Downrange-Crossrange-Altitude (DCA)

Results

- Attainable landing area and propellant & time cost map of associated region is computed using reachability analysis
- Initial discretization of state space
- Discretize region of interest
- Find optimal control law that steers the system from the initial condition to the target state
- Approximate the reachable set with an error of discretization step by solving following optimal control problem (OCP) for each grid point

\[ \text{Min } \frac{1}{2} \| x(t_f) - x_0 \|_2^2 \]
\[ \text{s.t. } x(t) = f(x(t), u(t)) \quad a.e. \text{ in } [t_0, t_f] \\
\]

Method

In this study, we apply an optimal-control-based algorithm for approximating nonconvex reachable sets of nonlinear systems [2].

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\]

Discretization of Optimal Control Problem

OCP is transcribed into NLP by SPARTAN (SHExFEX-3 Pseudospectral Algorithm for Reentry Trajectory AnalYsis) [3].

- Non-uniform collocation points to avoid the Runge phenomenon
- Exponential convergence w.r.t. number of collocation points
- Lagrange polynomials for interpolation of states, control inputs
- Jacobian of the associated NLP J is expressed as sum of 3 different contributors. Resulting sparse J is used by commercial off-the-shelf SQP solver.

\[ \text{lim}(J) = \left[ n(n+1) + 1 \right] \times \left[ (n+1)n + n(n+1) + 1 \right] \]

References: