Emulation of Multipath Fading Channels using the Monte Carlo Method

Peter Hoeher*

Information and Coding Theory Lab

University of Kiel

Kaiserstr. 2

D-24143 Kiel, Germany

Tel./ Fax.: +49 431 77572-402/ -403

Email: ph@techfak.uni-kiel.de

Alexander Steingaß

Institute for Communications Technology

German Aerospace Center (DLR)

P.O. Box 1116

D-82230 Oberpfaffenhofen, Germany

Tel./ Fax.: +49 8153 28-2864 /-1442

Email: Alexander.Steingass@dlr.de

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Abstract

A wide-sense-stationary uncorrelated scattering (WSSUS) multipath fading channel emulator based on the Monte Carlo method is proposed and compared with known stochastic and deterministic models. The model is intuitive, flexible, and suitable for implementation on a digital computer or in hardware. By making use of "controlled randomness", improved performance versus complexity can be achieved compared to known emulators. Some state-of-the-art channel models are reviewed.

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1 Introduction

MOBILE radio communication channels are well-modeled as linear time-varying multipath channels [1]-[4], [5, Chapter 11]. The simplest non-degenerate class of processes which exhibits uncorrelated dispersiveness in propagation delay and Doppler shift is known as the *wide-sense stationary uncorrelated scattering (WSSUS) channel*, introduced by Bello [1]. Any WSSUS process is completely characterized by the two-dimensional probability density function of the propagation delays and Doppler shifts, the so-called scattering function.

Numerous models based on Rice's sum of sinusoids have been proposed for emulating WSSUS processes, see for example [3, Chapter 1.7], [6]-[14]. These models are suitable for computer simulations or hardware implementations.

A decade ago, Schulze presented a *Monte Carlo model* (MC model) [8]. His model is recognized as being intuitive, flexible, and easy to implement. A single random parameter set approximating the scattering function is generated before the simulation run. In [10] (and Ref. [8] therein), we proposed to apply Schulze's model with multiple random parameter sets, and we derived the corresponding *equivalent discrete-time MC model*, which has been refined in [11]. Pätzold *et al.* recently have claimed that *deterministic models* are better than Schulze's MC model with respect to higher order statistical properties (such as the level crossing rate) [12]-[14]. However, other problems may be created with deterministic models as we will highlight in this paper.

In Section 2, we will (i) review the models by Schulze and Pätzold *et al.* and (ii) propose a new model, which is based on a Monte Carlo method with "controlled randomness". Thus, the performance-complexity trade-off can be improved with respect to state-of-the-art techniques as shown in Section 3. Finally, the conclusions are drawn in Section 4. The principles discussed here can easily be extended to obtain the corresponding equivalent discrete-time MC model, see [10, 11].

2 Emulation of the WSSUS Channel

Throughout the paper we make use of the complex baseband notation. Let $h(\tau, t)$ denote the impulse response of a WSSUS channel, where τ is the propagation delay and t is the absolute time. The autocorrelation function (acf) of $h(\tau,t)$ is given by

$$R_{hh}(\tau,\Delta t) = E[h(\tau,t)h^*(\tau,t+\Delta t)]. \tag{1}$$

The scattering function is obtained by taking the Fourier transform with respect to Δt:

$$S_h(\tau,f_D) = \int_{-\infty}^{\infty} R_{hh}(\tau,\Delta t)e^{-j2\pi f_D \Delta t}\, d\Delta t. \tag{2}$$

The scattering function is proportional to the two-dimensional probability density function $p(\tau,f_D)$ of the propagation delay, τ, and the Doppler shift, f_D [8, 10], where $0 \le \tau < \tau_{max}$ and $|f_D| < f_{D_{max}}$. The Doppler power spectrum and the delay power spectrum are obtained by integrating the scattering function with respect to τ and f_D, respectively. These results will be used in the following.

2.1 Known Monte Carlo Models

Schulze proved that

$$h(\tau,t) = \lim_{N\to\infty} \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \exp(j(\underbrace{\theta_n}_{phase} + \underbrace{2\pi f_{D_n} t}_{Doppler})) \cdot \underbrace{\delta(\tau-\tau_n)}_{delay} \tag{3}$$

provides an exact representation of the WSSUS channel for any given scattering function [8]. The continuous random variables θ_n $(0 \le \theta_n < 2\pi)$, τ_n $(0 \le \tau_n \le \tau_{max})$, and f_{D_n} $(-f_{D_{max}} < f_{D_n} < f_{D_{max}})$ must be generated according to the desired probability density functions $p(\theta) = 1/(2\pi)$ and $p(\tau,f_D) \sim S_h(\tau,f_D)$. An intuitive interpretation of (3) is that $h(\tau,t)$ is an incoherent superposition of N independent complex-valued echoes, where each echo is characterized by a random phase, θ_n, a random delay, τ_n, and a random Doppler shift, f_{D_n}, $1 \le n \le N$. Due to the factor $1/\sqrt{N}$ the average power is one. According to the central limit theorem the quadrature components of $h(\tau,t)$ are zero mean Gaussian random variables having the same variance. Therefore, $|h(\tau,t)|$ is Rayleigh distributed[1], if the quadrature components would be statistically independent. A good approximation is obtained for $N \ge 7$ [3, Chapter 1.7]. The time-selectivity of the channel is determined by the f_{D_n}'s and the frequency-selectivity is determined by the τ_n's, respectively. Flat fading is the special case

[1] A Rician channel may be modeled by adding a constant to the scattered component [8].

where $\tau_n = 0 \;\; \forall n$. The corresponding discrete version is obtained by setting $t = kT_s$, where $1/T_s$ is the sampling rate and k is the time index. For example, $1/T_s$ may be twice the symbol rate $1/T_{sym}$.

The different channel emulation techniques under investigation in this paper are based on (3), and fall into two model classes depending on how often the parameter set $\{\theta_n, f_{D_n}, \tau_n\}$ is generated:

- *"Single parameter-set Monte Carlo model"* (SPS MC model): A random parameter set is generated a priori, i.e., before the simulation run [8].
- *"Multiple parameter-set Monte Carlo model"* (MPS MC model): New random parameter sets are generated from time to time during the simulation, e.g. once for every frame or data block [10].

Emulators based on the MPS MC model are superior in that for similar statistics of the impulse response, $h(\tau, t)$, the computational effort can be improved. Example: Let N_s be the total number of parameter sets. For an MPS MC model with $N = 7$, $N_s = 10000$, and a block length of about 100 or more, the "real-time" complexity is similar to a SPS MC model with $N = 7$. However, to realize the impulse response with equivalent statistical properties, the SPS MC model would require $N = 70000$!

The SPS MC model outputs a periodic fading process (at least when the f_{D_n}'s are quantized) and produces a line delay-Doppler power spectrum. For an infinite number of parameter sets the MPS MC model, however, perfectly matches the desired scattering function and is non-periodic. Generating new parameter sets from time to time may be interpreted as perfect time or frequency hopping [10]. As opposed to SPS models and classical channel emulators based on filtering techniques, there is no lower limit on the Doppler spread. Unfortunately, the MPS MC model creates discontinuities in the temporal correlation. As a consequence, in the receiver the channel estimation or carrier recovery must be re-acquired after each draw of random parameters. We solve this drawback by puncturing a sufficient number of consecutive symbols in the beginning of each frame. For example, in a DPSK system we remove the first symbol from a bit error evaluation. The corresponding loss is generally over-compensated by the efficiency of the MPS MC approach.

An objectionable side-effect of Schulze's approach, Eqn. (3), may be the correlation between the quadrature components of $h(\tau, t)$, which occurs when the Doppler spread is non-zero and the number of echoes is finite. The cross-correlation function for flat fading is

$$R_{h_I h_Q}(\Delta t) = \frac{1}{2N'} \sum_{n=1}^{N'} \sin(2\pi f_{D_n} \Delta t), \tag{4}$$

where $N' = N \cdot N_s$. The cross-correlation is zero for $\Delta t = 0$ and converges to zero for $N' \to \infty$. Since N_s is large for MPS MC models, the cross-correlation is virtually zero. Pätzold *et al.* have avoided this possible problem by using N_I inphase components and, e.g., $N_Q = N_I + 1$ quadrature-phase components together with a deterministic parameter set [12]-[14]. This modeling may violate physical principles, but the cross-correlation function is always zero [12] as desired.

2.2 Generation of the Parameter Sets

As discussed in [10], θ_n, f_{D_n}, and τ_n may be generated by the well known transformation method:

$$v_n = g_v(u_n) = P_v^{-1}(u_n), \qquad 1 \leq n \leq N, \tag{5}$$

where v_n is a substitute for θ_n, f_{D_n}, and τ_n, respectively, $\{u_n\}$ is a set of random, uniformly distributed input variables produced by a random number generator ($u_n \in (0,1)$), and $g_v(u_n)$ is a memoryless nonlinearity, which is the inverse of the desired cumulative distribution function $P_v(v) = \int_{-\infty}^{v} p_v(\nu)\, d\nu$.

For illustration purposes, consider the Doppler power spectrum corresponding to two-dimensional isotropic scattering:

$$p_{f_D}(f_D) = \begin{cases} \frac{1}{\pi f_{D\,max}\sqrt{1-(f_D/f_{D\,max})^2}} & \text{if } |f_D| < f_{D\,max} \\ 0 & \text{else,} \end{cases} \tag{6}$$

where $f_{D_{max}}$ is the maximum Doppler frequency [2, 3]. Application of (5) gives

$$f_{D_n} = g_{f_D}(u_n) = f_{D\,max} \cdot \cos(\pi u_n). \tag{7}$$

An example for the delay power spectrum is shown in [10].

2.3 New Monte Carlo Model with Controlled Randomness

The statistics of the MC models depend on the particular realizations of the u_n's. If the randomly generated u_n's are not sufficiently uniformly distributed, which may occur when the number of echoes is small, the statistics become poor. To tackle this problem, we now present an efficient modification of the known Monte Carlo models. The goal is to emulate the WSSUS channel using (3), where N is as small as possible.

First, consider the approximation of $p(\tau, f_D)$. In order to improve the statistics, we divide the interval $(0,1)$ into N subintervals of length $1/N$ each. For each subinterval, we generate exactly one random variable u'_n, $1 \leq n \leq N$, where the set of u'_n's is uniformly distributed over $(0, 1/N)$. We define

$$u_n = u'_n + (n-1)/N. \tag{8}$$

Importantly, the set of u_n's is again uniformly distributed over $(0,1)$, as desired[2]. Hence, we have obtained a *Monte Carlo model with "controlled randomness"*. With this approach, the set of output random variables, $\{v_n\}$, more closely approximates the desired scattering function. Objectionable clustering of the parameters is now impossible. As a result of (8), the variates within the two parameter sets $\{\tau_n\}$ and $\{f_{D_n}\}$ are monotonically increasing or decreasing, and a permutation must be applied within one of the two sets. The principle of "controlled randomness" is applicable to both SPS MC models and MPS MC models. We favor the latter version for the reasons discussed in Section 2.1.

Now consider the approximation of $p(\theta)$. "Controlled randomness" should *not* be applied for the generation of $\{\theta_n\}$, since the echoes would nearly cancel out for $t = 0$, i.e. given short frames or slow fading conditions, the average power would be less than one. This can be seen best when N is large, because applying (8) in this case would result in the phases θ_n being nearly equi-spread. For the same reason, deterministic models [12]-[14] fail for very slow fading conditions, unless the time index k is shifted, i.e., the simulation is started with $t >> 0$.

[2]The assumption of equally-spaced subintervals may be relaxed, as long as the u_n's are uniformly distributed.

3 Performance versus Complexity Evaluation

The "real-time" complexity of all models under investigation is measured in terms of the number of echoes, N, required to accomplish a certain quality. After a parameter set is generated, only $4N$ additions, $2N$ table look-ups, but no multiplications are necessary in a hardware realization in order to compute one complex output sample. As opposed to classical channel emulators based on filtering techniques [5, Fig. 11-9], MC models are suitable for arbitrarily-shaped scattering functions, are flexible and tunable in the entire range $|f_{D_{max}}| \in (0, 1/(2T_s))$, and do not require acquisition time. Interpolation filters, which typically determine the complexity of classical channel emulators, are not necessary.

The performance criteria considered here are the accuracy of the acf (as proposed in [12]), and the accuracy of the bit error rate (BER) for a practical system (see for example [15]).

3.1 Autocorrelation Function

In the following investigations we want to emulate flat Rayleigh fading with the Doppler power spectrum given in (6). The corresponding desired acf is [3]

$$R_{hh}(\Delta t) = J_0(2\pi f_{D_{max}} \Delta t). \tag{9}$$

Given (3), for a single parameter set the emulated acf is

$$\hat{R}_{hh}(\Delta t) = \frac{1}{N} \sum_{n=1}^{N} \cos(2\pi f_{D_n} \Delta t). \tag{10}$$

For multiple parameter sets, N must be replaced in (10) by $N' = N \cdot N_s$. It is shown in [3] and [12] that the Bessel function, $J_0(.)$, can be represented as a Fourier series:

$$J_0(2\pi f_{D_{max}} \Delta t) = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} \cos(2\pi f_{D_n} \Delta t) \qquad \text{for } 0 \leq |\Delta t| < \infty, \tag{11}$$

where

$$f_{D_n} = f_{D_{max}} \cos\left(\pi \frac{n-0.5}{N}\right), \qquad 1 \leq n \leq N. \tag{12}$$

Note that (10) and (11) are equivalent for $N \to \infty$. For a finite number of echoes, the approximation is excellent if $0 \leq |\Delta t| \leq (N-1)/(4f_{D_{max}})$ and if the f_{D_n}'s are deterministic according to (12). This corresponds to the "method of exact Doppler spread" proposed in [12, 13], which therefore is nearly optimal with respect to the emulation of this particular desired acf. (In [12, 13] only positive Doppler frequencies have been considered, hence we have modified (12) slightly.)

In some applications it is sufficient to match the acf for certain arguments or in a certain range only. For example, for a DPSK signal transmitted over a flat-fading channel and detected by a conventional demodulator, the acf must only match at $t = T_{sym}$, where T_{sym} is the symbol duration. The tail of the acf is not relevant.

In Fig. 1 the acf of the desired Doppler power spectrum, Eqn. (6), and the acf obtained by the "method of exact Doppler spread" [12, 13] are plotted using (3), (10), and (12). The fit is nearly perfect for $f_{D_{max}}|\Delta t| \leq (N-1)/4$ by construction. In Fig. 2 the corresponding curves for the SPS MC model [8] are plotted, where different parameter sets are featured. For $f_{D_{max}}|\Delta t| < 0.3$ (which is the relevant range for most communication systems) the fit is good. In additional simulations, we verified that for MPS MC models (with or without controlled randomness) the emulated acf matches the desired acf for all Δt.

3.2 Bit Error Rate

Let us now investigate the BER of a binary DPSK system with conventional demodulation and assume a Rayleigh flat-fading channel with Doppler power spectrum (6). The intention of using this "simple" scenario is the existence of analytical BER curves, which serve as a reference.

In Fig. 3-5 the BER versus E_b/N_0 is plotted for the deterministic model proposed in [12], for the MPS MC model proposed in [10], and the MPS MC model with controlled randomness proposed here, respectively. E_b is the average energy per information bit and N_0 is the one-sided noise spectral density of an additive white Gaussian noise process. For the MC models, the BER was averaged over 10^5 blocks of length 100. New parameter sets were generated from block to block. In the deterministic model, the BER was simulated for one block of length 10^7; the initial time index was set to $k >> 1$. The theoretical BER of binary DPSK as a function of the fading rate given a perfect timing recovery

[4],

$$P_b = \frac{1}{2}\left(1 - \frac{J_0(2\pi f_{D_{max}} T_{sym})}{1 + N_0/E_b}\right), \tag{13}$$

is also plotted as a benchmark.

We conclude from Fig. 3 that the deterministic model [12] under investigation is poor with respect to the BER performance when $N_I = 10$ and $N_Q = 11$. (Different curves correspond to different permutations of the θ_n's.) This may be surprising, since the model was designed for the tested Doppler power spectrum and since it is known to perform better than the SPS MC model with respect to the acf, the level crossing rate, and the average duration of fades [13]. The MPS MC model [10] performs well for $N = 10$ to 20, see Fig. 4. Just $N = 7$ to 10 echoes (needed to comply with the central limit theorem) are sufficient for this particular scenario when using the MPS MC model with controlled randomness, see Fig. 5. The quality of the SPS MC model [8] depends on the actually generated parameter set: At least $N = 50$ to 100 echoes would be necessary to "guarantee" reliable results.

4 Conclusions

In this paper state-of-the-art channel emulators have been reviewed and a new Monte Carlo model with "controlled randomness" has been proposed. The new multipath fading channel emulator has improved performance versus complexity compared to known Monte Carlo models [8, 10] and deterministic models [12]-[14], particularly for a small number of echoes. For an infinite number of parameter sets the desired scattering function is perfectly matched and the emulator is non-periodic. The model is universal and applicable to arbitrary scattering functions. The concept proposed and investigated here may also be applied for the emulation of stochastic signals such as phase noise in oscillators.

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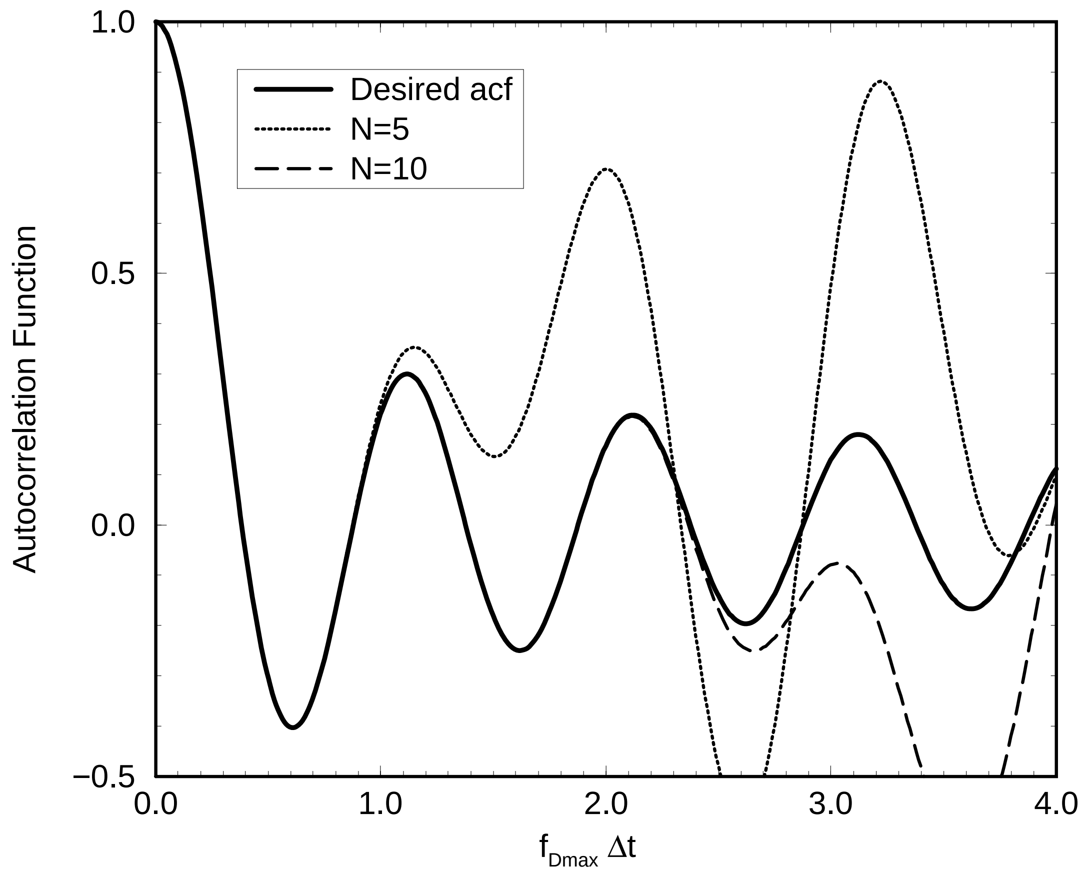

Figure 1: Desired autocorrelation function for 2-D isotropic scattering and its approximation for a deterministic model.

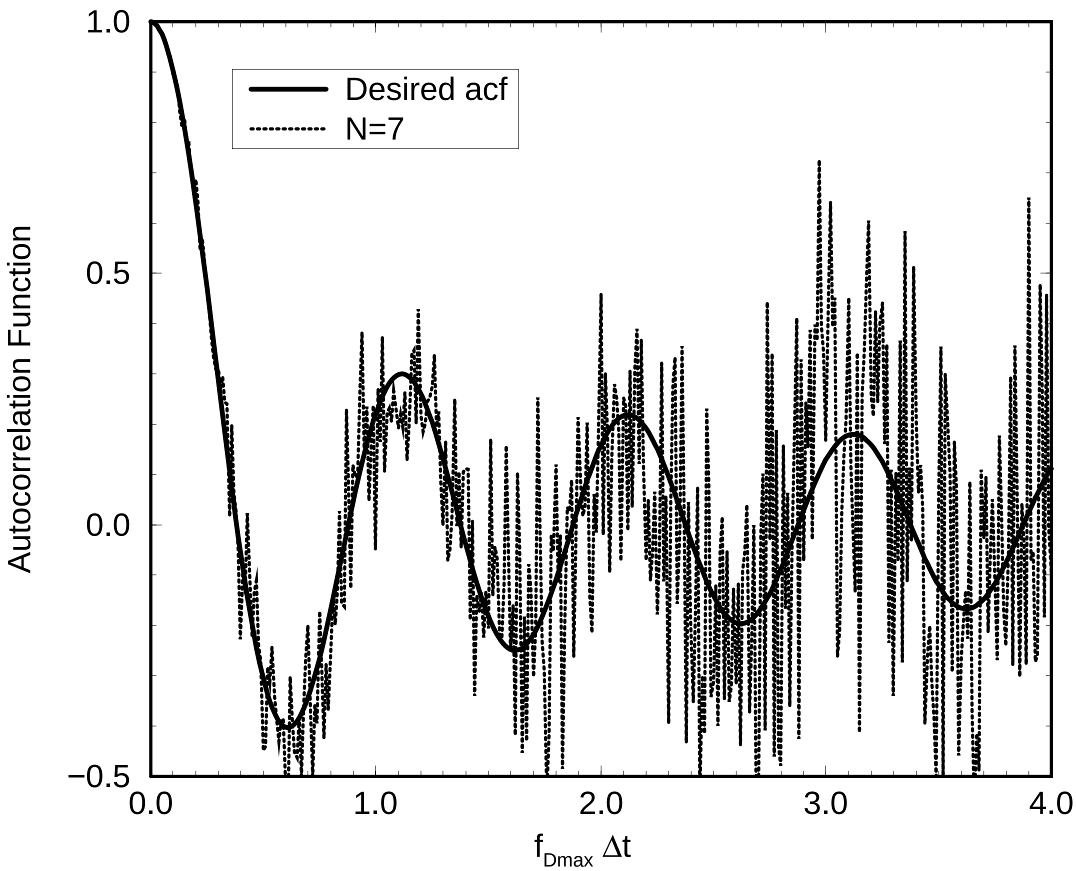

Figure 2: Desired autocorrelation function for 2-D isotropic scattering and its approximation for the "single parameter-set Monte Carlo model" featuring different parameter sets. "Multiple parameter-set Monte Carlo models" approach the desired autocorrelation function when $N \cdot N_s \to \infty$.

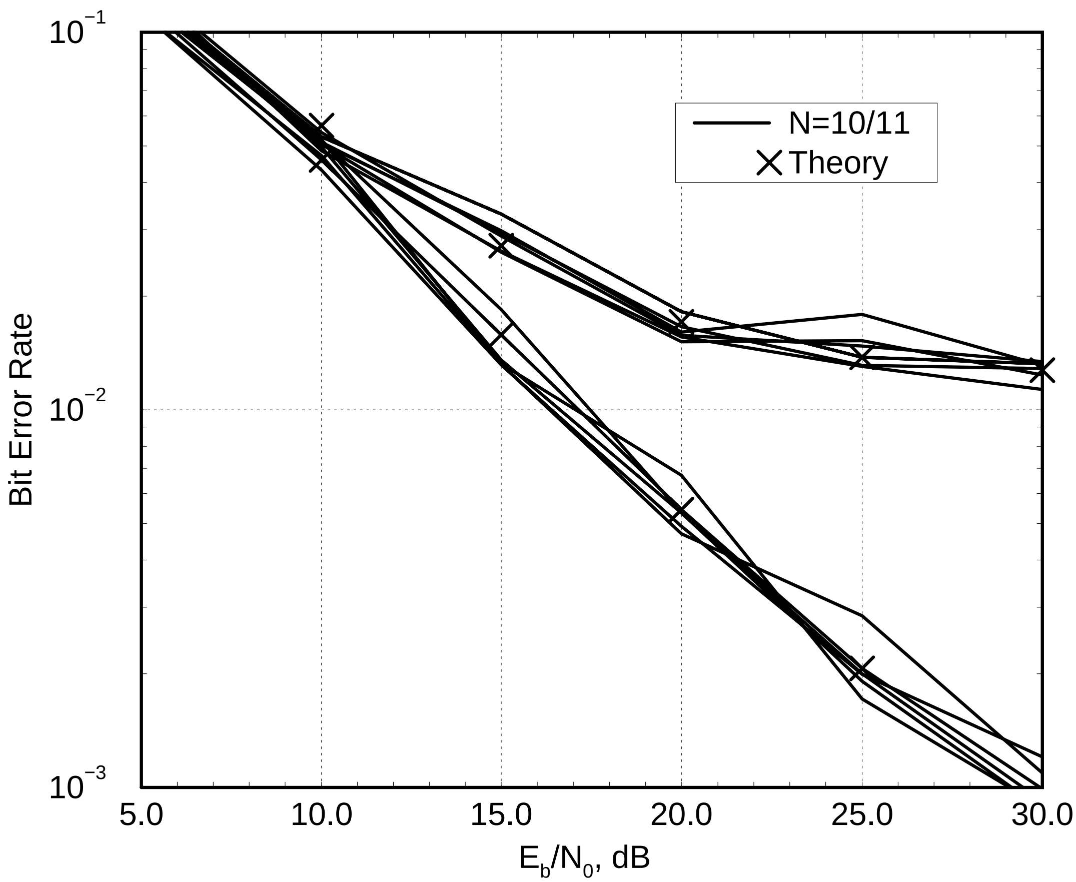

Figure 3: BER versus E_b/N_0 for a deterministic model. (2DPSK, Rayleigh fading, $f_{D_{max}}T_{sym} = 0.01$ and 0.05. Different curves correspond to different permutations of θ_n's.)

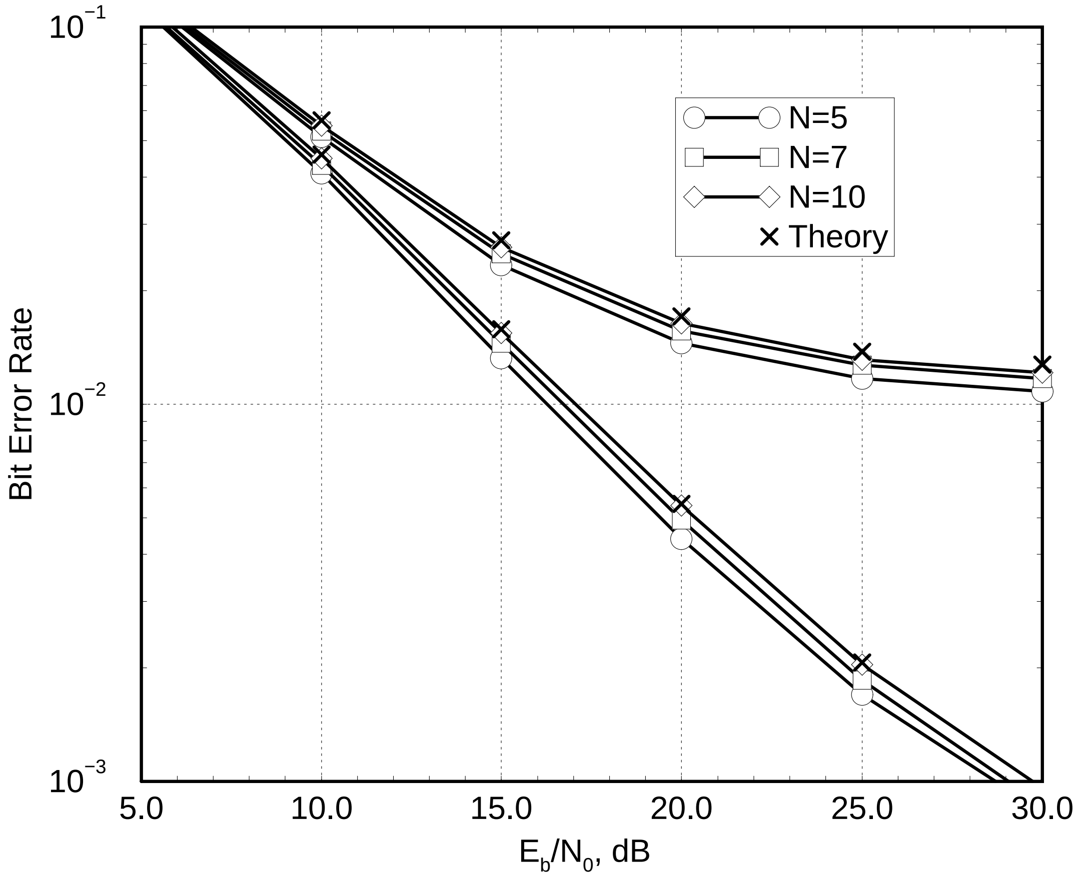

Figure 4: BER versus E_b/N_0 for the "multiple parameter-set Monte Carlo model". (2DPSK, Rayleigh fading, $f_{D_{max}}T_{sym} = 0.01$ and 0.05.)

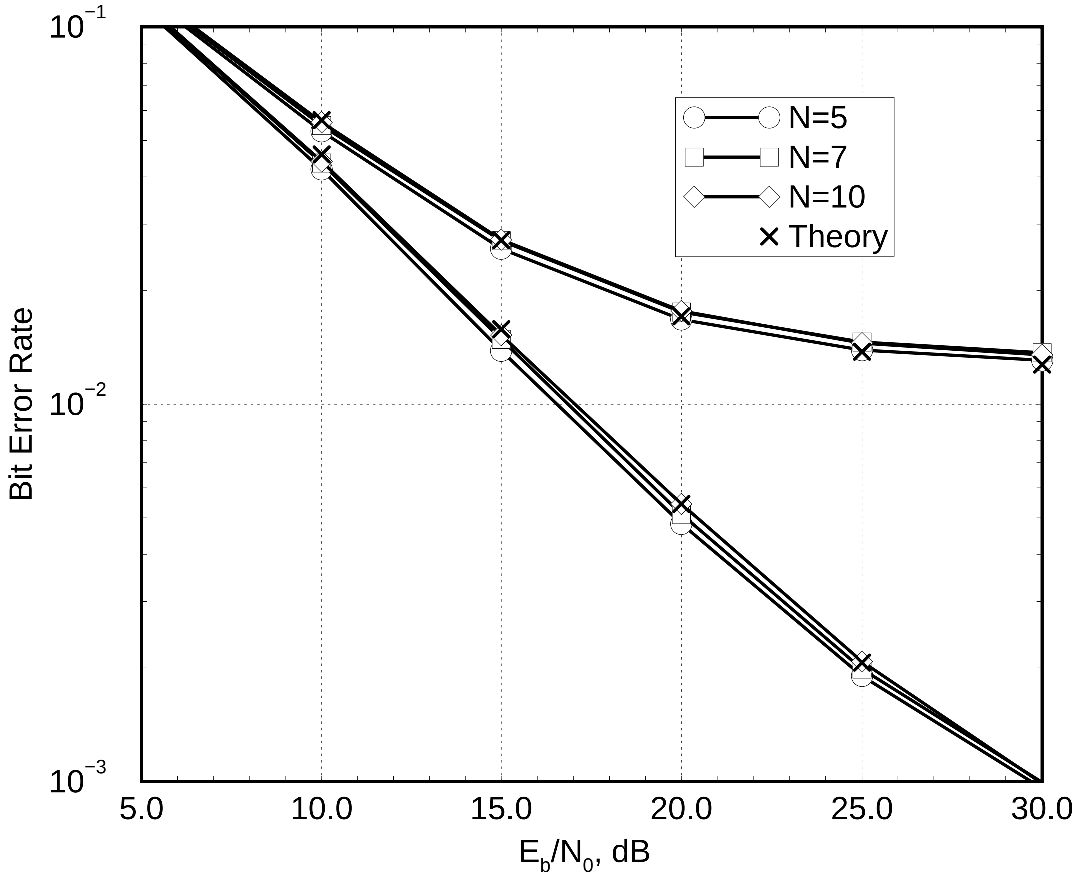

Figure 5: BER versus E_b/N_0 for the "multiple parameter-set Monte Carlo model with controlled randomness". (2DPSK, Rayleigh fading, $f_{D_{max}}T_{sym} = 0.01$ and 0.05.)