Higher Order Multigrid Algorithms for a 2D and 3D RANS-$k\omega$ DG-Solver

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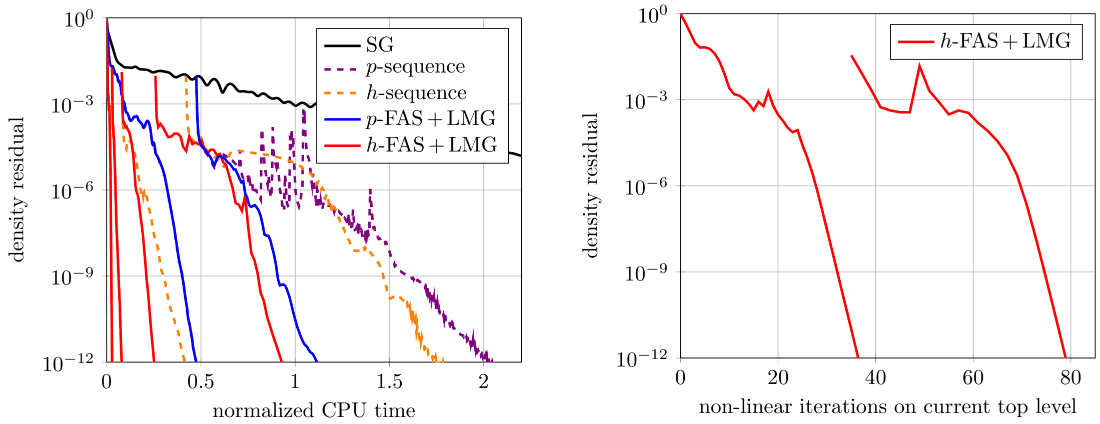
It has been shown that Discontinuous Galerkin (DG) finite element methods are suitable for CFD applications in the area of compressible aerodynamic flows [1, 4, 5, 2]. The Reynolds-averaged Navier–Stokes (RANS) equations in combination with suitable turbulence models such as the k–ω model of Wilcox [7] can still be considered as state-of-the-art for many applications in exterior aerodynamics. One of the main drawbacks of DG methods is the relatively high computation cost per degree of freedom.High Reynolds numbers, the associated highly stretched meshes typically used for an optimal resolution of turbulent boundary layers, and source terms present in the turbulence models all contribute to an increased stiffness of the resulting algebraic system of equations that has to be solved. Implicit operators are generally accepted as key component for increased efficiency of iterative algorithms. Several authors suggested strongly implicit schemes that are close to Newton's method [1, 4]. It is well-known that these implicit methods work best at the end of the iterative procedure in the regime of asymptotic convergence. Employing mesh and order sequencing helps to alleviate this problem by providing a good initial guess.

We will present several strategies to exploit hierarchies of coarse level problems in solver algorithms, including both level sequencing and linear as well as non-linear multigrid variations [6]. Based on either lower order discretizations or agglomerated coarse meshes the resulting algorithms can be characterized as either p- or h-multigrid, respectively. The only difference between these multigrid algorithms is the use of different coarse level DG discretizations and therefore transfer operators. All other ingredients like smoothers, timestep control, usage of a Galerkin-transfer, start up strategy, etc. will stay the same for both kinds of multigrids.

In this work non-linear h- and p-multigrid will be investigated and a linear h- and p-multigrid used as a preconditioner will be introduced. For the non-linear iteration we employ as underlying relaxation scheme a linearized Backward-Euler approach based on local pseudo-time steps that can be considered as a stabilized Newton's method and is also used predominantly as single-level solver [1, 4]. The resulting linear system is solved with a Krylov method. This method is preconditioned either by a line-Jacobi iterative scheme or a linear multigrid using the line-Jacobi scheme as a smoother. We will present 2D numerical examples in order to analyze the performance of the proposed algorithms with respect to both algorithmic convergence properties and run-time behavior in comparison with a single-level Backward-Euler scheme. Results indicate that the best preformacne is achieved with a combination of the linear multigrid as a preconditioner and a non-linear multigrid. Examples include a three element airfoil (L1T2) testcase [3] on a unstructured mesh with 23824 elements. Furthermore, the extension to 3D will be demonstrated on a simple streamlined body (BTC0).



(a) Computation of a $p = 2$ solution for the L1T2 case on an unstructured mesh with 23824 elements.

(b) Computation of a $p = 1$ solution for the 3D BTC0 case on a structured mesh with 6656 elements.

References

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