Mitteilung

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Application of Higher Order Multigrid Algorithms for turbulent flows

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Computational Fluid Dynamics (CFD) methods have advanced substantially in the past decades. Moreover, CFD tools have become essential in the design process and analysis of modern aircraft design. The last decade has seen an interest in high order numerical methods, in particular in the discontinuous Galerkin (DG) Finite Element method. The analysis of turbulent flows employing steady-state computations based on Reynolds-averaged Navier-Stokes (RANS) equations and a turbulence model might be considered as the work-horse in this field. Nevertheless, DG results are relatively rare for this particular application. One of the reasons for this might be the stiffness introduced by both the turbulence model equations and the highly stretched meshes typically used for an efficient resolution of turbulent boundary layers. In order to solve the RANS equations in combination with a turbulence model several authors suggested strongly implicit schemes that are close to Newton's method. A Backward-Euler method in combination with an iterative linear solver can be considered as standard approach to solve a nonlinear set of equations for DG [2].

Here, we focus on a combination of nonlinear multigrid algorithms using strongly implicit schemes as smoothers and linear multigrid algorithms to exploit hierarchies of coarse level problems in solver algorithms [3]. Based on either lower order discretizations or agglomerated coarse meshes the resulting algorithms can be characterized as either p- or h-multigrid, respectively. The only difference between these multigrid algorithms is the use of different coarse level DG discretizations and, therefore, transfer operators. All other ingredients like smoothers, timestep control, usage of a Galerkin-transfer [3], startup strategy, etc. will stay the same for both kinds of multigrid algorithms.

The proposed algorithms will then be applied to the DG discretizations of the steady-state RANS equations in combination with two different turbulence models: the Wilcox-$k\omega$ two equation model [4] and the negativ Spalart-Allmaras one equation turbulence model [1]. Results based on various combinations of multigrid algorithms are shown in comparison to a strongly implicit single grid solver. As a test case we consider the MDA 30P30N configuration which is a 2D high-lift three element airfoil and was recently considered as a test case for the Second International Workshop on High-Order CFD Methods in Cologne on May 2013.

In Figure 1 a comparison in normalized CPU time of a single grid Backward-Euler method and a 2 level nonlinear h-multigrid with a Backward-Euler smoother is shown. These computations are performed on a mesh with 8432 elements. This mesh is agglomerated resulting in an unstructured mesh with 4243 agglomerates. In order to converge a strongly implicit scheme in a satisfying behavior a good initial guess is needed. Therefore, a second order $(p = 1)$ solution of the RANS-$k\omega$ equations is computed with a single grid Backward-Euler solver on the agglomerated mesh with 4243 agglomerates. This converged solution is used as initial solution for a single grid Backward-Euler solver for a third order $(p = 2)$ discretization on the same mesh. These coarse level computations are marked with x in Figure 1. The resulting converged solution is used as initial solution for the algorithms used on the mesh with 8432 elements, represented in Figure 1 with a line which is unmarked and one marked with a •. The only difference between these lines is that the unmarked line uses in every iteration the agglomerated mesh in an h-multigrid sense, whereas the line marked with a • only uses the mesh with a 8432 elements to compute a converged solution.

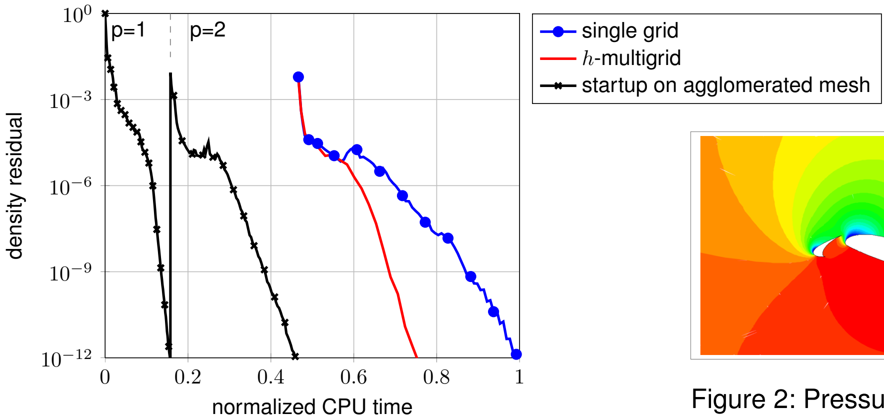

Figure 1: Computation of a third order $(p = 2)$ solution for the MDA airfoil with an h-startup strategy.

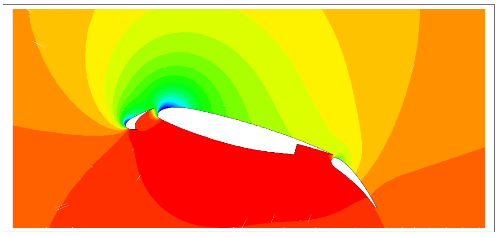

Figure 2: Pressure plot of a third order $(p = 2)$ solution on a mesh with 32k elements.

References

[1] S. Allmaras, F. Johnson, and P. Spalart. Modifications and clarifications for the implementation of the Spalart-Allmaras turbulence model. *ICCFD7*, 1902, 2012.

[2] R. Hartmann, J. Held, and T. Leicht. Adjoint-based error estimation and adaptive mesh refinement for the RANS and k-ω turbulence model equations. *Journal of Computational Physics*, 230(11):4268–4284, May 2011.

[3] U. Trottenberg, C. Oosterlee, and A. Schüller. *Multigrid*. Academic Press, 2001.

[4] D. C. Wilcox. Reassessment of the scale-determining equation for advanced turbulence models. *AIAA Journal*, 26(11):1299–1310, 1988.