Generalized Heat Equation and the Influence of the Leg Geometry on the Performance of a Thermoelectric Element

Knud Zabrocki[1], Gautier Laval[2], Wolfgang Seifert[3], Christophe Goupil[2,4], and Eckhard Müller[1,5]

Classical thermal energy balance equation for thermoelectric devices

Introduction

- Performance of thermoelectric devices – in the framework of continuum theory
- TE effects - interference of two irreversible processes: Heat transport and charge carrier transport
- Onsager-de Groot-Callen theory: Thermoelectrics as a kind of „field theory " in non-equilibrium thermodynamics
- Description via differential equations - thermal energy balance equation

Fig. 1 Influential personalities in macroscopic therory of thermoelectricity – Onsager – de Groot – Callen

Thermal energy balance

- Coupling of Fourier's and Ohm's law
- Transport coefficients - Onsager relations
- Local energy balance $\varrho_d c \frac{\partial T}{\partial t} + \nabla \cdot \dot{\mathbf{q}} = \mathbf{j} \cdot \mathbf{E}$
- Divergence of the heat flux different terms

$$\nabla \cdot \dot{\mathbf{q}} = \tau \, \mathbf{j} \cdot \nabla T + \mathbf{j} \cdot \mathbf{E} - \frac{\mathbf{j}^2}{\sigma_T} - \nabla \cdot (\kappa_j \nabla T)$$

- Representing Peltier, Thomson effects, Joule heating, Fourier heat conduction

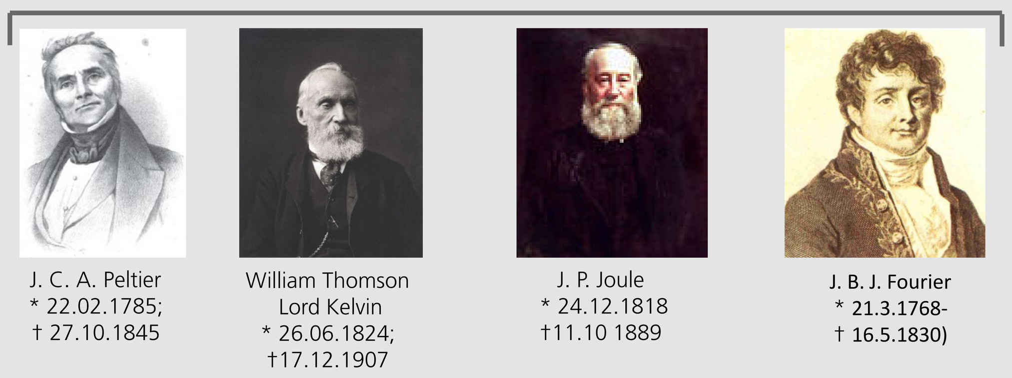

Fig. 2 Scientists - Non-equilibrium Thermodynamics

Performance of a TE element

- Performance depends on:
  - Material properties
  - Working/boundary conditions like junction temperatures and heat fluxes, load resistance, electrical current
  - Contact quality (resistance)
  - Coupling to the surrounding (convection, radiation)
  - *Geometry/shape of the TE elements*

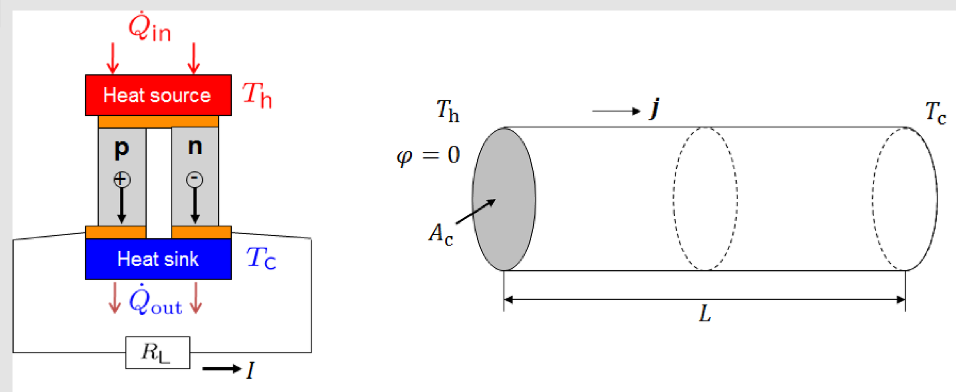

Fig. 3 Thermocouple and thermoelectric leg

Performance of thermoelectric elements

Ioffe-CPM approximation

- Constant properties model means no temperature (physical homogeneity) nor position dependence (chemical homogeneity) of the material properties
- Heat balance – analytic solution → Exact Performance values of a TE element
- Commonly 1D case with constant cross-sectional area used, here TE generator
- Heat flow (hs) $\dot{Q}_h = K\Delta T + I\alpha T_h - \frac{1}{2} I^2 R_{in}$
- Heat flow (cs) $\dot{Q}_c = K\Delta T + I\alpha T_c + \frac{1}{2} I^2 R_{in}$
- El. power: $P_{el} = \dot{Q}_h - \dot{Q}_c = (\alpha \Delta T - I R_{in}) I$
- Efficiency: $\eta = \frac{P_{el}}{\dot{Q}_h}$
- Thermal conductance: $K = \frac{A_c}{L} \kappa$
- Internal resistance: $R = \frac{L}{A_c} \varrho$

Optimum performance – Example

- Maximum power output:

$$I_{opt,p} = \frac{\alpha \Delta T}{2 R_{in}} \text{ and } P_{max} = \frac{(\alpha \Delta T)^2}{4 R_{in}} \qquad F = \frac{\alpha^2}{R_{in}}$$

- Maximum efficiency

$$I_{opt,\eta} = \frac{K \, \Delta T}{\alpha \, T_m} \left(-1 + \sqrt{1 + Z T_m} \right) \text{ and}$$

$$\eta_{max} = \frac{T_h - T_c}{T_h} \frac{\sqrt{1+ZT_m} - 1}{\sqrt{1+ZT_m} + {}^{T_c}/_{T_h}} \text{ with } Z = \frac{\alpha^2}{K \, R_{in}}$$

- Example: $T_h = 400\text{K}, T_c = 300\text{K}, L = 5\text{mm}, A_c = 1\text{mm}^2$

$\bar{\kappa}$ [W/(m·K)]	$\bar{\varrho}$ [Ω·m]	$\bar{\alpha}$ [μV/K]	$f = \bar{\alpha}^2/\bar{\varrho}$ [W/(m·K²)]	zT_m
1.468	$1.738 \cdot 10^{-5}$	222	$2.83567 \cdot 10^{-3}$	0.676

Tab 1: Example – averaged material properties

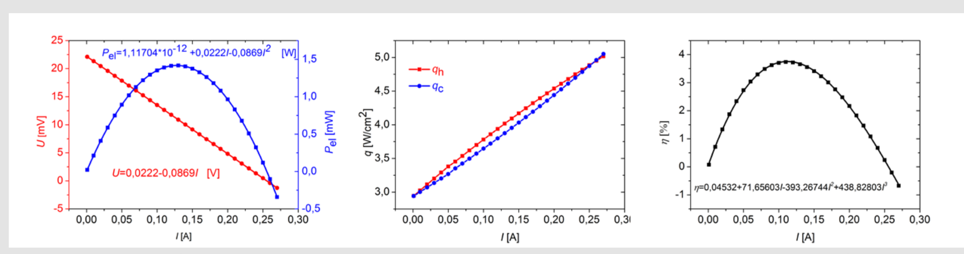

Fig. 4: Performance calculation (1D, const. A_c) for an example material

FEM simulation for non-trivial geometry

- Shaped elements – 2D or 3D calculation
- FEM software ANSYS workbench
- rotational symmetry 3D problem reduced to 2D

Method	U_α [mV]	R_{in} [mΩ]	I_{sc} [mA]	$I_{opt,p}$ [mA]	P_{max} [mW]	$I_{opt,\eta}$ [mA]	η_{max} [%]
Analytical	22.2	86.9	256	128	1.418	111	3.60
3D Sim.	22.2	86.9	256	124	1.375	112	3.75
2D Sim.	22.2	86.9	256	124	1.375	113	3.63

Tab 2: Comparison of methods

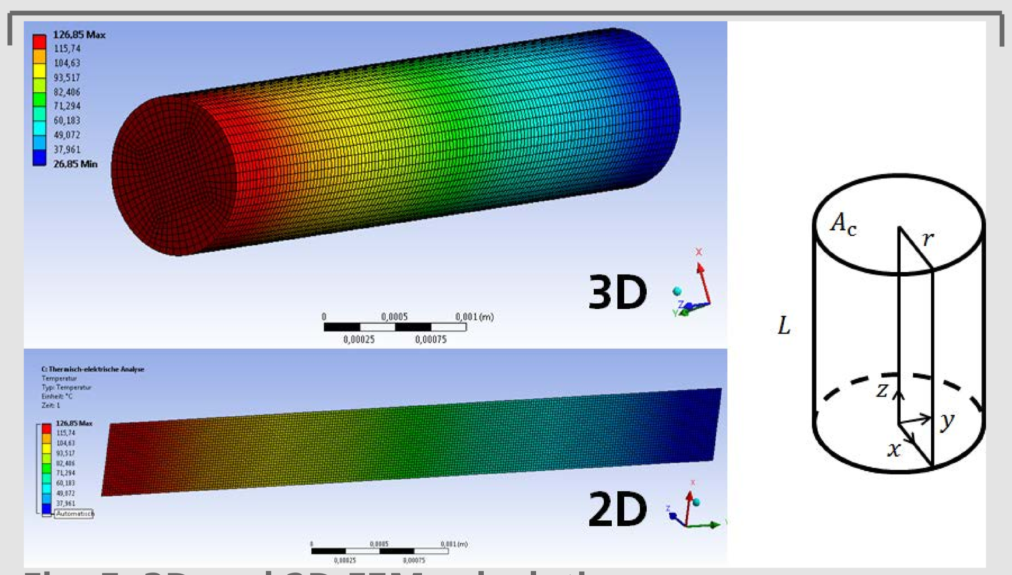

Fig. 5: 3D and 2D FEM calculation

Performance of elements with variable cross-section – Example: Truncated Cone

Quasi 1D approach

- Generalized thermal energy balance

$$\frac{\mathrm{d}}{\mathrm{d}z} \left[-\kappa A_c(z) \frac{\mathrm{d}T}{\mathrm{d}z} \right] = \frac{I^2 \varrho}{A_c(z)}$$

- Truncated cone

$$A_c(z) = A_{c,m} + \left(z - {}^{L}/_{2}\right) s_A$$

- $s_A = \frac{\mathrm{d}A_c(z)}{\mathrm{d}z}$ .... shape parameter
- Definition of a generalized aspect ratio

$$\Gamma = \int_0^L \frac{\mathrm{d}z'}{A_c(z')} = \frac{1}{s_A} \ln \left({}^{A_{c,L}}/_{A_{c,0}} \right)$$

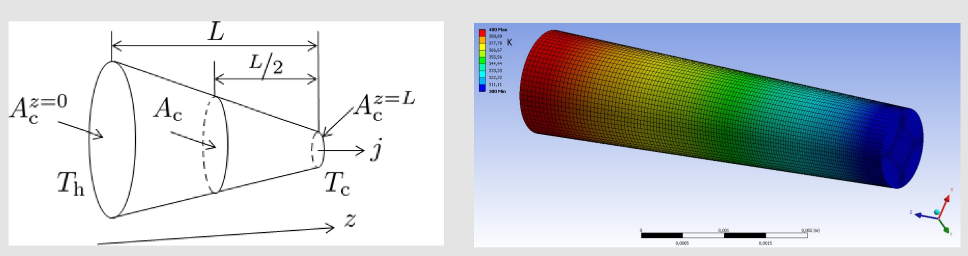

Fig. 6: Shaped element - Truncated cone.

FEM simulation of shaped elements

- Comparison of the analytical quasi-1D model and 2D FEM simulation
- Distribution of heat flux - Integration

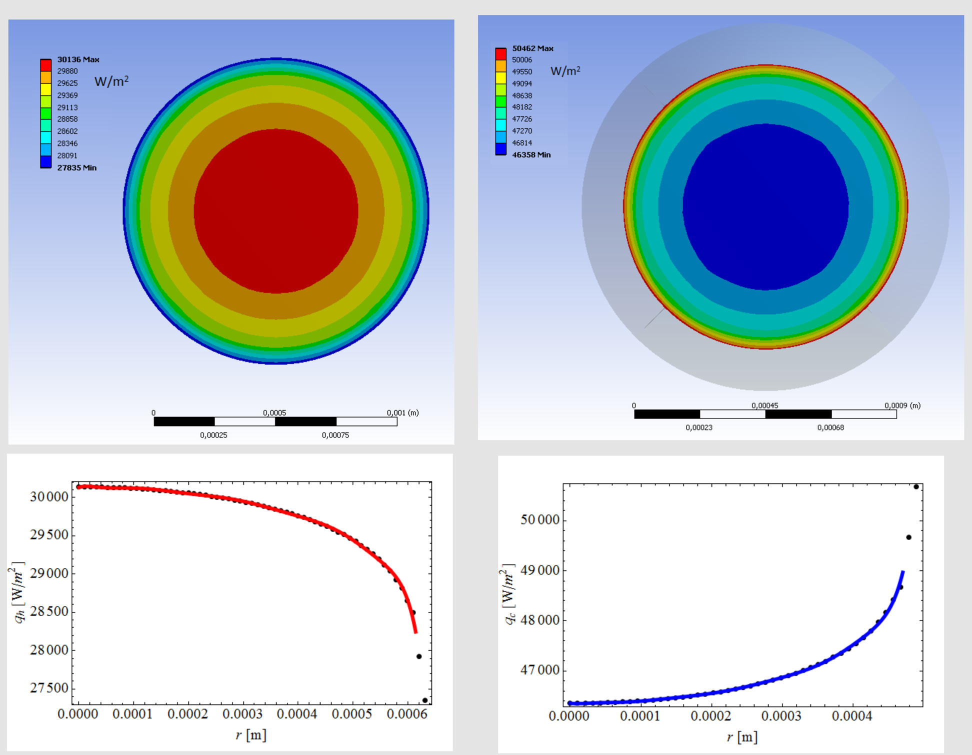

Fig. 7: Heat flux hot side (left) – cold side right.

Variation of the shaped parameter

- Differences between 1D analytical model and 2D simulation
- Efficiency independent of the shape
- Power output best for s_A=0 (cylinder), contradictions to [1]

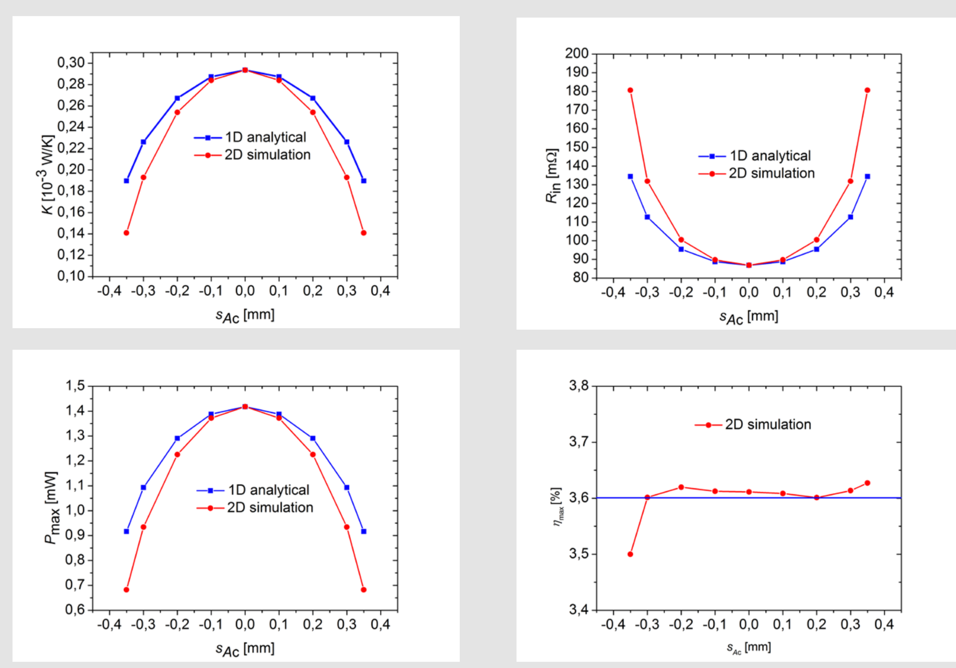

Fig. 8: Comparison of the performance between 1D analytical model and 2D simulation.

Affiliations
[1]Institute of Materials Research, German Aerospace Center (DLR), D-51170 Köln, Germany
[2]Laboratoire CRISMAT ENSICAEN, 6 Bd. Maréchal Juin, 14050, Caen, France
[3]Institute of Physics, University Halle-Wittenberg, D-06099 Halle, Germany
[4]Laboratoire Interdisciplinaire des Énergies de Demain LIED, Université Paris Diderot, France
[5]JLUGiessen, Institute of Inorganic and Analytical Chemistry, 35392 Giessen, Germany

Reference
[1] A.Z.Sahin and B. S. Yilbas Energy Conversion and Management 65 (2013) 26–32

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Bundesministerium für Wirtschaft und Technologie

DLR Deutsches Zentrum für Luft- und Raumfahrt

Contact
German Aerospace Center
Institute of Materials Research
51147 Cologne
Germany
Knud.Zabrocki@dlr.de
+49 (0) 2203/601-3359