COMPARISON OF NEW DESIGN GUIDELINES FOR COMPOSITE CYLINDRICAL SHELLS PRONE TO BUCKLING

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Keyword: Shell buckling, knock-down factor, composite materials, finite element models.

Summary. *Currently, imperfection sensitive shell structures prone to buckling are designed according to the NASA SP-8007 guideline, from 1968, using its conservative lower-bound curve. In this guideline the structural behavior of composite materials is not appropriately considered, since the imperfection sensitivity and the buckling load of shells made of such materials depend on the lay-up design. Due to the fact that this approach is outdated for preliminary design purposes, several authors are investigating less conservative methodologies. Some authors propose a new lower-bound curve approach based only on statistical analysis of experimental test on composite cylinders. The problem with this approach is that the range of applicability is limited to the database extension. Finite element models are also used by many researchers to characterize the behavior of cylindrical shell considering different types of material and geometrical imperfections. A representative finite element model allows studying a widespread area of possibilities from the design point of view. In this context a numerical investigation about the different methodologies to characterize the behavior of imperfection sensitive composite structures subjected to compressive loads up to buckling is presented in this paper. A comparative study is addressed between new deterministic methodologies, such as the "Single Perturbation Load Approach" proposed by the European project DESICOS and new statistical approaches based on experimental test on composite cylinders. The aim of this work is to define the range of applicability of these methodologies for unstiffened composite cylinders, advantage and disadvantage to use as a design tool, to provide means to calculate less conservative knock-down factors than the obtained with the NASA SP-8007 guideline.*

1. INTRODUCTION

The evolution of aerospace structures through the last decades has been pulled by the development of new materials, fabrication techniques and increasing payload requirements. Currently, one of the major problems is the design of thin walled structures, such as fuel tanks of launchers, which are prone to buckling. One can cite two major ways to find the ultimate load of structures prone to buckling. The first way is to predict the buckling load using fully non-linear finite element model considering the mid-surface imperfection, thickness imperfection, angle deviation, etc. This approach was used by Hilburger et al [1], Bisagni [2] and Degenhardt et al [3] and shows a good agreement with experimental results. One major problem with this technique is that usually measured imperfections are not available during pre-design stages. Furthermore the computational cost of these finite element models is too high to be used for pre-design stages, when faster tools are needed. The second way is to calculate the linear buckling load using linear bifurcation theory applied on a fast finite element model and then correct this value using a knock-down factor calculated from discrepancies between experimental and theoretical results. As an example, one can adopt the NASA SP-8007 guideline as a basis to calculate the knock-down factor. The NASA SP-8007 [4] design guideline was developed in the late 1960s and is still in use for the preliminary design of shell structures prone to buckling. This guideline is now outdated, the structures designed with this concept are "super" conservative and heavy, affecting the payload capability of the launcher and its final cost-benefit relationship. The major problem is that this guideline doesn't take into account the full potential of new materials, fabrication process and structural concepts. Due to that, several institutions such as DLR [5] [6] and NASA [7] are expending time and resources to develop a new methodology to support the aerospace industry during the design of new structures. In this context, one can describe two different approaches focused specifically on the design of cylindrical shells made in composite materials, with completely different bases, one deterministic and other statistical.

The so called "Single Perturbation Load approach" is a deterministic process to calculate the knock-down factor of cylindrical structures. The concept of single perturbation load approach (SPLA) was developed by Hünne et al [8], and uses the influence of radially applied load on the buckling load as an indication of imperfection sensitivity. With increasing radial load the buckling load is reduced, however, only until a certain radial load value, called P1. After P1 the buckling load remains nearly constant (see Figure 1). The SPLA defines the cylinder buckling load obtained at P1 as the design buckling load (P_{des}) which allows estimating the knock-down factor of the structure. Despite the advantages of this methodology, a reasonable computational effort to calculate the knock-down factor still is needed. Anyway, if further investigation provides the way to import the concept of the SPLA to an analytical or semi-analytical approach this methodology could be used for preliminary design.

On the other hand, one can find statistical approaches based on experimental data from cylindrical structures made in composite materials such as developed by Takano [9]. The author describes a statistical analysis based on experimental results obtained from the open literature, in order to calculate a new knock-down factor less conservative than the NASA SP-8007 approach. Takano defines two constant knock-down factors, called A-basis = 0.479 (with 99% probability and 95% confidence) and B-basis = 0.626 (with 90% probability and 95% confidence). It must be pointed out that the Takano's approach could be considered valid only in the range of the experimental data (R/t = 80 to 500; where R is radius and t is thickness of the cylinder). Furthermore one of the biggest assumptions of Takano is that the knock-down factors are independent of the radius over thickness ratio (see Figure 2). This assumption simplifies the methodology in such a way, that the only procedure needed to estimate the buckling load is to find the linear buckling load (using the linear bifurcation theory for example) and then multiply by the A-basis or B-basis calculated by Takano. Despite the advantages of Takano's methodology, one will show that for some cases Takano's approach can give more conservative results than NASA SP-8007.

Focused in these different approaches, this paper proposes a comparison between the NASA SP-8007 guideline, the SPLA method and Takano's approach to calculate the knock-down factor of cylindrical shells made in composite materials. The result of each methodology is compared against experimental results obtained from open literature. This comparison will allow one to address three major questions:

a) Is Takano's approach less conservative than the NASA SP-8007 guideline?
b) Is it realistic to assume a knock-down factor constant for different values of R/t ratio?
c) A deterministic approach such as SPLA, can it be used to calculate less conservative knock-down factors than statistical approaches?

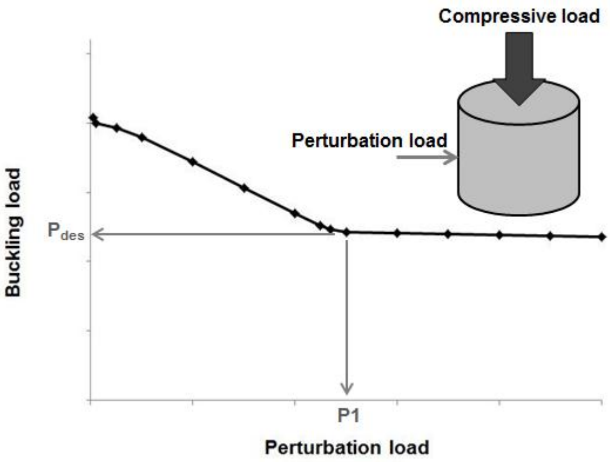


Figure 1: SPLA concept.

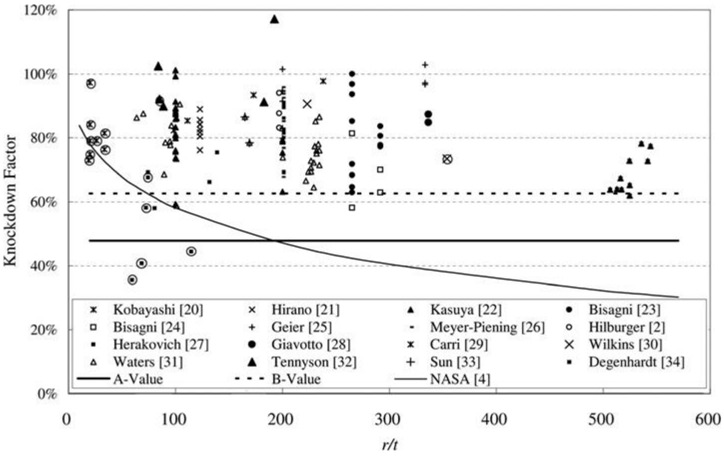


Figure 2: Comparison of Takano's knock-down factor (A-Value and B-Value) with experimental results [9].

2. STUDY CASES AND METHODOLOGIES

It is already statement that the buckling load of a perfect composite shell strongly depends on the laminate set-up, as shown by Zimmermann [10]. In addition, it is shown that the imperfection sensitivity also depends on it. Another parameter influencing imperfection sensitivity is the ratio of radius over thickness of the cylindrical shell, as shown in the NASA SP-8007 guideline [4]. One of the biggest assumptions made by Takano was that the knock-down factor remains constant for different values of radius over thickness ratio. This assumption is useful because it allows one to perform a fast calculation of the buckling load during the preliminary design of cylindrical shells. Still, as pointed out before, two major questions remain from this assumption: Is Takano's approach less conservative than NASA SP-8007 guideline? And on the other hand, is it realistic to assume the knock-down factor constant for different r/t values?

To answer these particular questions two benchmark cases were chosen from the open literature to perform the analysis and comparison of the statistical approach presented by Takano against non-linear finite element models considering initial mid-surface imperfection obtained from real measurements. The first cylinder called “Z33” is taken from previous studies from the European project “Design and Validation of Imperfection-Tolerant Laminated Shells” (DEVILS) project. This cylinder was originally designed and tested by DLR and published in the paper of Meyer-Piening et al [11] It was used by Hühne et al [12], Wullschleger [13] and is recognized as a benchmark case, due to its high imperfection sensitivity. The second benchmark called cylinder “Z22” is taken from the results of a European Space Agency (ESA) study, conducted at DLR and published by Degenhardt et al [3]. Three major differences between these two cylinders reveal them particularly appropriate for the investigation: a) The laminate set-up is different; b) The relation radius/thickness is 200 for cylinder Z33 and 515 for cylinder Z22; c) Due to the fact that these cylinders were fabricated at different times with different mandrels, the mid-surface imperfection pattern is completely different from each other; the geometry measurement performed by the authors at that time is shown in Figure 3 (magnified 100x).

The material properties (unidirectional carbon fiber prepreg with epoxy matrix), geometry and experimental buckling load for cylinders Z22 and Z33, taken from Degenhardt et al [3] and Meyer-Piening et al [11], respectively, are detailed in Table 1, Table 2 and Table 3.

	Z22 Cylinder	**Z33 Cylinder**
E_1 [GPa]	142.5	123.55
E_2 [GPa]	8.7	8.70
G_{12} [GPa]	5.1	5.7
ν_{12}	0.28	0.31

Table 1: Material properties for Z22 and Z33 cylinders.

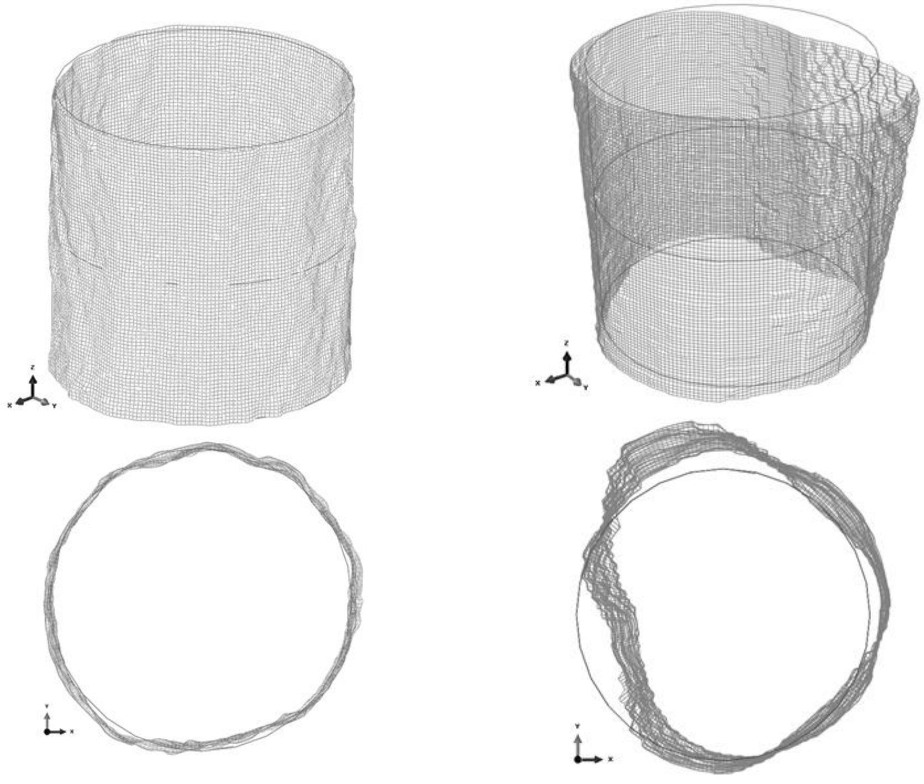
	Z22 Cylinder	**Z33 Cylinder**
Free length [mm]	500.0	510.0
Radius [mm]	250.3	250.625
Thickness [mm]	0.486	1.25
Lay-up [in-out]	[±24/±41]	[±0/±19/±37/±45/±51]

Table 2: Geometric parameters for Z22 and Z33 cylinders.

	Z22 Cylinder	**Z33 Cylinder**
Experimental buckling load [kN]	22.4	172.8

Table 3: Experimental buckling load for Z22 and Z33 cylinders.

The mid-surface geometry imperfection was measured after fabrication using non-contact scanning devices in both cylinders. The results taken from Degenhardt [3] and Wullschleger [13] for Z22 and Z33 cylinders, respectively, are shown on Figure 3. The nominal mid-surface imperfection is used in this paper to compare its effect on the knock-down factor value against the statistical approach proposed by Takano.



Z22 cylinder

Z33 cylinder

Figure 3 - Mid-surface imperfection pattern magnified in 100x.

2.1. Study proposal

To test the basis of Takano's approach in all its range of applicability using the benchmark cases it becomes evident the needed of "virtual specimens" capable to cover all the R/t range and not only two specific points defined by the original geometry and layup thickness of the benchmark cylinders. To do that it is proposed to adopt the characteristic imperfection pattern for each cylinder as the imperfection signature from the manufacturing processes and use the same imperfection pattern in all the virtual cylinders with different R/t ratio than the original. To change the R/t ratio the ply thickness for each case is increased or decreased, maintaining constant the cylinder radius. This methodology is consistent with the assumption of adopting the same imperfection pattern for all cases, since the same “virtual mandrel” could be used to fabricate all the cylinders.

2.2. Knock-down factor definition

To characterize the knock-down factor for each cylinder one can use the buckling load obtained through an eigenvalue analysis as a reference load (P_{cr}). Then, the knock-down factor (KDF) can be calculated as follows:

$$KDF = \frac{P}{P_{cr}} \qquad (1)$$

where P is the “design buckling load” or “real buckling load” and can be obtained from experimental results, SPLA, or any other methodology. This approach was commonly used by other authors to characterize the KDF as indicated by Hühne et al [8], Degenhardt et al [3] and Hilburger et al [1].

To calculate P_{cr} a finite element model is developed using the commercial software ABAQUS. Subspace iteration solver [14] is used to find the first eigenvalue for both cylinders that represents the first buckling load. The linear buckling load and the experimental KDF (calculated using data from Table 3) are shown on Table 4. More details for modeling setup, mesh and boundary condition are given in next sections.

	Z22 Cylinder	Z33 Cylinder
Linear buckling load	33.7 kN	198.2 kN
Experimental KDF	0.664	0.871

Table 4 - Linear buckling load obtained from eigenvalue analysis and experimental KDF for Z22 and Z33 benchmark cylinders.

2.3. NASA SP-8007 guideline

The design procedure presented in NASA SP-8007 [4] was originally proposed for isotropic shells, being extended for orthotropic composites by using correction factors. The guideline presents analytical equations for predicting the buckling load. In this case the correction factor is used in the squared form (“γ^2”), but modern applications use directly the “γ” value applied to the buckling load of the perfect shell, usually obtained by linear buckling analysis. The formula for “γ” was originally proposed by Seide et al [15] and later modified by Weingarten et al [16], to the form presented in Eq. (2). This formula is the lower-bound curve of a set of test results, as shown in Figure 4. It is important to notice the equivalent thickness which is used when applying the methodology for composite materials, calculated using the bending and extensional stiffness of the laminate obtained from the ABD matrix.

$$\gamma = 1 - 0.902\left(1 - e^{-\phi}\right) \qquad (2)$$

where, for isotropic material:

$$\phi = \frac{1}{16}\sqrt{\frac{R}{t}} \qquad (3)$$

For orthotropic material:

$$\phi = \frac{1}{16}\sqrt{\frac{R}{t_{eq}}} \qquad (4)$$

$$t_{eq} = 3.4689\sqrt[4]{\frac{D_{11}D_{22}}{A_{11}A_{22}}} \qquad (5)$$

where D_{11} and D_{22} are the bending stiffness and A_{11} and A_{22} are the extensional stiffness in the axial and circumferential directions, respectively, obtained through the ABD matrix.

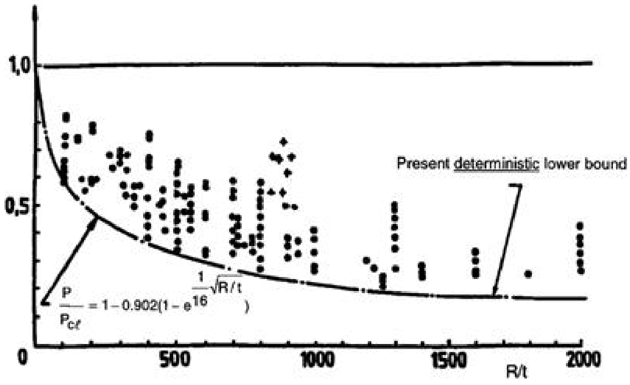


Figure 4 – Test data for isotropic cylinders subjected to axial compression (modified from Arbocz and Starnes Jr. [17])

Many authors have proved that the NASA SP-8007 guideline has given conservative estimations for the buckling load of imperfect shells (see Arbocz & Starnes Jr. [17], Hilburger et al [1], Hühne et al [8] and [6], Degenhardt et al [5]). This can be observed if one calculates the knock-down factor of the study cases using NASA´s approach and compares with the experimental results, as presented in Table 5 and Table 4, respectively. The results show that the KDF obtained using the NASA guideline is 52% and 45% more conservative than the experimental KDF for the Z22 and Z33 cylinders, respectively.

	Z22 Cylinder	Z33 Cylinder
NASA KDF	0.317	0.476

Table 5 – KDF obtained from NASA SP-8007 guideline for Z22 and Z33 benchmark cylinders

3. COMPARISON OF METHODOLOGIES TO DETERMINE THE KDF

The finite element software ABAQUS is used to perform the numerical simulations. One finite element model is developed for each cylinder to perform the study considering mid-surface imperfection and different R/t ratios. For all studies Newton-Raphson iteration solver is used with artificial damping. This methodology is less computationally demanding than dynamic relaxation methods and can handle the instability phenomena when the structure reaches the buckling load. The major inconvenience with this approach is to define the correct damping factor for each model, since this is not a physical parameter and can only be estimated after a convergence analysis. The final mesh parameters used in this paper are presented on Table 6.

	Z22 Cylinder	Z33 Cylinder
Element type	S4R	S4R
Element size	≈ 6.5 mm	≈ 7.9 mm
Number of elements	18240	12672
Damping factor	1.10^{-7}	4.10^{-7}
Initial increment size	0.01	0.02
Minimum incr. size	1.10^{-6}	1.10^{-6}
Maximum incr. size	0.01	0.02
Max. number of incr.	10000	10000

Table 6 - Finite element models setup.

The boundary condition used in both cases is clamped on top and bottom edge. For all analyses the model is loaded using 1.0 mm axial shortening applied at the upper edge.

3.1. KDF variation for different ratios of radius over thickness.

Based on the original configuration of Z22 and Z33 cylinder several analyses are performed in order to cover the range of applicability of Takano’s approach, changing the ply thickness of each laminate in order to obtain a different R/t ratio. The results are presented in Figure 5 together with the 99% probability and 90% probability knock-down factors from Takano’s approach. The black circles represent the original R/t ratio for Z22 and Z33 cylinder. Arrows indicate the directions where the thickness is changed to achieve different values of R/t.

From the analysis of Figure 5 it can be noticed that:

a) For cylinder Z22 the reduction of R/t ratio doesn't affect the knock-down factor, which remains nearly constant between 0.82 and 0.87;
b) For cylinder Z33 increasing the R/t ratio decreases the knock-down factor from 0.9 (R/t = 100) to 0.58 (R/t = 300) and for R/t above 300 the KDF remains nearly constant;
c) From R/t higher than 300 the KDF obtained for the Z33 cylinder is lower than the 90% probability KDF proposed by Takano.

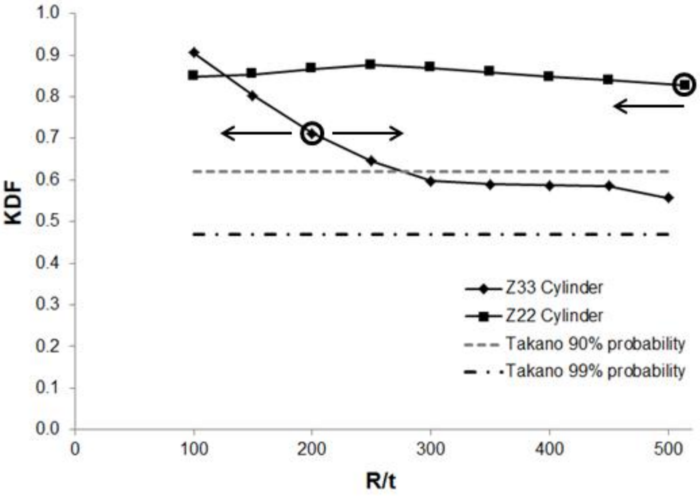


Figure 5 – KDF variation for different ratios of radius over thickness for Z22 and Z33 cylinders.

Due to the different behavior verified for both benchmark cases, further investigations are addressed to check if the imperfection pattern extracted form real measurements from cylinder Z33 is more critical than the imperfection pattern from cylinder Z22. To do that it is proposed to interchange the imperfection pattern between the cylinders. The results of these new analyses are presented on Figure 6 together with the 99% probability and 90% probability knock-down factors from Takano's approach. The black circles represent the original R/t ratio for Z22 and Z33 cylinder. Arrows indicate the directions where the thickness is changed to achieve different values of R/t.

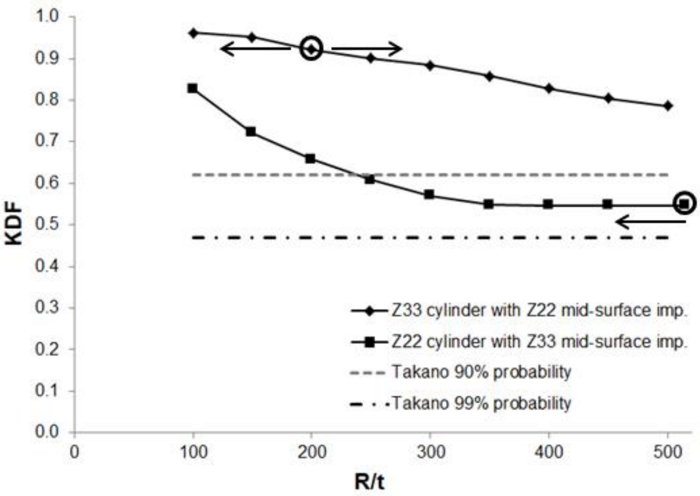


Figure 6 – KDF variation for different ratios of radius over thickness for Z22 and Z33 cylinders with interchanged mid-surface imperfections.

From the analysis of the results presented in Figure 6 it can be pointed out that:

a) For cylinder Z33 increasing the R/t ratio decreases the knock-down factor from 0.96 (R/t = 100) to 0.78 (R/t = 500)and the values of KDF are significant higher than the previous study;
b) For cylinder Z22 increasing the R/t ratio decreases the knock-down factor from 0.82 (R/t = 100) to 0.54 (R/t = 350) and for R/t above 350 the KDF remains nearly constant;
c) From R/t higher than 200 the KDF obtained for the Z22 cylinder is lower than the 90% probability KDF proposed by Takano.

Making a comparison between Figure 5 and Figure 6 it becomes clear that the mid-surface imperfection measured from cylinder Z33 is more critical than the mid-surface imperfection measured on cylinder Z22, since lower KDFs are obtained with the cylinder that includes the mid-surface imperfection from cylinder Z33. Also it’s clear that the KDF doesn’t remain constant for different values of R/t ratio.

3.2. Comparison between NASA SP-8007, SPLA, Takano’s approach and experimental knock-down factors

Different cylinder configurations (radius, thickness, layup, material, etc.) are evaluated in order to verify the findings of the previous study changing the R/t ratio only changing the ply thicknesses. The SPLA method is used to check if the KDF remains constant for different R/t ratios. To do that it is proposed to use the same experimental results, extracted from the open literature by Takano in his statistical approach [9]. A total of 48 different cylinder configurations are analyzed with the respective geometrical characteristics and material properties presented in Table 7 and Table 8, respectively. More details about these cylinders can be found in Takano [9]. The experimental KDF and also the KDF calculated using the NASA guideline and the SPLA are presented in Table 7.

							Knock-down factor		
N°	Layup sequence (right most is top layer)	t	R	L	B.C.	Author	Exp.	NASA	SPLA
1	(20/-20/90)	0.42	100	200	C1	[18]	1.23	0.442	0.805
2	(0/45/-45/90)	0.578	100	200	C1	[18]	0.94	0.509	0.855
3	(30/-30/-30/+30/90/90)	0.899	100	200	C1	[18]	0.86	0.567	0.727
4	(0/60/-60/-60/60/0)	1.017	100	200	C1	[18]	0.83	0.581	0.737
5	(20/-20/0/0/40/-40)	0.814	100	300	S4	[19]	0.75	0.549	0.581
6	(20/-20/40/-40/0/0)	0.814	100	300	S4	[19]	0.69	0.540	0.601
7	(40/-40/20/-20/0/0)	0.814	100	300	S4	[19]	0.83	0.552	0.880
8	(0/90/0/90)s	1	100	300	S4	[20]	0.90	0.576	0.889
9	(0/0/0/0/90/90/90/90)	1	100	300	S4	[20]	1.01	0.581	0.908
10	(-20/-20/20/20)	0.5	100	300	S4	[20]	0.75	0.470	0.865
11	(-20/-20/-20/-20)as	1	100	300	S4	[20]	0.87	0.581	0.894
12	(-20/20/-20/20)as	1	100	300	S4	[20]	0.76	0.581	0.704
13	(-45/45/45/-45)	0.5	100	300	S4	[20]	0.63	0.470	0.746
14	(-45/-45/45/45)	0.5	100	300	S4	[20]	0.82	0.470	0.684
15	(-45/-45/-45/-45)as	1	100	300	S4	[20]	0.78	0.581	0.714
16	(-45/45/-45/45)as	1	100	300	S4	[20]	0.60	0.581	0.710
17	(-70/70/-70/70)s	1	100	300	S4	[20]	0.71	0.581	0.569
18	(-70/70/-70/70)	0.5	100	300	S4	[20]	0.70	0.470	0.568
19	(-70/-70/-70/-70)as	1	100	300	S4	[20]	0.88	0.581	0.662
20	(-70/70/-70/70)as	1	100	300	S4	[20]	0.72	0.581	0.568
21	Fabric (0/45/-45/0)	1.32	350	540	C1	[21] [22]	0.66	0.434	0.655
22	Fabric (45/-45)s	1.32	350	540	C1	[21] [22]	0.92	0.424	0.785
23	Unidir (45/-45)2s	1.2	350	540	C1	[21] [22]	0.76	0.408	0.782
24	Unidir (90/0)2s	1.2	350	540	C1	[21] [22]	0.75	0.403	0.961
25	(60/-60/0/0/68/-68/52/-52/37/-37)	1.25	250	510	C1	[23] [11]	0.80	0.464	0.560
26	(51/-51/45/-45/37/-37/19/-19/0/0)	1.25	250	510	C1	[23] [11]	0.92	0.476	0.949
27	(30/-30/90/90/22/-22/38/-38/53/-53)	1.25	250	510	C1	[23] [11]	0.89	0.464	0.578
28	(51/-51/90/90/40/-40)	0.75	250	510	C1	[23]	1.03	0.379	0.601
29	(39/-39/0/0/50/-50)	0.75	250	510	C1	[23]	0.97	0.379	0.789
30	(49/-49/36/-36/0/0)	0.75	250	510	C1	[23]	0.97	0.390	0.940
31	(-37/37/-52/52/-68/68/0/0/-60/60)	1.25	250	510	C1	[11]	0.93	0.464	0.718
32	(38/-38/68/-68/90/90/8/-8/53/-53)	1.25	250	510	C1	[11]	0.83	0.458	0.611
33	(0/0/19/-19/37/-37/45/-45/51/-51)	1.25	250	510	C1	[11]	0.86	0.476	0.623
34	(-45/45/0/0/0/0/45/-45)	1.016	200	355.6	C1	[1]	0.94	0.468	0.874
35	(-45/45/90/90/90/90/45/-45)	1.016	200	355.6	C1	[1]	0.88	0.468	0.589
36	(-45/45/0/90/90/0/45/-45)	1.016	200	355.6	C1	[1]	0.83	0.456	0.636
37	(0/90/90/0)	1.04	350	550	C1	[24]	0.87	0.384	0.932
38	(45/-45/-45/45)	1.04	350	550	C1	[24]	0.85	0.384	0.748
39	(0/45/-45)s	0.85	190.5	381	C1	[25]	0.91	0.447	0.666
40	(45/-45/0/90)s	1.01	203.2	355.6	C1	[26]	0.74	0.452	0.636
41	(+45/-45/0/0/0/0/-45/45)s	1.95	203	355.6	C1	[26]	0.90	0.575	0.740
42	(0/0/45/45/-45/-45/90/90)	1	83.85	284.7	C1	[27]	1.02	0.620	0.836
43	(0/0/45/45/-45/-45/0/0)	0.94	83.82	269.2	C1	[27]	1.03	0.586	0.671
44	(0/90/90/0)	0.46	83.57	282.7	C1	[27]	0.90	0.463	0.855
45	(90/0/0/90)	0.43	83.57	267.7	C1	[27]	1.18	0.452	0.973
46	(90/0/0/90)	0.49	83.31	152.4	C1	[28]	0.79	0.475	0.976
47	(0/90/90/0)	0.51	83.31	152.4	C1	[28]	0.86	0.481	0.845
48	(24/-24/41/-41)	0.46	250.71	500	C1	[3]	0.71	0.307	0.584

Table 7 – Geometric characteristics and KDF for several different composite cylinders.

In Table 7 *t* is the thickness in [mm], *R* is the radius in [mm], *l* is the cylinder length in [mm], *B.C.* is the boundary conditions used in the finite element model (C1 denotes clamped and S4 denotes simply supported condition). The experimental KDF is calculated dividing the experimental buckling load by the linear buckling load obtained through a finite element model analysis. For the cases that more than one experimental test were done, the average KDF is informed. The *NASA KDF* is calculated using the NASA SP-8007 guideline as described in the previous section. The *SPLA KDF* is calculated using the SPLA methodology presented in chapter 1.

Author	Material	E_{11} (MPa)	E_{22} (MPa)	G_{12} (MPa)	G_{13} (MPa)	G_{23} (MPa)	ν_{12} (-)
[18]	CFRP (1)	127,600	8,504	4,020	4,020	2,410	0.332
[18]	CFRP (2)	132,000	8,840	4,250	4,250	2,550	0.330
[18]	CFRP (3)	130,200	8,720	4,160	4,160	2,496	0.331
[18]	CFRP (4)	119,600	8,050	3,660	3,660	2,196	0.334
[19]	T300/S305	105,000	8,740	4,560	4,560	2,736	0.327
[20]	T300A/Epoxy	136,705	8,169	4,746	4,746	2,848	0.316
[21] [22]	CFRP fabric	52,000	52,000	2,350	2,350	2,350	0.302
[21] [22]	CFRP unidirectional	113,000	9,000	3,820	3,820	2,292	0.730
[29] [11]	CFRP	123,550	8,708	5,696	5,696	3,418	0.319
[1]	AS4/3502	127,500	11,300	5,990	5,990	3,594	0.300
[24]	Kevlar fabric	23,450	23,450	1,520	1,520	1,520	0.200
[25]	Graphite-Epoxy Narmco 5505/T300	149,623	9,928	4,482	4,482	2,689	0.280
[26]	AS4/3502 (40)	127,630	11,307	6,005	6,005	3,603	0.300
[26]	AS4/3502 (41)	132,790	11,307	6,247	6,247	3,748	0.299
[27]	3M SP288 T300	141,349	9,653	4,068	4,068	2,441	0.260
[28]	AS/3501-6	145,486	10,756	5,792	5,792	4,882	0.290
[3]	IM7/8552	175,300	8,600	5,300	5,300	3,180	0.300

Table 8 – Material properties for different composite cylinders extracted from open literature.

From Table 7 it can be seen, as expected, that the KDF obtained using the NASA SP-8007 guideline and Takano’s 99% probability curve (KDF = 0.479) are more conservative than the experimental KDF. Also Takano’s 90% probability curve (KDF = 0.626) gives conservative KDFs than obtained through experimental tests for most of study cases excepting the cylinder N° 16. Figure 7 shows a comparison between the NASA SP-8007 guideline and the Takano’s approach. It can be seen that Takano’s 99% probability curve gives more conservative results than NASA approach for ratio of radius over thickness smaller than 170. For higher ratios of R/t the NASA guideline becomes more conservative than Takano’s 99% curve.

The results of the SPLA methodology presented on Figure 7 reflect that the KDF depends not only of the ratio of radius over thickness, but also depends of other parameters like layup and material properties.

On the other hand the SPLA methodology gives less conservative KDFs than Takano’s 99% probability curve and it can be noticed also that the SPLA methodology gives more conservative KDF than NASA guideline for cylinders N°17 and 20.

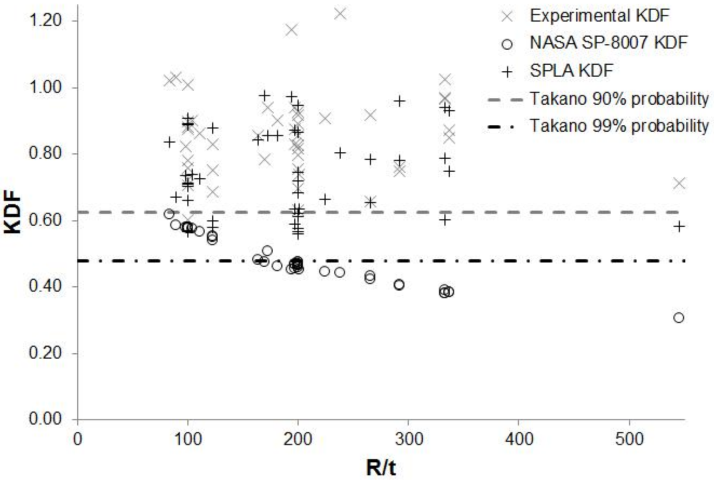


Figure 7 – KDF from test results, NASA SP-8007, SPLA and Takano's approach for cylinders listed in Table 7.

Moreover, it is clear that the SPLA as presented here cannot be used as a design guideline by itself since its fails for some cases presented in Table 7. In order to guarantee a robust design, the approach must be combined with a specific stochastic procedure, which considers "non-traditional" imperfections such as thickness variation, material properties, among others (proposed by Degenhardt et al [3] [5]) and also accounts for load asymmetry, as proposed by Wagner & Hühne [30]. The outcome of this methodology will be a new KDF that results of the combinations of the KDF from the SPLA methodology considering load asymmetry and another KDF from the stochastic procedure [31].

4. CORRECTION FOR TAKANO'S APPROACH

From the results presented on Table 7 it can be seen that for cylinders with R/t ratio smaller than 170 Takano's approach with 99% probability (KDF = 0.479) becomes more conservative than the NASA SP-8007 guideline, currently in use. Furthermore one can compare the complete NASA knock-down curve for the benchmark cases Z22 and Z33 cylinders for different ratios of radius over thickness and Takano's approach. The results presented in Figure 8 show that the Takano's 99% probability curve becomes more conservative than NASA guideline for R/t smaller than 200.

Although NASA guideline gives too conservative KDF and is not specifically developed to deal with composite structures, it is widely used un the industry and it becomes clear that if Takano's approach is applied for the design of cylindrical composite shells prone to buckling, an additional analysis must be performed to check if the KDF given by the NASA guideline is less conservative than Takano KDF. If the NASA guideline gives a KDF higher than 0.479, the NASA guideline should be used.

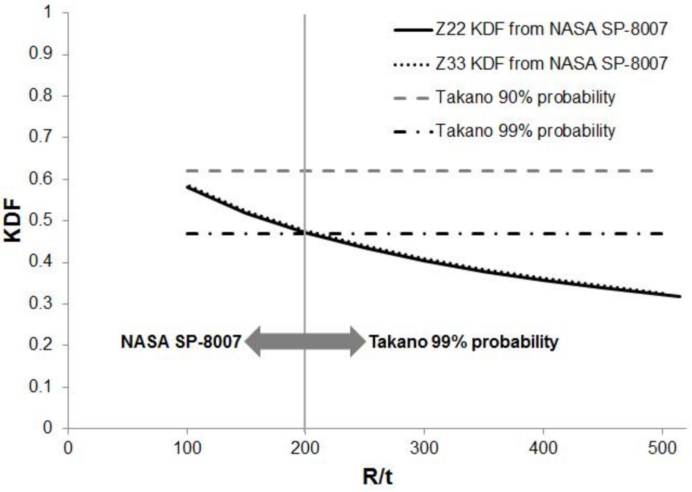


Figure 8 – KDF from NASA SP-8007 and Takano approach for Z22 and Z33 benchmark cases. Note that Takano approach becomes more conservative than NASA guideline for R/t<200.

5. CONCLUSIONS

A detailed numerical study with experimental validation has been developed to characterize the buckling loads of imperfection sensitive composite cylindrical shells. A comparison between the NASA SP-8007 guideline (currently in use for design of thin-walled structures prone to buckling), the single perturbation load approach and the statistical knock-down factors calculated by Takano [9] is presented. The results show that Takano 99% probability knock-down factor is more conservative than the NASA guideline for lower values of radius over thickness ratio and - based on the general understanding that the NASA guideline is too conservative - a correction for Takano's approach is proposed. On the other hand, the SPLA gives less conservative results than Takano 99% probability KDF but also gives higher KDF than the experimental results for some cases, and it cannot be used as a guideline by itself without further investigations, in order to understand the causes of such a behavior. Also it's noticed that the KDF for composite cylindrical shells is depending not only on the radius over thickness ratio but also depends on the layup and material properties, as reflected by the different values of KDF obtained using the SPLA method.

6. ACKNOWLEDGMENTS

The research leading to these results has received funding from the European Community's Seventh Framework Programme (FP7/2007-2013) under Priority ERDF (European Regional Development Fund), Grant Agreement Number WA3-80123539 and collaboration form the European Community's Seventh Framework Programme (FP7/2007-2013), Grant Agreement Number 282522 (www.DESICOS.eu). The information in this paper reflects only the author’s views and the European Community is not liable for any use that may be made of the information contained therein.

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