Imperfection Sensitivity of Antisymmetric Cross-Ply Cylindrical Shell Under Axial Compression Using Hui's Postbuckling Theory

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Abstract

This study deals with the use of Hui's Postbuckling Theory in antisymmetric cross-ply cylindrical shell under axial compression. The Hui's postbuckling theory, also termed the Improved Koiter's postbuckling theory was first applied to buckling of structures such as axial postbuckling of cylindrical panels and postbuckling of beams (Hui and Chen 1986, Hui 1988). In this theory, the postbuckling b coefficient, as developed by Budiansky and Hutchinson (1964) is evaluated at the actual applied load rather than at the classical buckling load as Koiter's general stability theory. The results are compared with ABAQUS simulation results. A significant positive shift of the postbuckling b coefficient is found which indicates that the Koiter's general theory first published in 1945, has significantly overestimated the imperfection sensitivity of the structure. Also, compare with the Koiter's general stability theory, the valid region is significantly increased by using Hui's postbuckling theory.

Introduction

Koiter(1945) was the first one to present a rigorous postbuckling theory which is capable of taking into account the presence of geometric imperfections and interactions between buckling modes, and which can be called Koiter's general stability theory. This theory is based on an asymptotic approach but has several limitations. (i)it was assumed that the shape of the imperfection should be identical to the buckling mode. (ii)the theory is valid asymptotically only for sufficiently small values of the imperfection amplitude, usually few percentage of the shell thickness. In order the solve the second limitation, we apply the improved Koiter's theory, which is evaluating the postbuckling b coefficient under the actual applied load rather than the classical buckling in Koiter's general theory. In order to apply the Hui's postbuckling theory, two special care should be taken, (i) the differential equations for the second-order field are solved under actual applied load; (ii) the formula to calculating the postbuckling b coefficient also need to be evaluated at the actual load. A least square curve-fit of the improved knock down curve is take place for the imperfection amplitude between 0% and 25% of the shell thickness to get the improved b coefficient. The result of Hui's postbuckling theory are comparing with the result of Koiter's general theory and ABAQUS simulation.

Governing Equations and First Order Field

The governing differential equations for the initial postbuckling of imperfect antisymmetric cross-ply laminate cylindrical shell are nonlinear Donnell type equilibrium and compatibility equations, written in terms of the out-of-plane displacement W and the stress function F, which are (see Hui and Du 1987),

$$L_D^*(W) + L_B^*(F) + \frac{1}{R}\mathrm{F}_{,XX} = F_{,XX}\,W_{,YY} + F_{,YY}\,W_{,XX} - 2F_{,XY}\,W_{,XY} \qquad (1)$$

$$L_A^*(F) - L_B^*(W) - \frac{1}{R}W_{,XX} = (W_{,XY})^2 - W_{,XX}\,W_{,YY} \qquad (2)$$

The total displacement and the stress function can be express as follow,

$$w = w_0 + \xi w_I + \xi^2 w_{II}$$
$$f = f_0 + \xi f_I + \xi^2 f_{II} \qquad (3)$$

by applying the non-dimensional quantities to eq.(1) and (2). we can get the GDEs for first order field

$$L_d^*(w_I) + L_b^*(f_I) + f_{I,xx} + p w_{I,yy} + \sigma w_{I,xx} - 2\tau w_{I,xy} = 0 \qquad (4)$$

$$L_a^*(f_I) - L_b^*(w_I) - w_{I,xx} = 0 \qquad (5)$$

The differential operator L_a^*, L_b^*, and L_d^* is defined by Hui and Du(1987). General solution for the first order field can be represent in the following separable form,

$$w_I(x,y) = w_c(\mathrm{x})\cos(Ny) + w_s(x)\sin(Ny) \qquad (6)$$

$$f_I(x,y) = f_c(\mathrm{x})\cos(Ny) + f_s(x)\sin(Ny) \qquad (7)$$

where $N = n(h/R)^{1/2}$, n is the number of circumferential full-waves. The clamped boundary is applied in both end of the cylindrical shell.

$$w = w_{,X} = U_{,YY} = V_{,Y} = 0 \; at \; x = 0, L/(Rh)^{0.5} \qquad (8)$$

Calculation details can be found in Hui and Du(1987).

Second Order Filed and Postbuckling b Coefficient

Substituting the equations of the total out-of-plane and total stress function into the governing Donnell type equations and collecting the terms involving ξ^2, then we can get the equilibrium and compatibility equations for the second order field is,

$$L_a^*(w_{II}) + L_b^*(f_{II}) + f_{II},_{xx} + p_a w_{II},_{yy} + \sigma_a w_{II},_{xx} - 2\tau_a w_{II},_{xy} = f_I,_{yy} w_I,_{xx} + f_I,_{xx} w_I,_{yy} - 2 f_I,_{xy} w_I,_{xy} \quad (9)$$

$$L_a^*(f_{II}) - L_b^*(w_{II}) - w_{II},_{xx} = (w_I,_{xy})^2 - w_I,_{xx} w_I,_{yy} \quad (10)$$

Note that instead of using p_c, σ_c and τ_c to calculate the second order field, we replace them by the actual applied load p_a, σ_a and τ_a respectively. The general solution for the second order field is,

$$w_{II}(x,y) = w^*(x) + w_A(x)\cos(2Ny) + w_B(x)\sin(2Ny) \quad (11)$$

$$f_{II}(x,y) = f^*(x) + f_A(x)\cos(2Ny) + f_B(x)\sin(2Ny) \quad (12)$$

The postbuckling b coefficient can be written in terms of applied load, fist order field and section order field. For simplicity, the formula will not present in here, details can be seen in Hui and Du (1987). Note that the formula for the b coefficient should also be evaluated at the actual applied load. Then we can get the b coefficient as a function of applied load. Least square curve fit is taken place to get the improved b coefficient.

Result and Discussion

In the example, we are using $_{in}[0^o{}_t/90^o{}_t]_{out}$ laminate cylindrical shells with the following material and geometric parameters,

$$\frac{E_1}{E_2} = 10, \frac{G_{12}}{E_2} = 0.5, v_{12} = 0.25, t = 0.05, R = 30, L = 50$$

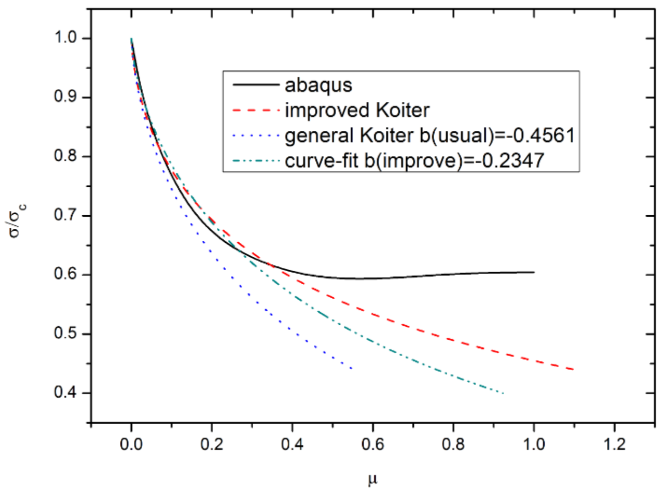

Fig.1Knock down curve for$_{in}[0^o{}_t/90^o{}_t]_{out}$ antisymmetric cross-ply cylindrical shell under axial compression

Fig. 1 shows the Knock down factor calculated by ABAQUS, Hui's postbuckling theory, Koiter's general theory and curve fit of the improved b coefficient. From the figure, we can see that the curve for the Hui's postbuckling theory can fit the ABAQUS result of the imperfection up to 39% of the shell thickness. The Koiter's general theory is only valid when the imperfection of the cylinder around 10% of the shell thickness. We can see that the knock down curve for the improved b coefficient is also valid when the imperfection up to 30% of the shell thickness. The general b coefficient is -0.4561 compare with the improved b coefficient which is -0.2347. We can find out a 50% positive shift of the postbuckling coefficient by using the Hui's postbuckling theory, this means that the general Koiter's theory is significantly overestimate the imperfection sensitivity.

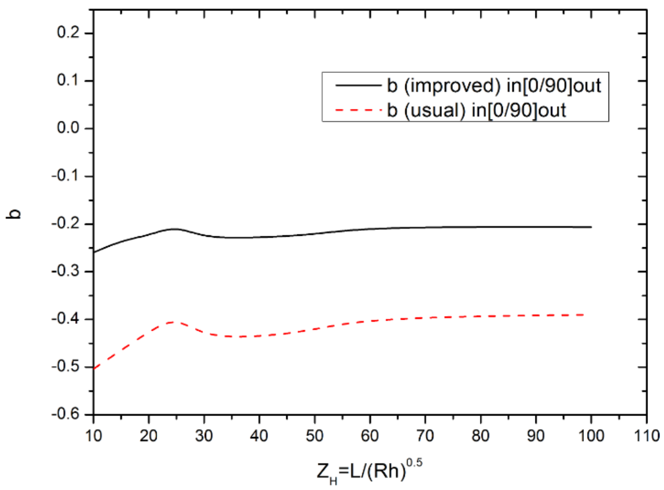

Fig.2 b coefficients versus the reduced-Batdorf parameters for $_{in}[0^o{}_t/90^o{}_t]_{out}$ antisymmetric cross-ply cylindrical shell under axial compression

Fit. 2 shows that there is always an about 50% positive shift from the usual b to the improved b when Z_H change from 10 to 100, which means the Koiter's general stability theory always significantly over estimate the imperfection sensitivity.

Conclusion

We successfully compare the postbuckling behavior of an antisymmetric cross-ply cylindrical shell by using Koiter's general theory, Hui's postbuckling theory and ABAQUS simulation. Compare with the Koiter's general stability theory, the valid region is significantly increased by using Hui's postbuckling theory. Also a significant positive shift of b coefficient is found.

Reference

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