A QUANTITATIVE ASSESSMENT OF RANDOM FIELD MODELS IN FINITE ELEMENT BUCKLING ANALYSES OF COMPOSITE CYLINDERS

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Abstract

Large scatter characterizes the collapse load of thin-walled structures due to their imperfection sensitivity. Random fields can be used to include imperfections in a finite element model. Principal component analysis and analytical covariance functions are matched to available correlation information of geometrical imperfection measurements. The random fields are realized by Karhunen-Loève expansion combined with Monte Carlo methods. The results of the different covariance models are compared with the deterministic collapse loads of the measured imperfections in the FE-model. This approach isolates the effect of the different covariance models from other inaccuracies such as boundary conditions, loading imperfections, etc. The results show that random fields models can improve predictions and the understanding of the structural behavior of thin-walled structures, especially by using data based PCA. Caution is recommended with regard to analytical covariance functions since they may fail to capture the complete behavior of the structure.

1. INTRODUCTION

The buckling load of thin-walled structures is known to be highly sensitive to imperfections. Imperfections in the loading, material properties, boundary conditions, geometry, etc. affect the buckling load of, in particular cylindrical, shells [1, 2]. In current practice, knockdown factors are used to account for these uncertainties in industrial applications.

Knockdown factors are derived from experimental data from the 1960ies and take the effects of all imperfections into account [3] in a cumulative way. This makes it very hard to reduce the knockdown factor if one can reduce the uncertainty or imperfection of one or more design variables. This may lead to overly conservative designs.

Nowadays, finite element software is widely used to determine the structural behavior of a given structure [4]. Detailed prediction of the buckling behavior of thin-walled structures requires a detailed and correct description of the imperfections. One needs a robust probabilistic framework and models that represent the imperfections to be modeled [5, 6].

Random fields have been used to model imperfections. Geometry, thickness, material, loading, etc. have all been the subject of investigations by means of random fields [7, 8, 9]. Most of this research was based on the use of exponential covariance functions that were fitted to measurement data in a limited number of cases.

It appears that the verification of such models is lacking in the literature. Usually, these models have been compared to the experimental failure loads. However, by comparing with the experimental failure load, the quality of the complete finite element model is evaluated, not the quality of the random field model. In such a comparison, it is tempting to attribute the differences to other uncertainties that were not taken into account, e.g. loading or boundary conditions. In contrast to this, the comparisons reported in this paper were done with respect to the deterministic imperfections applied to the finite element model. The differences between the models are now reduced to the way the imperfections were created; in this way, the effect of different descriptions of the random variations is isolated. This admits the assessment of the accuracy of the stochastic models in capturing the random influences as well as their predictive power.

2. PROBABILISTIC FRAMEWORK

In this paper, the analysis of the data and the simulation of random fields are based on the Karhunen-Loève expansion as well as on principal component analysis. A direct Monte Carlo simulation approach was used to generate the samples for the finite element computations. Due to the rather high computational cost, the analysis has to be performed with small sample size N. In such a situation it is mandatory to use correlation control in order to improve the empirical independence of the generated random variables. These ingredients will now be described in more detail.

The Karhunen-Loève expansion decomposes a second order stochastic process U into a set of orthonormal deterministic functions, based on the eigenvalues and eigenvectors of its covariance functions:

$$U(\mathbf{x}, \omega) = g(x) + \sum_{n=0}^{\infty} \sqrt{\lambda_n}\, \xi_n(\omega) \phi_n(x)$$

where ξ_n is a sequence of random variables determined by the process, $g(x)$ is the mean function and λ_n and $\phi_n(x)$ are the eigenvalues and eigenvectors, respectively, of the eigenvalue problem

$$\int_W C(x_1, x_2)\, \phi_n(x_2) dx_2 = \lambda_n \phi_n(x_1)$$

where W is the domain and $C(x_1, x_2)$ is the covariance function. The Karhunen-Loève expansion converges in the mean square sense; in numerical approximations, it is truncarred after a finite number M of terms, taking account of the largest eigenvalues. It is assumed here that the process U is Gaussian. Then the random variables $\xi_n(\omega)$ are independent and identically distributed according to a standard normal distribution.

A direct Monte Carlo approach was used for the simulations. In this approach, an artificial sample of size N of the set of input variables is generated and the computational model is evaluated for each realization of the input. This results in a sample of size N of the output variable, which can be further analyzed statistically. This simple approach was chosen because it allows easy implementation of a random field generator and can be used in combination with the available, deterministic, finite element packages such as ABAQUS or MARC to perform a non-linear buckling analysis [10].

When the sample size is small, as usually required in large FE calculations to save computational cost, and when the number of variables involved is appreciable, the multivariate output of a random number generator typically is weakly to moderately correlated, but not independent. To improve the performance of the analysis and reduce the number of required realizations, correlation control was applied. Correlation control is an empirical method that was proposed by Iman and Conover [11] that consists in a suitable rearrangement of the generated sample and leads to an approximately uncorrelated sample.

3. STOCHASTIC PROBLEM FORMULATION

The Karhunen-Loève method delivers an expansion of the covariance functions into orthogonal deterministic functions. These orthogonal functions can be derived in various ways. Section 3.1 explains how analytical functions can be used to assemble a covariance matrix. Section 3.2 shows how, by using principal component analysis, the eigenfunctions can be obtained more directly.

3.1. Covariance functions

Since the covariance matrix models contain the information of the spatial distribution of the imperfections, adequate modeling of the covariance matrix is key in producing trustworthy results. In the past, a variety of covariance models have been applied. It appears that a quantifiable comparison has not yet been reported in the literature.

In theory, every function that produces a symmetric, positive definite matrix can be used to assemble a covariance matrix. In practice, exponential or Gaussian covariance functions usual suffice to model the empirical data.

The exponential and Gaussian covariance functions for a homogeneous and isotropic random field are defined by

$$C(h) = \sigma^2 \exp(-|h|/\mathrm{L_c})$$

and

$$C(h) = \sigma^2 \exp(-h^2/\mathrm{L_c^2}),$$

respectively, where h is the distance between two points and $\mathrm{L_c}$ is the correlation length. The correlation length determines the variation of the random field over the structure. Since the geometrical imperfections that will be described by random fields are not necessarily isotropic, a slightly different representation is necessary for orthotropic random fields. If one assumes that the variation along different axes is independent, the correlation structure can be split along, e.g., the axial and circumferential direction:

$$C(r, \theta) = \sigma^2 c_r(r) c_\theta(\theta)$$

Here, $c_r(r)$ represents the correlation function along the axial direction and $c_\theta(\theta)$ the correlation function along the circumferential direction. An important issue in industrial applications with irregular geometry is the distance calculation over the surface. For surfaces with a sufficiently simple analytical expression, this is a trivial task. If such a representation is lacking, the assembly of the autocovariance matrix becomes computationally expensive.

One of the main advantages of using covariance functions is that one does not necessarily need imperfection data of the specific structure to assemble a covariance matrix and perform a random field analysis. Available imperfection data can be extrapolated to structures with different dimensions or even shapes. Whether these covariance functions can be an adequate representation of the imperfection will be investigated in Section 5 of this paper.

3.2. Principal component analysis

Principal component analysis is a statistical technique that extracts the main modes from a set of observations that, together, account for the major portion of the variance present in the observations [12]. Just as with the Karhunen-Loève method, these modes can then be used in a truncated expansion for simulating random fields with the same statistical distribution as the observed imperfections.

The covariance matrix $C = C(x_i, x_j)$ is directly calculated from the measured data at the points (x_i, x_j) by

$$C(x_i, x_j) = \mathrm{E}\left[U(x_i), U(x_j)\right] - \mu_i \mu_j$$

where $\mu_i = \mathrm{E}[U(x_i)]$ is the mean of the field at point x_i. Solving the eigenvalue problem $CV = VD$, the eigenvectors in V will form the basis of the expanded process, with the eigenvalues in D representing the weight of each eigenvector. Again, only a limited number M of elements of the basis will be included to represent the random process. PCA has been widely applied in computational statistics, e.g. in data mining and image compression. The main disadvantage of this approach for use with random fields is the necessity for measurement data. It is hardly possible to extrapolate parameters from data available elsewhere and match it to the case at hand.

4. STRUCTURAL ANALYSIS

To evaluate the random field models, a finite element model to determine the failure load of a thin-walled cylinder was created. In this comparative study, the measured geometrical imperfections were used as input of the FE model on the one hand; on the other hand, simulated imperfections based on the different random field models were entered in the FE calculations.

When investigating the buckling behavior of thin-walled structures, large displacements arise. A nonlinear analysis is necessary to make a prediction of the failure load. Because of the large instability of the problem and the necessity to decrease the user input during the analysis as much as possible a loading-driven arc-length approach was used. Failure of the structure was defined by the occurrence of a load-drop of 20% compared to the highest load reached during the previous steps. The highest load reached during the analysis is defined as the failure load. The analysis was ended when failure occurred; no post-buckling analysis was performed. This allows one to assume that the strains remain small and therefore stay in the elastic regime.

Thin-walled structures are very sensitive to the applied boundary conditions and loading. Since the main goal of this research is the comparison of the random field models and not the accurate modeling of the real failure load, it was decided to use easy to model boundary conditions and loading. The evaluation of the model was done with respect to the results of the same finite element model where the measured imperfections were applied. This means that the only differences come from the models of the imperfections, but none are from the discrepancies between reality and the finite element model.

Boundary conditions are applied at the top and bottom of the cylinder. For the bottom nodes, all the translational degrees of freedom are constrained. At the top the same boundary conditions are applied, but the axial direction is left free. In this direction, a compressive load is applied. The load is applied at the center of the cylinder and connected to all the top nodes using a multi-point constraint. A mesh convergence analysis showed that a mesh with around 11.000 elements was a good compromise between numerical accuracy and analysis time.

The numerical model was implemented in ABAQUS. Since a thin-walled cylinder needs to be modeled, the S4 and S4R elements in the ABAQUS library are the most appropriate. A comparative study showed that the gain in computational efficiency of the S4R element did outweigh the gain in accuracy of the S4 element, thus the S4R element was chosen.

The empirical data for this research was made available by [13] and consisted of scans of the geometry of 10 cylinders made of composite material.[1] From these measurements, it was possible to derive the local imperfect radius of the cylinder at around 200.000 points per cylinder. Since the measurement mesh was too fine for the finite element computations and the correlation analysis, the data had to be broken down to the coarser FE mesh. Using the imperfect radius of the measured point closest to each grid point of the finite element description was judged to be enough. The imperfections were applied onto the finite element model using the *IMPERFECTION option provided by ABAQUS. This option allows one to include imperfections in the finite element model without getting errors about distorted elements.

To set the stage, Figure 1 shows a comparison of the different buckling loads for each cylinder, using the FE model without imperfections, the FE model with the measured imperfections added in a deterministic way, and the experimentally measured buckling loads.

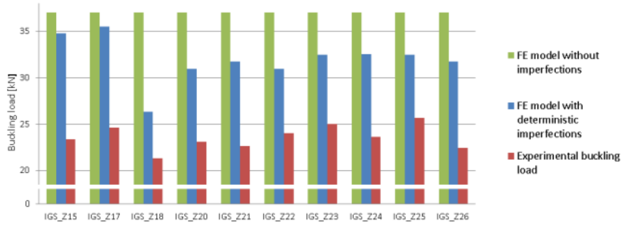

FIGURE 1. Comparison of the failure loads showing the discrepancies between finite element failure load predictions with and without geometrical imperfections and the experimental buckling load

5. RESULTS

5.1. Setting up the random field model

The primary goal of this research was to assess the predictive quality of different approaches to modeling and creating random fields as representations of geometrical imperfections. The comparisons following hereafter are made with respect to the failure loads of the finite element model with the measured geometrical imperfections directly applied to it. For reasons explained in the introduction, they are not compared directly to the experimental buckling loads.

After investigation of the probability functions of the measurements it was found that the assumption of a Gaussian distribution was acceptable. A more problematic observation was the large variation of the standard deviation of the local radii. Investigations showed that these can vary up to almost a factor ten. A closer investigation revealed two sets of cylinders with distinctly different standard deviations. A visual investigation of the imperfections confirms the existence of these sets. Figure 2 shows the imperfect radius for two different cylinders which are both representative for a set.

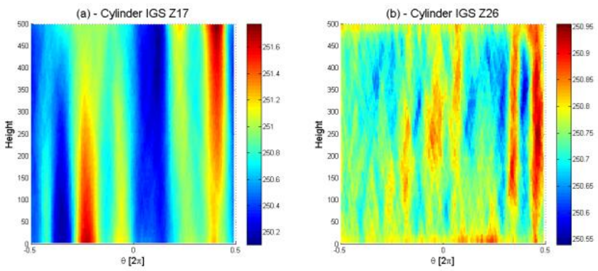

FIGURE 2. Geometrical imperfections of two cylinders. Cylinder IGS Z17 is part of set 1, cylinder IGS Z26 is part of set 2, data from [13]

One can see that the two sets exhibit different imperfection structures, which is also confirmed by the calculation of the correlation functions in the next section.

[1] The nominal values were: radius $250\ mm$, height $500\ mm$, thickness $0.5\ mm$, four layers, lay-up $24°, -24°, 41°, -41°$.

Based on this observation it was decided to divide the available sample in two different sets and to form the analysis separately. The average mean and standard deviation of each set are reported in Table 1.

Set	Sample Size	Mean	Standard Deviation
Set 1	3	250.78	0.321
Set 2	6	250.73	0.068

TAB 1. Average mean and standard deviation of the geometrical imperfections per set

The visual inspection also revealed the existence of an outlier. Because it was impossible to make any assumptions about the reason for the deviating imperfection shape, it was decided to remove this realization from the available sample. This reduced the sample size from 10 to 9.

5.2. Theoretical covariance models

As one can see in Figure 2, the imperfections are direction dependent. This observation was confirmed by calculating the correlation functions independently. Figure 3 shows the correlation functions in axial and radial direction.

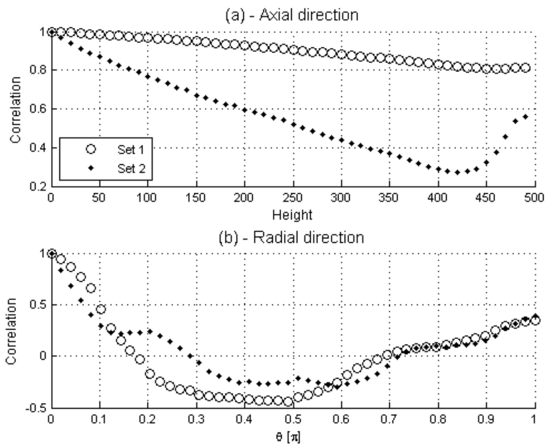

FIGURE 3. Empirical correlation functions of the geometrical imperfections in axial and radial directions

The correlation is higher in the axial direction then in the radial direction. In axial direction, the cylinders are clearly different; set 2 has a lower correlation than set 1. It is less obvious from Figure 2 that the correlation function in the radial direction is similar for both sets.

Direction	Correlation Function
Axial and Radial	$c\ (r) = \exp(-\lvert r\rvert/\mathrm{CL_{ax}})$
Axial and Radial	$c(r) = \exp(-r^2/\mathrm{CL_{ax}^2})$
Radial	$c\ (\theta) = \exp(-\lvert\theta\rvert/\mathrm{CL_{rad}})\cos(\lvert\theta\rvert/P)$
Radial	$c(\theta) = \exp(-r\theta^2/\mathrm{CL_{rad}^2})\cos(\lvert\theta\rvert/P)$

TAB 2. Fitted correlation functions

The theoretical correlation functions shown in Table 2 have been fitted to the empirical correlation functions. The exponential correlation function gave the best fit in axial direction. The Gaussian correlation function did not fit well at $r = 0$ since the slope of the Gaussian correlation function is zero at this point. In radial direction, the mix of the cosine and the exponential correlation function gave the best fit.

Using these theoretical correlation functions, $N = 100$ random fields were generated and applied to the finite element model to evaluate the failure load. Figure 4 exemplarily shows the resulting failure loads in a histogram compared to the deterministic failure loads of the imperfections.

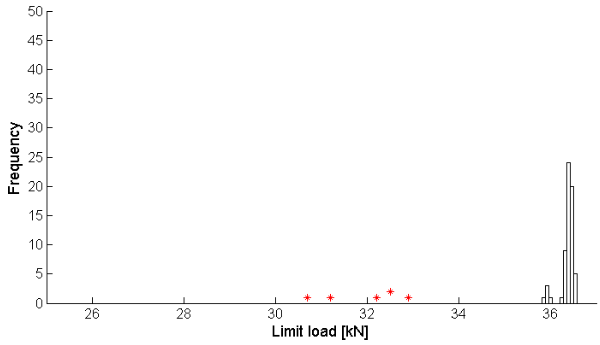

FIGURE 4. Histogram of the failure load of a theoretical covariance function with independence assumed along axial direction $c\ (r) = \exp(-\lvert r\rvert/\mathrm{CL_{ax}})$ and radial direction $c\ (\theta) = \exp(-\lvert\theta\rvert/\mathrm{CL_{rad}})\cos(\lvert\theta\rvert/P)$; stars indicate the results of deterministic FE-calculations

It becomes clear that the theoretical correlation model conspicuously overestimates the failure load. Further experiments showed that artificially increasing the standard deviation would reduce the average failure load, but increase the spread of the failure load. Further, the effects of choosing different correlation functions turned out to be small, especially when compared with the effect of the standard deviation.

5.3. Principal component analysis

As an alternative to the theoretical covariance functions, PCA is proposed. When using PCA, the assumption enters that the given measurements are a valid (and complete) representation of the main characteristics of the imperfections. As explained in Section 3.2, PCA obtains the eigenvectors and eigenvalues directly from the covariance matrix of the available measurements. These eigenvectors and eigenvalues can then be used in an orthogonal expansion to assemble the random fields.

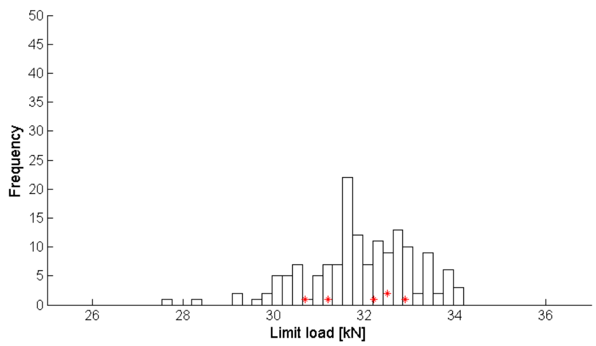

FIGURE 5. Histograms of the failure load of a PCA-analysis; stars indicate the results of deterministic FE-calculations

A typical result from the Monte Carlo simulation using PCA is displayed in Figure 5. As one can see from this figure, the predictions are much better when using PCA to obtain the eigenvectors and eigenvalues, which captures the characteristics of the random field more closely from the available measurement data.

A large number of further numerical experiments with various combinations of covariance functions and values for the standard deviation confirmed the findings.

6. CONCLUSIONS

Different covariance models have been evaluated to check their validity to represent general geometrical imperfections. The models with random fields have been compared with the failure loads of the finite element model when the measured imperfections are applied. By comparing to the deterministic imperfect finite element failure loads, the effect of the geometrical imperfection is isolated from other influences, such as boundary or loading imperfections.

It was shown that the theoretical covariance models overestimate the failure load and do not provide an accurate prediction of the failure load. The results of the PCA are better, but only work well when the sample size is large. In this case, PCA is able to capture the lowest failure loads.

An overall conclusion is that it is important to validate stochastic models before applying them to engineering problems. A mere comparison of the computational results with the experimental buckling loads to evaluate correlation models may be misleading, especially for shells where different sources of imperfections can have a large influence on the resulting failure load. A better approach consists in a comparison with the measured imperfections applied to the finite element model.

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