

## **Analysis of Structures under Uncertainty**

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### **ABSTRACT**

Theories of uncertainty are applied to a large range of fields, and they have a special interest on engineering, because of safety and risk concerns related to human lives. This concern is very high on aerospace applications, where safety is truly the most critical requirement. The most common approach to deal with uncertainties due to lack of knowledge and inherent variability on aerospace structures is through the application of safety factors, which are design margins against failure. Safety factors are applied on all scales from simple components to more complex systems. Within this paper, inherent variabilities are expressed through statistical distributions, which is an alternative approach to traditional safety factors. Structures are interpreted as systems whose reliability level should be assessed. The range of application of these theories is large, and two examples related to aircraft structures are discussed in this work. In first example, the reliability analysis is applied to estimate the inspection interval for a typical structural aluminum joint, and in the second one, the reliability a composite structure under progressive damage until collapse is evaluated. The results obtained are presented and discussed.

**Keywords:** Aerospace Structures, Uncertainty, Reliability.

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## 1. INTRODUCTION

Development of new aircraft generally requires a detailed verification of the structure characteristics concerning static strength, fatigue resistance and damage tolerance. Such a work is carried out in order to comply with certification requirements established by national certification agencies as FAA (Federal Aviation Administration), EASA (European Aviation Safety Agency) and others. Every material, manufacturing process and structural design that is proposed by the aircraft manufacturer (OEM) requires background information (for example, material allowable and geometric tolerances).

Some aerospace materials such as 2000 and 7000 aluminum alloys have been used during decades, so that their specifications have been defined by MMPDS [1] and other organizations, while the OEM main tasks has been (i) to assure that the material supplier is following these specifications and (ii) to dimension the aircraft structure according to the same specified values. It should be noted that many aluminum alloy specifications are actually established by the OEMs, and these values are usually tighter than the allowable ones from MMPDS.

Taking e.g. MMPDS properties, they result from test samples and are often available in A-, B- and S- basis for each alloy. While the S-basis reflects the minimum values specified by governing industry specifications, the A-basis usually comes from a large amount of tests performed, where 99% of the population exceeded the property value with 95% of confidence, and the B-basis follows a similar approach, where 90% of the population exceeded the property value with 95% of confidence. Hence, these properties may be interpreted as deterministic values, although they result from statistical treatments. The fatigue (S-N) curves presented in MMPDS are also obtained from statistical treatments and reflect a least square fitting. Many companies apply a scatter factor ([2] and [3]) in order to assure that at least 90% of the population exceeds the property with 95% of confidence.

Regarding crack propagation in metallic materials, most of the  $da/dN$  vs.  $\Delta K$  curves represents average trends obtained from tests with standard specimens. Usually, from these curves, the analyst obtains an average crack propagation behavior. Uncertainties in this process are also accounted for when the inspections intervals are defined by applying a safety factor, which may vary depending on the loading conditions and structural categories.

Composite materials are also subjected to large scatter of their properties. A good basis for statistical analysis and recommendations for testing samples and statistical treatment of data can e.g. be taken from MIL-HDBK-17 [4]. Further, due to the nature of composite material manufacturing process, there is generally a larger scatter compared to metallic materials that must be accounted for in order to assure appropriate assembly. The scatter of composite allowable is appropriately accounted for throughout the A-basis and B-basis, which were above described.

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Uncertainties are therefore intrinsic in the aircraft structural design through variations in material properties, dimensions, loading, environment etc. Many approaches (and the concept of A-basis and B-basis may be included) substitute statistical variations by critical values and may become apparently deterministic approaches. However, from the point of view of damage tolerance, as discussed in [13], statistically accounting for these uncertainties may contribute to a more consistent maintenance plan that may significantly reduce the costs of aircraft operation.

This paper addresses the application of the risk analysis to the design of aircraft metallic and composite structural systems. It deals with a probabilistic approach for the damage tolerance analysis, which herein eventually aims at improving the parameters for establishing maintenance plans and at bringing advances concerning aircraft structural integrity. In the applications, uncertainties applied to metallic and composite structures are discussed. Comparisons with the safety-factor design process are also presented.

## **2. THEORETICAL BACKGROUND**

### **2.1 Uncertainty**

Uncertainty in engineering is a rich research field. Several authors devoted to research on this field have presented definitions and classifications of uncertainty.

Uncertainty in engineering can be classified as irreducible (or aleatory), reducible (epistemic) and error. Irreducible uncertainty is an inherent variation associated to a physical system or the environment under consideration. Reducible uncertainty is a potential deficiency in any phase or activity of the modeling process that is due to lack of knowledge. Error is a recognizable deficiency in any phase or activity of modeling and simulation that is not due to lack of knowledge [5].

Besides the deterministic approach based on safety factors, there are several non-deterministic methods to consider uncertainty in engineering, such as: probabilistic, fuzzy-sets, and convex-sets / interval arithmetic's [6].

In this paper, only inherent uncertainties are considered and the analysis of uncertainties is undertaken through probabilistic approaches.

### **2.2 Structure as a system**

The performance of a mechanical structure could be evaluated as an engineering system. In order to understand this assumption, one should review a modern definition of system: "Simply stated, a system is an integrated composite of people, products, and processes that provide a capability to satisfy a stated need or objective" [7], or "An aggregation of end products and enabling products to achieve a given purpose" [8]. In this context, Halligan [9] defines "Systems Engineering" as a process to manage and design engineering systems.

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Although these definitions are broad and applied to many types of systems, one should observe that any system shall perform at least one function, and this function is the objective or purpose of the system. This characteristic is always found in any structural system.

Reliability is typically used as a measure of performance of systems. The most typical definition of reliability is based on probability of failure: "The probability that a component part, equipment, or system will satisfactorily perform its intended functions under given circumstances, such as environmental conditions, limitations as operational time, and frequency and thoroughness of maintenance, for a specified period of time" [10]. But this definition of reliability based on the frequency of occurrence of (failure) events is not unique. One may find also a non-probabilistic definition of reliability: "In the non probabilistic formulation of reliability, a system is reliable if the range of performance fluctuations is acceptably small" [11].

Main difficulties, arising from probabilistic structural analysis is the number of design variables that should be taken into account, as well as the limited knowledge of the variation of such variables is also currently limited. In terms of engineering, the number of variables should be reduced so that the problem can be solved within reasonable time, and some distributions of design variables should be assumed, due to the lack of data. But such assumptions have also consequences, as described by Thoft-Christensen [12]: "A real structural system is so complex that direct exact calculation of the probability of failure is completely impossible. The number of possible different failure modes is so large that they cannot all be taken into account; and even if they could all be included in the analysis, exact probabilities of failure cannot be calculated".

In this paper, the probabilistic approach is adopted to the evaluation of the reliability of structural systems. A case study on the reliability of a damage tolerant structure is reviewed [13], and a case study is introduced about the reliability of a composite structure (idealized as) system.

### **2.3 Material progressive failure**

This topic is of interest for both metallic and composite materials, but herein an example from composite materials is explored. Material progressive failure analysis is currently available in several commercial finite element solvers, such as MD NASTRAN, ABAQUS, ANSYS, or GENOA.

The implemented failure criteria are usually macromechanical or micromechanical. According to the respective techniques, the material ply properties can be degraded selectively if matrix or fiber failure occurs, and the analysis continues until total failure of the laminate (collapse). No solver "restart" is necessary on this non-linear material analysis procedure. Progressive failure is not only useful to capture the residual strength of the structure, but it may also be used to map the failure path of the structure.

The standard post processing of material progressive failure usually gives information for each element, which ply has failed (damaged). Figure 1 illustrates an example, in which for element id = 1, ply 1001 is failed but ply 1002 is not failed.

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ELEMENT ID	PLY ID	INTEG. POINT ID	FAILURE THEORY	MAXIMUM FAIL. INDEX	STRENGTH RATIO	FAILURE MODE	TOTAL DAMAGE
1	1001	1	MAXSTRES	1.647E-01	6.071E+00	4	9.990E-01
		2	MAXSTRES	1.647E-01	6.071E+00	4	9.990E-01
		3	MAXSTRES	5.136E-01	1.947E+00	1	9.990E-01
		4	MAXSTRES	5.136E-01	1.947E+00	1	9.990E-01
	1002	1	MAXSTRES	8.534E-01	1.172E+00	1	0.000E+00
		2	MAXSTRES	8.534E-01	1.172E+00	1	0.000E+00
		3	MAXSTRES	8.487E-01	1.178E+00	1	0.000E+00
		4	MAXSTRES	8.487E-01	1.178E+00	1	0.000E+00

Figure 1 – Standard post processing of material progressive failure within MD NASTRAN.

Consider a finite element model with several thousands elements. In order to know if a ply failed in all elements, one may need to verify the failure status element by element and ply by ply using current standard output. In terms of system analysis, this standard output should be processed to give more useful information: for each load step, which ply is safe, damaged, or failed considering all elements that share the same ply. Figure 2 illustrates a laminate system of three plies and its corresponding failure path. Using this technique, not all possible failure paths have to be statistically verified, but only those paths identified through progressive failure analysis.

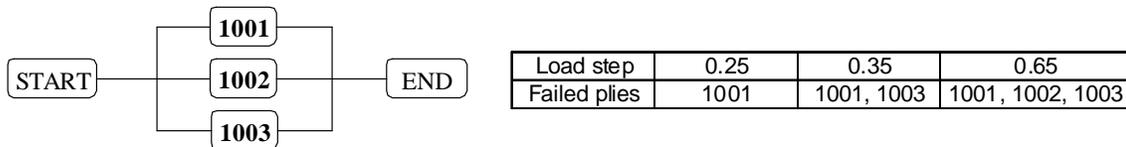


Figure 2 – System of three plies and corresponding failure path.

An assumption adopted is that the failure path mapping through material progressive failure analysis can be used to calculate the reliability of a structural composite system.

### 3. UNCERTAINTIES TYPICAL WITH DESIGN OF METALLIC STRUCTURES

#### 3.1 Static analysis

As an example to show the variabilities in static properties, qualification tests were performed for a sample of one hundred 7050-T7451 aluminum test specimens in order to verify the ultimate tensile allowable. Figure 3 shows the frequency of occurrence of ultimate tensile stress varying from 62.93 ksi to 68.30 ksi. By postprocessing this data, a mean value of 65.85 ksi can be found for a Normal distribution, while a pseudo-mean value of 65.36 ksi can be found for a three-parameter Weibull distribution. The A-basis value obtained from this analysis was 62.88 ksi and the B-basis value was 64.17 ksi.

From this simple example, a procedure can be drawn on how to obtain static material allowable from statistical data. The observed variability on the above graphic is typical for other material properties, which justifies the need of a statistical description.

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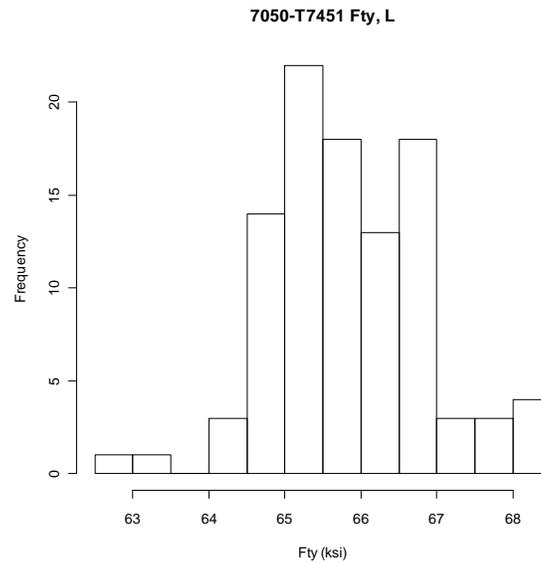


Figure 3 – Distribution of experimental static tensile ultimate stress (Fty).

### 3.2 Fatigue and Damage Tolerance Analysis

This investigation aims at an estimation of inspection interval for a typical structural aluminum joint. A fastened butt joint configuration was taken as example of application of uncertainties in damage tolerant analysis, as outlined in Figure 4(a). This joint was tested in order to verify the fatigue life of fuselage circumferential joints subjected to different fastener interference levels. The material is Al 7475 plate alloy. The S-N curve was obtained for certain load levels, from which one level was selected for this analysis. By taking the fastener row with the highest load transfer, a crack propagation model was idealized and NASGRO software [16] was used for crack growth analysis. Figure 4(b) shows the NASGRO model that was used for idealization of the critical region. A complete description of this problem can be found in [13].

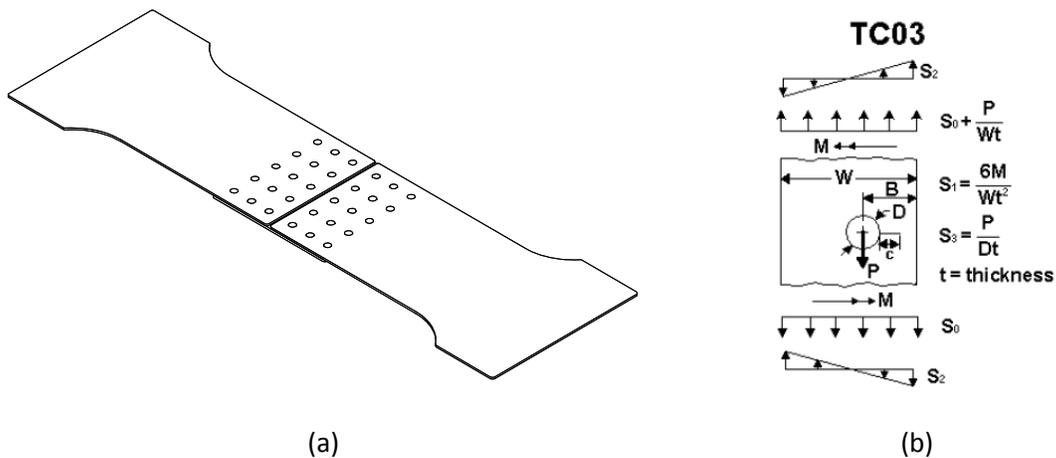


Figure 4: (a) Outline of test specimen, (b) NASGRO input configuration corresponding to most loaded row.

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**Geometric Variables:** for the configuration shown in Figure 4(b), some level of variability was assumed for the main geometry dimensions, as follows: (a) the plate thickness is equal to 2.0 +/- 0.1mm, the hole diameter (D) is equal to 10.0 +/- 0.05mm, the plate width (W) is equal to 100.0 +/- 1.0mm and the offset (B) is equal to 40.0 +/- 1.0mm. These are theoretical values, where for example the changes in interference levels due to variations in the hole diameter are not accounted for. Actual aircraft structure values (reflecting the manufacturing quality) may be obtained by means of GD&T (geometrical dimensioning and tolerance) evaluation results.

**Loading Variables:** the statistical representation of the loading behavior would require a series of assumptions that were deemed undesirable by the authors in the scope of the present work. For example, if circumferential joints in fuselages are being evaluated, the spectrum loading due to gusts and maneuvers (symmetric and non-symmetric) should be compared to the pressurization load at each section where circumferential joints are present along the fuselage. Here, for the sake of simplification, the nominal stress  $S_0$  was assumed as 100 +/- 10 MPa for this work, and the resulting normal distribution was used.

**Material Variables:** from the S-N curve at the selected load level, the EIFS (equivalent initial flaw size) distribution was obtained. However, as long as the amount of experimental data was scarce, a Monte Carlo simulation was performed from this data, and the resulting initial flaw ( $A_0$ ) lognormal distribution was used.

The crack propagation ( $da/dN \times \Delta k$ ) behavior was defined by the Modified Forman Equation and input to NASGRO in tabular format. Although this equation presents a range of variables and fitting parameters, the general material behavior is according to the Paris Equation, as follows:

$$\frac{da}{dN} = C(\Delta K)^n$$

Where  $\Delta K$  is the stress intensity factor variation,  $C$  and  $n$  are fitting parameters. In order to represent usual variability in crack growth behavior, the  $C$  coefficient was also defined through a lognormal distribution.

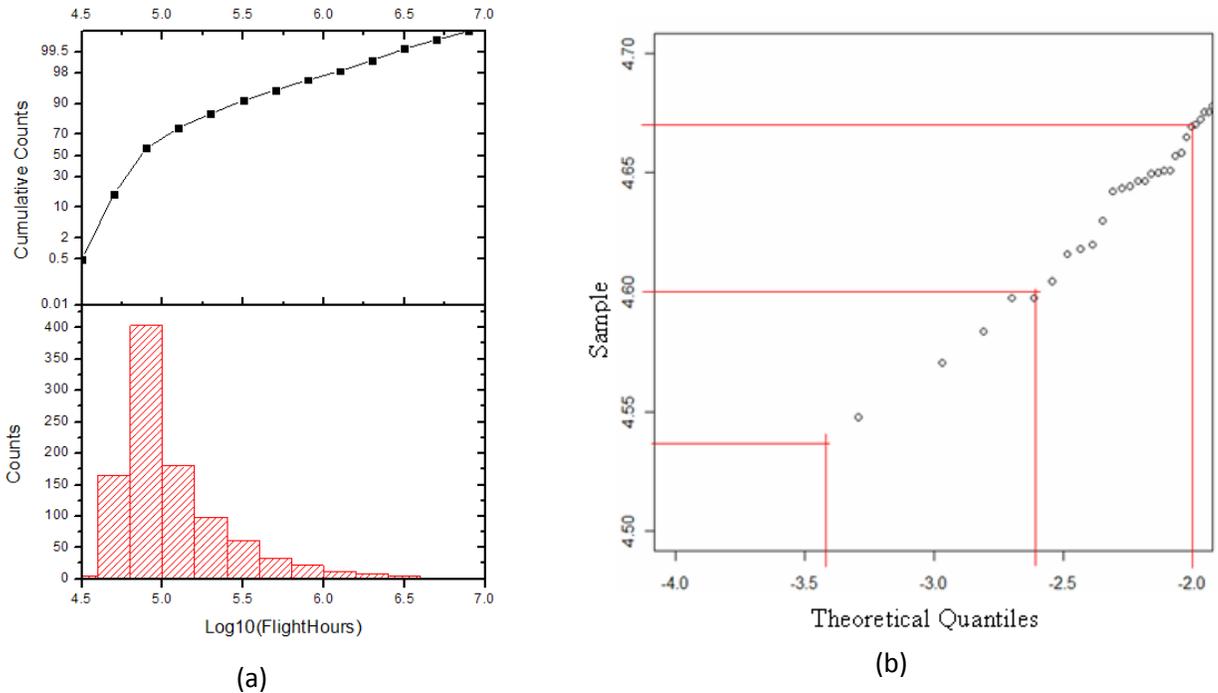
**Risk Analysis:** the results from the risk analysis by many runs of NASGRO and subsequent statistical analysis in terms of flight hours (where one hour corresponds to one flight cycle) are shown in Figure 5. This figure presents the plots in logarithm scale and includes a table showing the number of hours as function of the reliability.

As a comparison, performing a deterministic crack propagation analysis where all parameters are set as the average values from the input distributions, the total number of flight hours in this case is 101,849, and assuming a safety factor of three, the corresponding inspection Interval will be 33,950 flight hours.

From the statistical analysis performed, it is now possible to determine the reliability for this inspection interval, and it is equal to 99.9%. If a reliability of 95.5% (equivalent to two standard

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deviations) is acceptable for this component, the new inspection interval may be equal to 46,774 flight hours. This represents an increase of 37.8% in terms of flight hours, which would represent a significant economy for the airline operator.



- (a) Probability density function result for the number of flight hours, in logarithm scale.
- (b) Correlation between theoretical quantiles and sample results, showing the corresponding number of flight hours for 50.0%, 95.5%, 99.1% and 99.9%.

Total Flight Hours	Log10	Quantile	Reliability
39,811	4.60	-2.6	99.1%
46,774	4.67	-2.0	95.5%
101,849	5.01	0.0	50.0%
33,950	4.53	-3.4	99.9%

Figure 5: General analysis results

**4. UNCERTAINTIES TYPICAL WITH DESIGN OF COMPOSITE STRUCTURES**

A composite structure has several sources of uncertainty. These sources are due to composite material properties inherent variations, and manufacturing process inherent variations such as geometric, angle, thickness, and pre-stress variations. Loading and constraint boundary conditions also may have large variation, and usually play an important role within uncertainty analyses. Some of these variations might be reduced in some extent by improvements of manufacturing and engineering processes, but they could never be completely eliminated. Therefore, a trade-off between the degree of tolerated uncertainties and economic constraints is part of engineering concerns.

In this section, a composite plate submitted to tensile load is analyzed until collapse under material progressive failure. The composite plate is defined herein as a structural system of parallel sub-systems, in which each ply of the laminate is a sub-system, which components are

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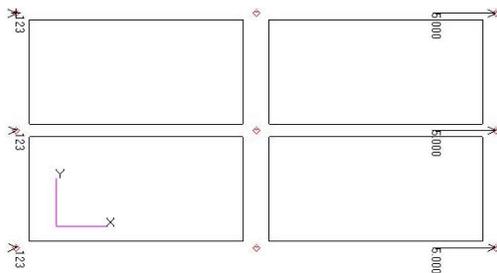
fiber and matrix. If no ply is failed, the system is defined as “Safe”. If at least one ply is failed, the system is defined as “Damaged”. When all plies of the system are failed, the system assumes a “Failed” status. The sequence of ply failure is defined as failure path of this system.

Firstly, the structure is analyzed with the nominal values of the variables, i.e., the values that should be used in case of deterministic design. The result of this simulation is used to compare with a non-deterministic analysis. Nevertheless it is always recommended to perform a preliminary deterministic analysis, because one may be able to check the problem set up and get a better feeling of the model, before a possible time consuming non-deterministic analysis.

Secondly, variations are added to input variables, and a numerical experiment is set up and performed. The added variations are assumed uniform in the interval, in which the nominal value is also the mean value. The results of the progressive failure of the structure are observed for each numerical experiment, in which the structure is always analyzed until collapse.

**4.1 Deterministic analysis**

The composite plate is modeled as a finite element model with four shell elements. The translations XYZ (123) are fixed at one end, and the model is pulled at the other end with a total displacement of 5.00 mm on the X direction. Figure 6 shows the finite element model, and Table 1 lists the lay up of the laminate. The laminate property was input as PCOMPG, using MD NASTRAN nomenclature. This allowed that each finite element that shares the same ply could have the same ply ID, which is called a global ply.



**Figure 6 – Finite element model.**

Ply ID	Angle
1001	90.
1002	0.
1003	45.
1004	-45.
1005	-45.
1006	45.
1007	0.
1008	90.

**Table 1 – Laminate stacking sequence.**

This modeling technique is convenient, because the solver gives as standard answer the failed (global in this case) ply for each finite element at each load step of the progressive failure analysis. The failure of an individual finite element, as well as the failure of each ply across all finite elements is monitored. If a ply is failed in one or more finite elements, but not in all finite elements sharing this ply, it will receive the “Damaged” status. As a consequence, the ply “Failed” status is assumed only when the ply is failed in all finite elements, which share it.

Table 2 lists the nominal values of the input variables, which are used in the deterministic case, as well as the corresponding variation assumed, for the non-deterministic case. Variables from E1 to FB are unidirectional layer material properties of AS4-3501 (carbon epoxy).

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<b>Random Uncertainties</b>	<b>Mean</b>	<b>Max</b>	<b>Min</b>	<b>Unit</b>
Load	5	6	4	mm
Layer angle	0	2.5	-2.5	Degree
Layer thickness	0.25	0.263	0.238	mm
E1	142000	149100	134900	MPa
E2	10300	10815	9785	MPa
G12	7200	7560	6840	MPa
n12	0.27	0.284	0.257	
F1t	2280	2394	2166	MPa
F1c	1140	1197	1083	MPa
F2t	57	59.9	54.2	MPa
F2c	228	239.4	216.6	MPa
F3t	57	59.9	54.2	MPa
F3c	228	239.4	216.6	MPa
F12	71	74.6	67.5	MPa
F23	71	74.6	67.5	MPa
F31	71	74.6	67.5	MPa
FB	7.1	7.46	6.75	MPa

Table 2 – Input variables with a uniform distribution.

Table 3 lists the failure sequence according to the load step, where “PID” is the global ply identification number (PCOMPG), and step is the load step. Notice that this table summarizes the “Failed” status of each global ply at a given load step, and not a damaged intermediate status, although this status was also mapped.

<b>PID</b>	<b>step (%)</b>	<b>mm</b>
1001	0.25	1.25
1002	0.65	3.25
1003	0.35	1.75
1004	0.35	1.75
1005	0.35	1.75
1006	0.35	1.75
1007	0.65	3.25
1008	0.25	1.25

Table 3 – Summary of Failed Ply per Load Step.

From the table above, one can verify the following failure path for this laminate:

- a) at load step 0.25 (1.25mm): plies 1001 and 1008;
- b) at load step 0.35 (1.75mm): plies 1003, 1004, 1005 and 1006;
- c) at load step 0.65 (3.25mm): plies 1002 and 1007.

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#### 4.2 Non-deterministic analysis

As already discussed, a laminate can be idealized as a structural system of parallel sub-systems, in which each ply of the laminate is a sub-system. Figure 7 shows this modeling idealization, by the use of a reliability block diagram (RBD) of the layup.

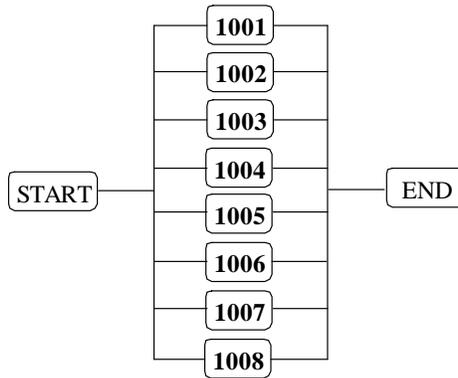


Figure 7 – Reliability block diagram (RBD) of laminate layup.

Parallel systems are also referred as systems of redundant units. At least one unit must succeed for the system to succeed. In the case where the failure rate of one ply affects the failure rates of others, conditional probabilities must be considered. This is the case for structural composite systems. The unreliability of the parallel system represented at the RBD of Figure 7 can be calculated as [14]:

$$Q_s = P(X_1 \cap X_2 \dots \cap X_8) = P(X_1) \cdot P(X_2 / X_1) \dots P(X_8 / X_1 X_2 \dots X_7)$$

Where,

- $Q_s$ : unreliability of the system;
- $X_i$ : event of failure of ply  $i$ ;
- $P(X_i)$ : probability of failure of ply  $i$ .

The reliability can be calculated as:

$$R_s = 1 - Q_s$$

This non-deterministic analysis was run for 996 experiments, created by Sobol technique implemented in modeFRONTIER [15]. As observed in the deterministic case, due to layup and loading boundary condition, the results for ply 1001 are similar to the results for ply 1008; the results for ply 1003 are similar to the results for plies 1004, 1005, and 1006; and the results for ply 1002 are similar to the result for ply 1007. For this reason, only the results for plies 1001, 1002 and 1003 are presented in the following, which are representative of the failure path.

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In average, it was verified that ply 1001 fails at load step 1.247mm with a standard deviation of 0.093mm; ply 1003 fails at load step 1.641mm with a standard deviation of 0.096mm; and ply 1002 fails at load step 3.419mm with a standard deviation of 0.160mm. Comparing this result with the deterministic case, one can verify that the deterministic result is in the range of variation of the non-deterministic methodology within two standard deviations. Although the mean value and standard deviation are good estimators of the sample, the usage of CDF (cumulative density function) is more useful to evaluate the reliability of the system.

Figure 8, Figure 10 and Figure 12 illustrate the probability density function (PDF) of ply 1001, 1002 and 1003 respectively. The continuous line is a curve fitting of current data, and the dashed line is a normal distribution approximation. Figure 9, Figure 11, and Figure 13 illustrates respectively the cumulative density function (CDF) of ply 1001, 1002, and 1003 respectively.

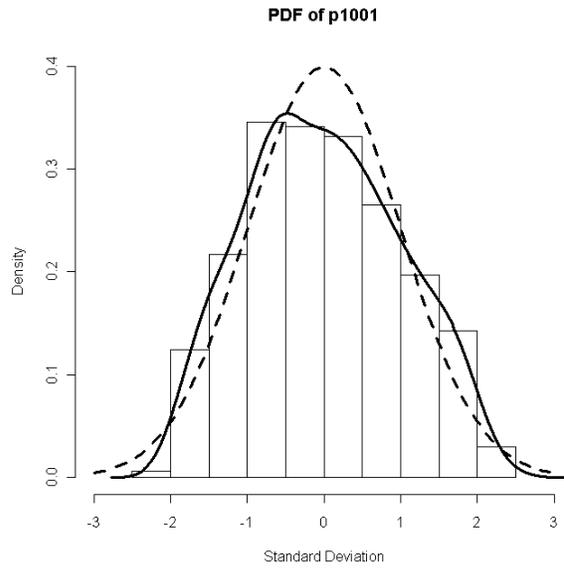
Using the CDF of ply 1001, one can conclude that it has a reliability of 93.98% at load step 1.1mm, and a reliability of 5.22% at load step 1.4mm. Using CDF of ply 1003, one can conclude that it has a reliability of about 89.46% at load step 1.5mm, and a reliability of 2.71% at load step 1.8mm. One should have in mind that the reliability of ply 1003 is a conditional reliability, because ply 1001 already failed. Similarly, using CDF of ply 1002, one can conclude that it has a reliability of about 98.52% at load step 3.1mm, and a reliability of 4.12% at load step 3.7mm. Once more, one should have in mind that the reliability of ply 1002 is a conditional reliability, because plies 1001 and 1003 already failed. Table 4 summarizes these results.

It is also given in Table 4 the reliability level of the deterministic analysis. A load step equal to 1.25mm corresponds to a reliability of 45.38%, a load step equal to 1.75mm corresponds to a reliability of 12.25%, and a load step equal to 3.25mm corresponds to a reliability of 82.83%. The reliability of last ply corresponds also the reliability of the ultimate load of the laminate system. Instead of applying a safety factor e.g. of 50% for the ultimate load as a design criterion, one can establish e.g. a desired reliability level e.g. of 99.7% (three standard deviations). The ultimate load might be equal to 2.98mm using this criterion, instead of 2.17mm based on a deterministic safety factor of 50% on the load step of 3.25mm.

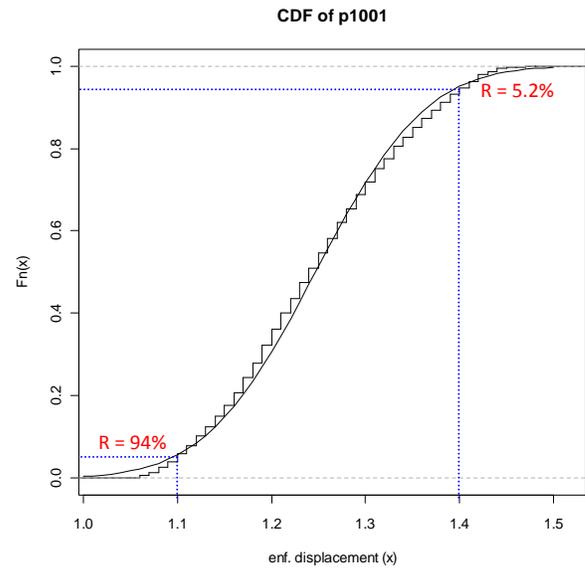
Load (mm)	Failed PID	Reliability
1.10	<b>1001</b>	93.98%
<b>1.25</b>		<b>45.38%</b>
1.40		5.22%
1.50	<b>1003</b>	89.46%
<b>1.75</b>		<b>12.25%</b>
1.80		2.71%
3.10	<b>1002</b>	98.59%
<b>3.25</b>		<b>82.83%</b>
3.70		4.12%

**Table 4 – Reliability level per load step (enforced displacement).**

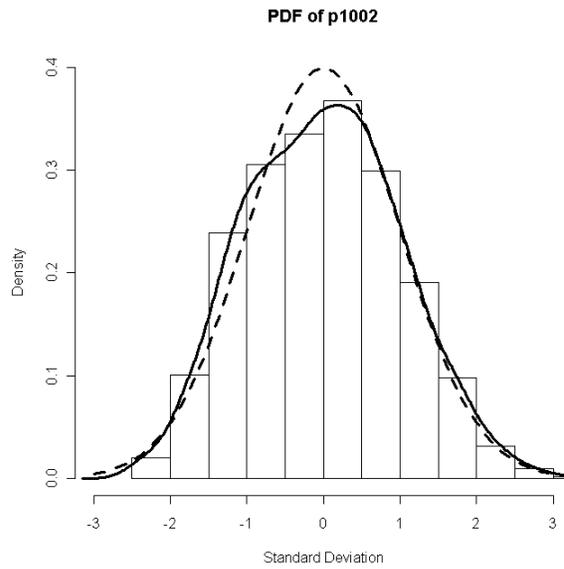
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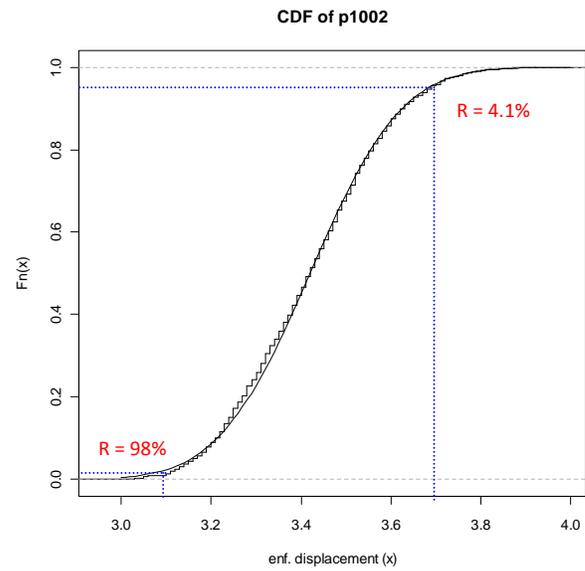
**Figure 8 – PDF of ply 1001.**



**Figure 9 – CDF of Ply 1001.**



**Figure 10 – PDF of ply 1002.**



**Figure 11 – CDF of Ply 1002.**

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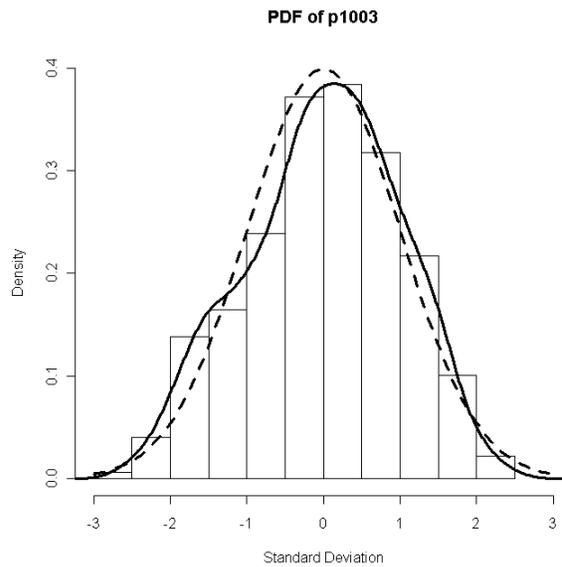


Figure 12 – PDF of Ply 1003.

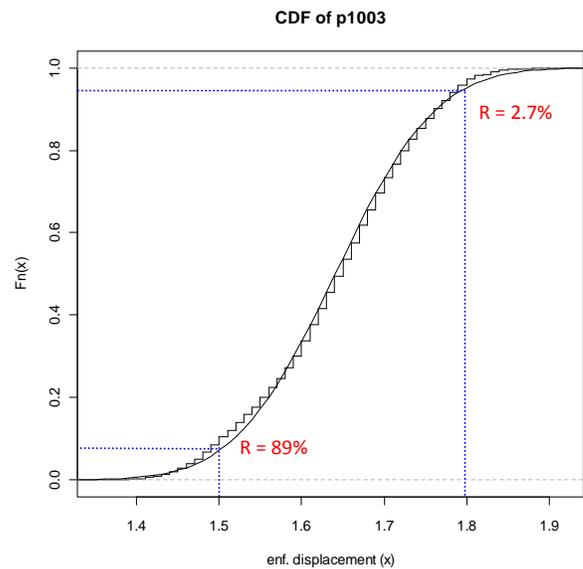


Figure 13 –CDF of Ply 1003.

Figure 14 and Figure 15 show the correlation matrix between plies 1001, 1002, 1003 and some input variables. Notice that the loading boundary condition was the most influential input variable. The number of input variables can be reduced on subsequent studies based on the results of such correlation matrices.

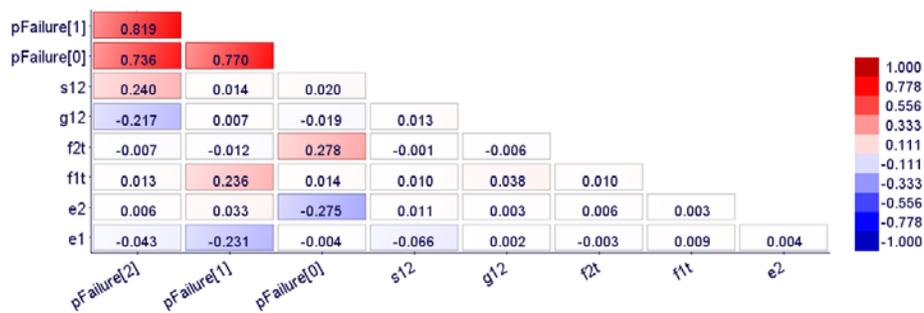


Figure 14 – Correlation matrix: ply failure and material variables (e1, e2, f1t, f2t, g12, s12).

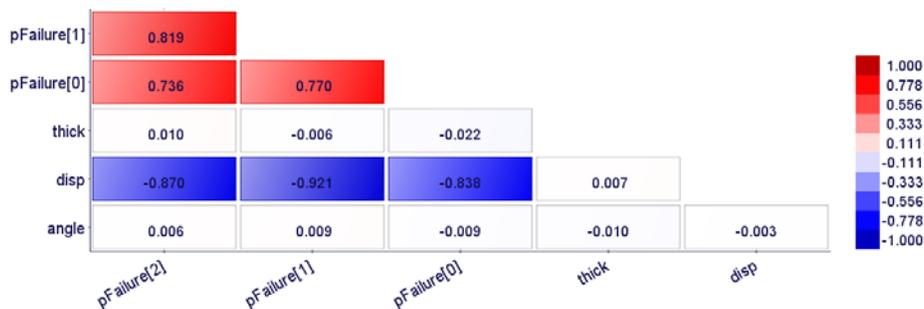


Figure 15 – Correlation matrix: ply failure, displacement, angle and thickness.

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Although variations of failure path could be possible, they were not observed in this case study, i.e. the same failure path of the deterministic analysis was observed on all non-deterministic analysis. This result was expected due to the simplicity of the problem and assumptions made.

As can be observed, a non-deterministic analysis is more informative compared to a deterministic analysis. The additional information provided by such an analysis can be used for a robust design of the structural system. It can also be concluded that material progressive failure analysis can be an important technique to assess composite structures from a system engineering point of view.

### **5. CONCLUSIONS**

Understanding and managing uncertainties is a primary concern in engineering design, especially in applications where human lives are involved as on aerospace structure design. There are different approaches to undertake uncertainty in engineering design. The most typical approach is well-known as Safety Factor, which is undoubtedly a powerful concept, and corresponds also to the current paradigm of aerospace structures. But this deterministic approach is not the only one, and should not be necessarily the most efficient one to manage uncertainties to achieve a robust design with reduced weight.

This work explored two applications based on the probabilistic approach. In the first one, a metallic structure is considered, its reliability based on a probabilistic approach assessed, and compared to the deterministic solution obtained through the safety factor design process. In the second application, a composite structure is submitted to a progressive failure, and again, its reliability based on a probabilistic approach is assessed and compared to its counterpart deterministic solution.

In both cases, the additional information provided by the non-deterministic case could be demonstrated, so that the reliability level against a specified (or required) failure level can be stated. In terms of a damage-tolerant design, this result can be integrated in an optimization loop in order to increase the robustness of a structure, and by this improve the performance of complex systems. Also maintenance intervals can be predicted more precisely in case of available input data.

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