Stochastic Modeling of Non-Stationary Channels

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***Abstract*—This paper addresses stochastic modeling of non-stationary small scale fading channels. Referring to the WSSUS assumption for simplified modeling, we model the opposite cases of scatterers correlated in delay and Doppler frequency. Correlated scatterers can be easily incorporated in a sum-of-sinusoids based tapped delay line, thus, enabling a simple realization of a non–stationary channel impulse response.**

***Index Terms*—stochastic modeling, non-stationary, tapped delay line**

I. Introduction

Channel models have been widely used for simulative performance estimation of wireless systems. In the past, system performance simulation mainly utilized a Wide Sense Stationary Uncorrelated Scattering (WSSUS) based channel model with a constant Power Delay Profile (PDP) and Doppler Power Spectrum (DPS), the latter often following the Jakes spectrum shape, see e.g. [1]. Then, the increasing system bandwidth enabled a higher channel resolution and motivated efforts for more accurate channel modeling. So, the geometry based channel models became very popular, see for example [2]. For navigations systems, which often utilize sophisticated tracking algorithms, the geometric concept is very interesting as it accurately emulates the continuous changes in the propagation channel. The geometry based models are referred to as stochastic, when a certain geometry is not exactly reproduced but equivalent scatterers are placed in the simulated environment. However, the resulting path delays and Doppler frequencies are entirely determined by the scatterers positions and by the transmitter (Tx) and receiver (Rx) movements[1].

The same results can be actually achieved by directly placing scatterers in the delay–Doppler frequency plane. The scatterer (τ, ν) coordinates can be chosen according to some available statistics. Then, the correspondence between the scatterers placement and a simple Tapped–Delay–Line (TDL) model is straightforward. Especially for the evaluation of an average system performance of a communication link, a simplified TDL model is often preferable. However, in a non-stationary channel, the geometry–based modeling remains accurate, whereas a WSSUS based TDL neglects the changes in the channel characteristics. In the latter, slow alteration can be considered by choosing a new set of fixed channel parameters for each new data frame, i.e., simulation run. Furthermore, a non-stationarity can be induced by utilizing tap persistence matrices to model the limited tap lifetime, see e.g. [3].

[1]For simplicity, we consider in the following that only Rx moves.

The aim of this paper is to provide a simple generic TDL channel model though capable of realistic modeling of non–stationary channels. Our approach bears on the scatterer based channel representation and the Doppler–variant impulse response also known as the spreading function. Referring to the work of Bello [4], we induce the non-stationarity by incorporating moving and hence correlated scatterers in the model. For modeling, we draw on the TDL which utilizes Rice's sum-of-sinusoids method [5], [6]. Furthermore, we assume the knowledge of the parameters of the system involved and the availability of the channel statistics. The latter can be gained from channel measurements or by assessing possible environment scenarios. Beside the Rx velocity, the smallest distance expected between the moving Rx and the potential reflecting surfaces is also supposed to be known.

II. Time Variant Channel Characteristics

In most description of the multipath propagation channel the focus is set on the time-varying Channel Impulse Response (CIR) $h(\tau; t)$, given as a sum of shifted Dirac pulses $\delta(\tau - \tau_\ell)$, each multiplied by its own fading process $a_\ell(t)$,

$$h(\tau; t) = \sum_{\ell} a_\ell(t) \cdot \delta(\tau - \tau_\ell(t)). \tag{1}$$

The simplest channel that exhibits time and frequency selective behavior is a WSSUS channel [4] characterized by uncorrelated channel taps with different propagation delays τ_ℓ and stationary fading processes $a_\ell(t)$. The autocorrelation function of the channel impulse response characterized by Uncorrelated Scattering (US) is represented with Dirac pulses on the delay axis,

$$R_h(\tau_\ell, \tau_k; t, t + \Delta t) = R_h(\tau_\ell; t, t + \Delta t)\delta(\tau_\ell - \tau_k). \tag{2}$$

In the following, we discuss the channel stationarity by regarding only one channel tap $h(\tau_\ell; t) = a_\ell(t)$ with a Rayleigh distributed amplitude. Assuming a finite observation time, one channel tap can be represented by a sum of N_H harmonic functions with complex Gaussian distributed amplitudes a_n and random phases ϕ_n,

$$a_\ell(t) = \sum_{n=0}^{N_H - 1} a_n \cdot e^{j\phi_n} e^{j2\pi\nu_n t}. \tag{3}$$

A stochastic process $a_\ell(t)$ is said to be wide sense stationary if the first two moments do not depend on the absolute time t.

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Assuming an uniformly distributed phase, $\phi_n \in [0, 2\pi]$, yields

$$\mathrm{E}[e^{j\phi_n}] = 0, \tag{4}$$

where E[.] denotes the expectation value, which implies

$$\mathrm{E}[a_\ell(t)] = 0. \tag{5}$$

In the autocorrelation function

$$R_a(\Delta t) = \mathrm{E}[a_\ell^*(t) \cdot a_\ell(t + \Delta t)] = \tag{6}$$

$$\sum_{n=0}^{N_H-1} \sum_{m=0}^{N_H-1} \mathrm{E}[a_n^* \cdot a_m]\mathrm{E}[e^{j[2\pi(\nu_m-\nu_n)t+2\pi\nu_m\Delta t]+j(\phi_m-\phi_n)}],$$

the cross-product cancels out when $m \neq n$ yielding

$$R_a(\Delta t) = \sum_{n=0}^{N_H-1} \mathrm{E}[|a_n|^2]\mathrm{E}[e^{j2\pi\nu_n\Delta t}]. \tag{7}$$

Thus, the autocorrelation function also does not depend on time t, hence $a_\ell(t)$ in (3) is wide sense stationary.
Having independent Gaussian distributed amplitudes, it also follows that $\mathrm{E}[a_n^* \cdot a_m] = 0$ and $\mathrm{E}[a_n^2] = \sigma^2$. Under the assumption of a certain Doppler frequency probability density function $p_\nu(\nu)$, we could also calculate the autocorrelation function using

$$\mathrm{E}[e^{j2\pi\nu_n\Delta t}] = \int_{-\infty}^{+\infty} p_\nu(\nu)e^{j2\pi\nu\Delta t}\mathrm{d}\nu. \tag{8}$$

Applying the Fourier transform regarding the time variable t, we obtain the Doppler variable impulse response or the spreading function $S(\tau;\nu)$,

$$S(\tau;\nu) = \int_{-\infty}^{+\infty} h(\tau;t)e^{j2\pi\nu t}\mathrm{d}t. \tag{9}$$

Now, from the definition of the delay Doppler power density and assuming the stationary process $a_\ell(t)$, we obtain

$$\begin{aligned} R_S(\tau_\ell;\nu,\mu) &= \qquad\qquad (10) \\ \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} & a_\ell^*(t)a_\ell(t+\Delta t)e^{j2\pi((\mu t-\nu t)+\mu\Delta t)}\mathrm{d}t\mathrm{d}\Delta t \\ &= \int_{-\infty}^{+\infty} R_a(\Delta t)e^{j2\pi\mu\Delta t}\mathrm{d}\Delta t \cdot \int_{-\infty}^{+\infty} e^{j2\pi(\mu-\nu)t}\mathrm{d}t \\ &= R_S(\tau_\ell;\mu)\cdot\delta(\mu-\nu). \end{aligned}$$

Thus, the assumption that the fading processes which describes the time alteration of a channel tap is stationary is equivalent to the assumption that the scatterers with different Doppler frequencies generating the corresponding tap process are uncorrelated. Furthermore, a stationary channel exhibits a constant dispersion in delay and Doppler frequency described by the scattering function $R_S(\tau;\mu)$. However, due to the receiver movement but also due to the movements of other objects in the vicinity, the environment and thus the channel characteristics will change. For the resulting non-stationary channel, the scattering function does not exist and the instantaneous channel dispersion can be best observed in the spreading function.

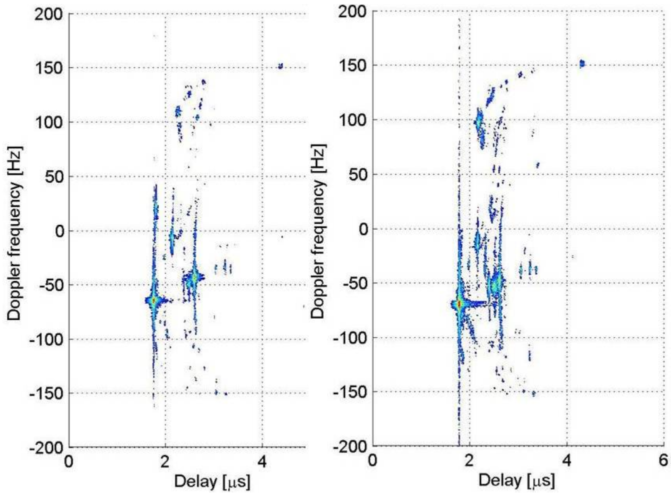

Fig. 1. Example of two spreading functions which evaluation intervals are shifted by 2 seconds

Practically, when we deal with the channel characteristics, we either have some digital data available from channel measurements, or we are about to generate subsequent channel impulse responses by some chosen channel model. The corresponding spreading function can be calculated as the Discrete Fourier Transform (DFT) of CIR in respect to t within an observation interval. The observation time determines also the resulting Doppler frequency resolution.
By observing the subsequent spreading functions, obtained by calculating the DFT of the CIRs after shifting the observation interval, reveals the (non-)stationarity of the channel. Figure 1 shows an example of changes observed in the spreading function when the observation interval is shifted by 2 seconds. The data used for evaluation were collected in the measurements at 5.2GHz [7] with a receiver moving at 30km/h.

III. Correlated Scatterer

When evaluating the spreading function from measurement data, the scatterer typically do not occur as Dirac pulses, but exhibit a spread due to the superposition of different reflections, which arrive at the receiver with similar delays and Doppler shifts. Beyond this, the finite bandwidth of the Rx filter causes widening in τ. The widening in Doppler frequency is caused by the finite observation time. In the case of moving Rx, significantly noticeable changes in the angle between the reflected path and the receiver direction manifest additionally in a wider Doppler frequency of the scatterer [8]. This effect depends on the relative Rx position, Rx heading and on the observation interval. Furthermore, in a sequence of subsequent spreading functions, a scatterer caused by the same reflecting object may change its $(\tau;\nu)$ position, amplitude and shape. Analogously, a sequence of spreading functions reveals changes in the receiver velocity and heading. It reveals also reflections that suddenly appear/disappear and indicates the reflecting objects positions if the corresponding scatterer results from a single and not from multiple reflections.

In general, we can assume that two distinct scatterers observed in one spreading function are caused by distinct reflections and thus uncorrelated. However, by shifting the observation interval, we may observe two scatterers in different positions, however caused by same reflections and thus, correlated in their amplitudes. Hence, the US assumption becomes void if the scatterer shift in delay direction exceeds the sampling time of the system considered for the simulations. Analogously, the WSS characteristics is related to the scatterer movement in the Doppler direction.

When Rx moves, the reflecting objects which remain behind seem to move away, the corresponding scatterers delays increase and Doppler frequencies decrease towards $-\nu_{\max} = -f \cdot v/c$, where c denotes the speed of light and v the Rx velocity. Conversely, the reflecting objects in front of Rx get closer and their scatterers move to smaller delays. The Doppler frequencies change from the maximum value for frontal reflections to zero when reflections come from beside. The fastest change in the delay will experience the scatterers of the reflections which are directly in front or behind the moving vehicle. Within the time interval T, the maximum change in delay is given by

$$\delta_\tau = v \cdot T/c. \tag{11}$$

The fastest change in the angle of arrival of the incoming path, and thus in the Doppler frequency of a scatterer, occurs when the receiver is passing by the corresponding reflecting object. Assuming that the moving vehicle passes by a reflecting surface at distance d, the maximum change in the scatterer Doppler frequency within time T is given by

$$\delta_\nu = \frac{v^2 \cdot f \cdot T}{c \cdot d}, \tag{12}$$

i.e., the resulting change in the Doppler frequency δ_ν is inversely proportional to the distance between the reflector and the receiver route. Thus, the closer Rx moves to a reflecting surface or object, the more noticeable becomes the non–stationarity of the channel.

During the Rx movement, all persistent scatterers will experience a shift in delay and in Doppler frequency. In general, when observed over a longer period of time, the scatterer position in the (τ, ν) plane describes a parabolic function. Two limiting cases, given by the frontal and the sideways reflections, occur rather temporary and can be approximated with one fixed dimension and another given as a function of time.

In a scenario where the reflecting object is far ahead/behind the moving Rx, the corresponding scatterer Doppler frequency will remained unchanged, approximately at $\pm\nu_{max}$, whereas the change in delay can be modeled as a linear function of time,

$$\tau(t) = \tau(0) \mp t \cdot \delta_\tau. \tag{13}$$

The change in the Doppler frequency of the scatterer related to the reflecting object the Rx is just passing by can be

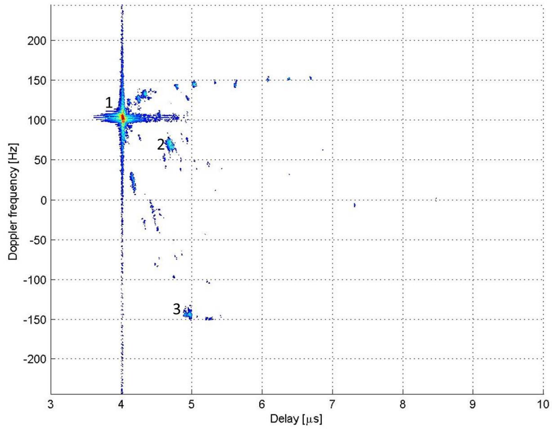

Fig. 2. Example of a spreading function with three persistent scatterers marked with 1,2 and 3

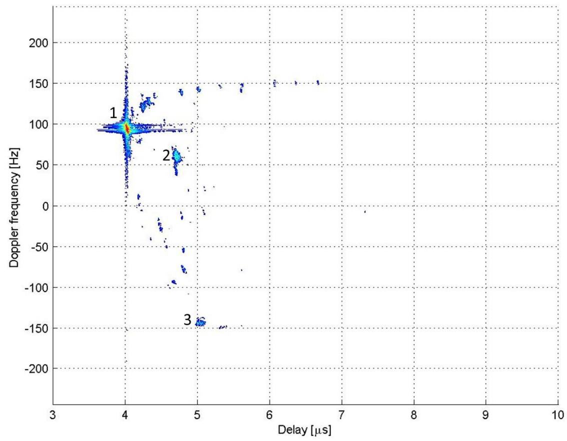

Fig. 3. Change in the spreading function shown in Fig. 2 observed after 2s, reveling the shift of the three persistent scatterers marked with 1,2 and 3

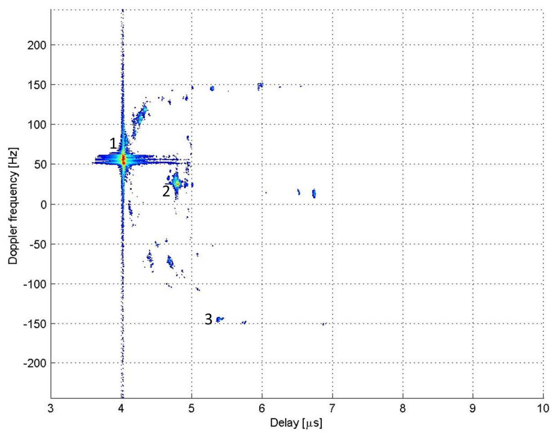

Fig. 4. Change in the spreading function shown in Fig. 3 observed after 8s, reveling the shift of the three persistent scatterers marked with 1,2 and 3

expressed as

$$\nu(t) = \nu(0) - t \cdot \delta_\nu, \tag{14}$$

while the change in delay is rather negligible.

Figures 2-4 show again examples of the spreading function evaluated from the measurements in [7]. Three persistent scatterers are identified in Fig. 2: The one marked by '1' represents the direct path from Tx to Rx, the one marked by '2' is related to a reflector close to the Tx and the scatterer '3' belongs to a reflector which is far behind the moving Rx. Figure 3 shows the scatterers positions after 2s and Figure 4 again 8s later. Due to the Rx movement towards objects resulting in scatterers '1' and '2', their positions shift to lower Doppler frequencies is noticeable in Figs. 3 and 4. The third reflector remains far behind Rx, and therefore the corresponding scatterer position shifts only in delay direction. However, for a system bandwidth B and thus, delay resolution $\Delta\tau = 1/B$, the change in the delay of one scatterer, and consequently also in the tap delay, will be noticeable only after M data samples with $M > \frac{c}{v}$.

A change in the Doppler frequency of one scatterer, on the other hand, yields an alteration of the tap fading function. Considering the Doppler frequency being a function of time (14), the tap fading process becomes

$$a_\ell(t) = \sum_{n=0}^{N_H-1} a_n e^{j\phi_n} e^{j2\pi(\nu_n(0) - t\cdot\delta_\nu)\cdot t}. \tag{15}$$

Calculating the autocorrelation according to (6), we now obtain

$$R_a(\Delta t) = \sum_{n=0}^{N_H-1} \mathrm{E}[|a_n|^2]\mathrm{E}[e^{j2\pi(\nu_n(0)-\Delta t\delta_\nu)\Delta t}]e^{-j4\pi\Delta t\delta_\nu\cdot t} \tag{16}$$

which is now time dependent.

IV. Channel Model

We assume that all parameters needed for the generation of the channel function are available, i.e., chosen according to some statistics. Thus, the tap amplitudes and delays, number of scatterers per tap, the Doppler shift and spread of each scatterer are set. In the TDL model, the ℓ-th tap, $\ell=0,\ldots,N_L-1$, is described by the delay τ_ℓ, the amplitude A_ℓ and an independent fading process $a_\ell(iT)$ with $|a_\ell(iT)| = 1$, where iT denotes the i-th time sample. The channel output is the sum of weighted delayed copies of the channel input signal $x(iT)$,

$$y(iT) = \sum_{\ell=0}^{N_L-1} A_\ell \cdot \alpha_\ell(iT) \cdot x(iT - \tau_\ell). \tag{17}$$

A non-stationary channel is commonly described with time variant tap amplitudes and delays, $\{A_\ell(iT)\}$, $\{\tau_\ell(iT)\}$. Following the idea of a constantly moving scatterer, the change in the scatterer delay can be expressed as

$$\tau(iT) = \tau(0) \mp i \cdot \delta_\tau, \tag{18}$$

taking the minus sign for the scatterer with a large positive Doppler shift and plus sign for the scatterer with a large negative Doppler shift.

Using Rice's representation of a random process that follows the normal distribution from [5], [6], which is often referenced as the sum-of-sinusoids method (see e.g. [9]), we model $\alpha_\ell(iT)$ by

$$\alpha_\ell(iT) = \sum_{n=0}^{N_H-1} C_{\ell,n} e^{j\left(2\pi\nu_{\ell,n} iT + \varphi_{\ell,n}\right)}. \tag{19}$$

Here, the amplitudes $C_{\ell,n}$ are determined by the power spectrum $w(\nu_{\ell,n})$,

$$C_{\ell,n} = \sqrt{2w(\nu_{\ell,n})\Delta\nu_\ell}, \tag{20}$$

and $\varphi_{\ell,n}$ is a random phase angle distributed uniformly over the range $(0, 2\pi)$. N_H and $\Delta\nu_\ell$ are chosen such that they cover the frequency range of interest, hence,

$$\Delta\nu_\ell = \frac{4\sigma_{\nu\ell}}{N_H} \text{ and } \nu_{\ell,n} = \nu_{D\ell} + \left(n - \frac{N_H}{2}\right)\Delta\nu_\ell, \tag{21}$$

where the Doppler shift $\nu_{D\ell}$ and the Doppler spread $2\sigma_{\nu\ell}$ of the ℓ-th tap are chosen according to some statistics. According to [9], [10], this method yields a good approximation of the Gaussian PSD already for a moderate number of harmonics N_H, e.g. $N_H = 25$, but the drawback is that the resulting fading process repeats with the period $2/\Delta\nu_\ell = N_H/2\sigma_{\nu\ell}$. However, this is of no concern as long as this period exceeds the simulation interval.

Furthermore, one tap can be composed of few scatterers, i.e., reflections from distinct objects. Assuming that the ℓ-th tap is composed of $N_S(\ell)$ uncorrelated scatterers with different Doppler shifts $\nu_{D\ell}^{(s)}$ and spreads $\sigma_{\nu\ell}^{(s)}$, the corresponding fading coefficient α_ℓ is computed as follows:

$$\alpha_\ell(iT) = \sum_{s=0}^{N_S(\ell)-1} \sum_{n=0}^{N_H-1} C_{\ell,n}^{(s)} e^{j\left(2\pi\nu_{\ell,n}^{(s)} iT + \varphi_{\ell,n}^{(s)}\right)}. \tag{22}$$

Drawing on the depiction of the moving scatterer, the non-stationarity can be simply incorporated in the model by setting

$$\nu_\ell^{(s)}(iT) = \nu_\ell^{(s)}(0) - i \cdot \delta_{\nu_\ell^{(s)}}. \tag{23}$$

Furthermore, the choice of δ_ν according to (12) concerns only the scatterers with an initial Doppler shift close to zero. For all other scatterers, the Doppler increment can be neglected or an adequately small value can be taken.

V. Conclusion

In this paper, we show a simple way of stochastic modeling of non-stationary channel based on the idea of moving and thus correlated scatterers. We propose to model the change in delay and Doppler frequency of scatterers as a linear function of time, assuming that the simulation interval is short enough. Hereby, the scatterer with a small Doppler shift is expected to exhibit a change in the Doppler frequency and the scatterer with a high Doppler shift will rather experience a delay altering. Furthermore, the relative movement of a scatterer in delay direction during an observation interval of interest depends only on the transmitter/receiver velocity and is often negligible. The relative movement of a scatterer on the Doppler axis depends also on the carrier frequency and on the distance from the sideways reflecting objects and thus becomes more noticeable when the latter is small. The moving scatterer model can be easily incorporated in the sum-of-sinusoids based TDL model for small scale fading channels.

REFERENCES

[1] COST 207, "Digital land mobile radio communications," Office for Official Publications of the European Communities, Final report, Luxembourg, 1989.

[2] A. F. Molisch, A. Kuchar, J. Laurila, K. Hugl, R. Schmalenberger, "Geometry–based directional model for mobile radio channels – principles and implementation," *Euro. Trans. Telecommunications*, vol. 14, pp. 351-359, 2003.

[3] D. W. Matolak, I. Sen, W. Xiong, "The 5GHz Airport Surface Area Channel Part I, Measurement and Modeling Results for Large Airports," *IEEE Trans. Vehicular Technology*, Vol. 57, Issue 4, pp. 2014-2026, July 2008.

[4] P. A. Bello, "Characterization of Randomly Time-Variant Linear Channels," *IEEE Tran. Communications Systems*, vol. 11, no 4, pp.360-384, December 1963.

[5] S.O. Rice, "Mathematical analysis of random noise", Bell Sys. Tech. J., vol. 23, pp. 282332, July 1944.

[6] S.O. Rice, "Mathematical analysis of random noise", Bell Sys. Tech. J., vol. 24, pp. 46156, January 1945.

[7] S. Gligorevic, R. Zierhut,T. Jost,W. Wang, "Airport Channel Measurements at 5.2GHz," *Proc. 3rd European Conference on Antennas and Propagation (EuCAP-2009)*, Berlin, March 2009.

[8] S. Gligorevic, T. Jost, M. Walter, "Scatterer Based Airport Surface Channel Model", *Proc. Digital Avionics Systems Conference (DASC) 2009*, October 2009, pp. 4.C.2-1-4.C.2-10.

[9] M. Paetzold, *Mobile fading Channels*, John Wiley & Sons, LTD, 2002, ISBN 0471 49549 2.

[10] M. Paetzold, U. Killat, "A Deterministic Digital Simulation Model for Suzuki Processes with Application to a Shadowed Rayleigh Land Mobile Radio Channel", IEEE Trans. Vehicular Technology, vol. 45, no. 2., pp. 461468, May 1996.