On the Stability of Contention Resolution Diversity Slotted ALOHA (CRDSA)

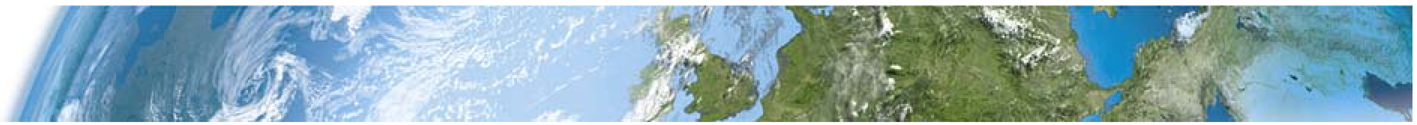
Christian Kissling

German Aerospace Center (DLR)

Overview

Overview

- Introduction
- Framework for Stability Analysis
- Stability Results
- Stability Comparison with SA
- Conclusions



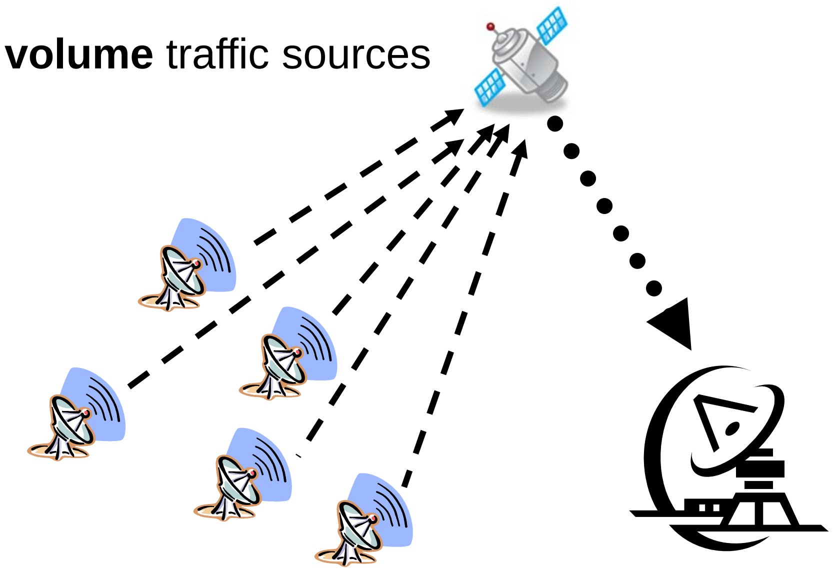
Introduction

Focus of this presentation

- Random Access techniques
  - Interesting for **bursty**, **unpredictable** and **small** size traffic and **delay critical** messages
  - Initial **Logon-signalling**

- Demand Assigned Multiple Access

  Efficient for **predictable** and **large volume** traffic sources

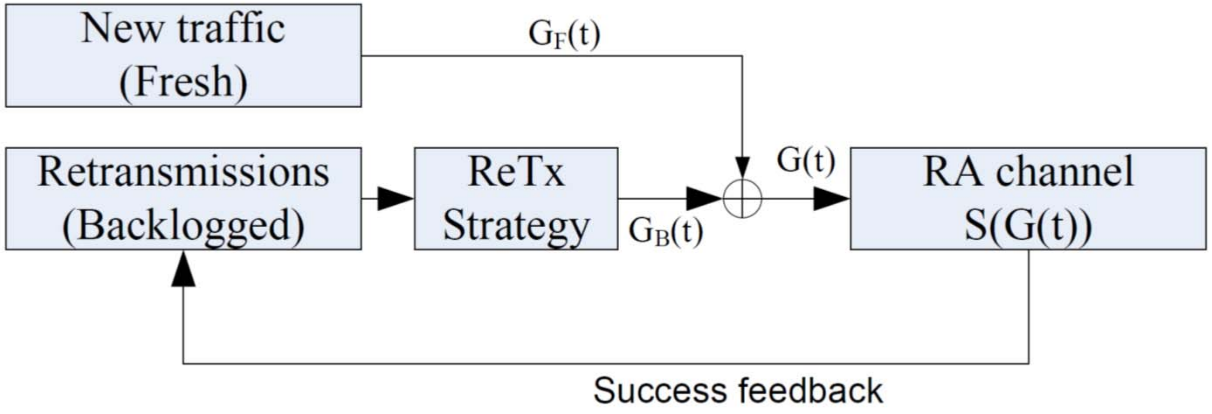


- **Packet loss due to collisions**

- **Throughput** is **load dependent**

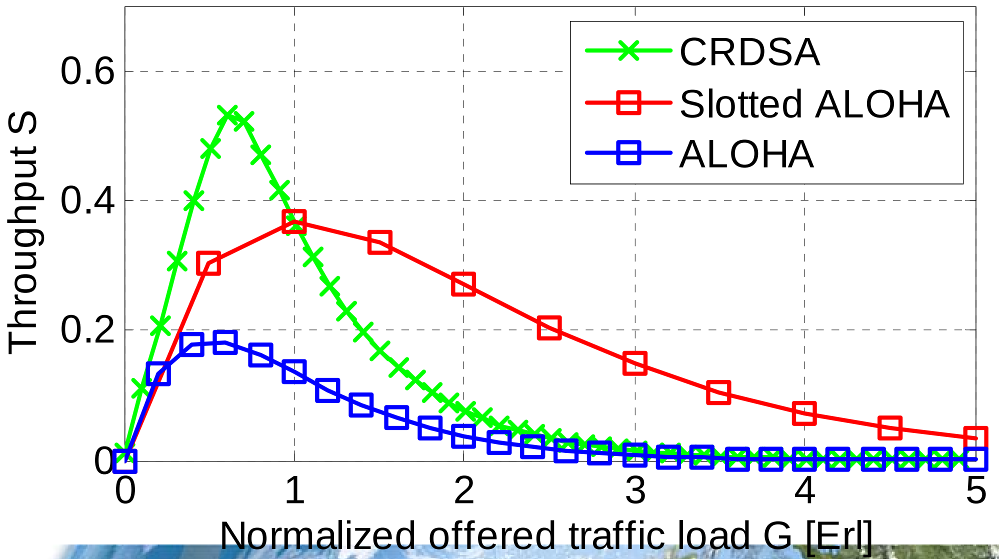
Retransmission Mechanisms

- **Retransmission** (ReTx) mechanism ensure a **reliable** packet **delivery**
- **Attempt** packet **retransmission** with **probability** p_r in every transmission opportunity (geometric distribution)
- **Total load** determined by two components:
  - **New** offered **traffic** (*fresh* traffic) fluctuates statistically
  - **Retransmissions** add on top of fresh transmissions (*backlogged* traffic)



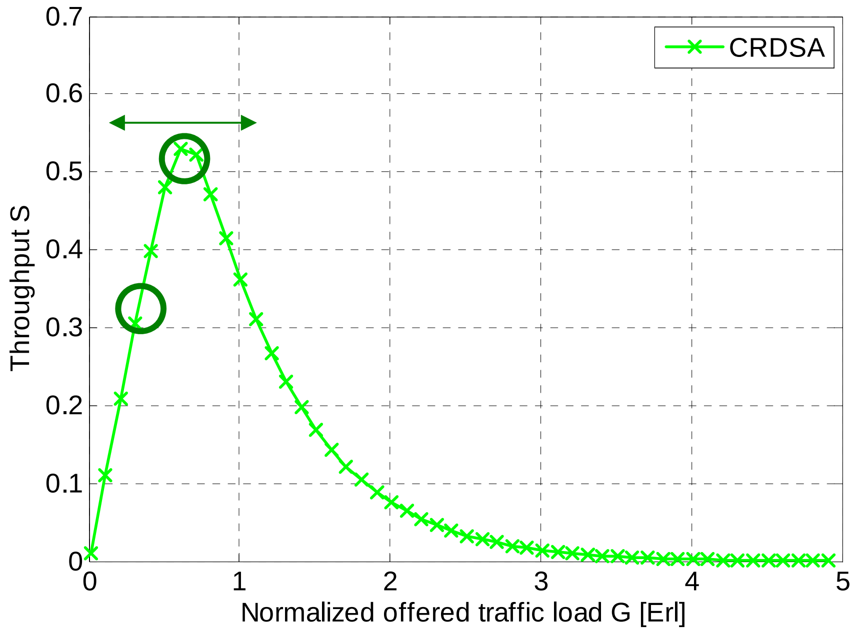
Retransmission Mechanisms (2)

- Problem:
  - Overall load determines the collision probability and throughput
  - Total load depends on user population M, traffic generation probability p_0 and retransmission probability p_r



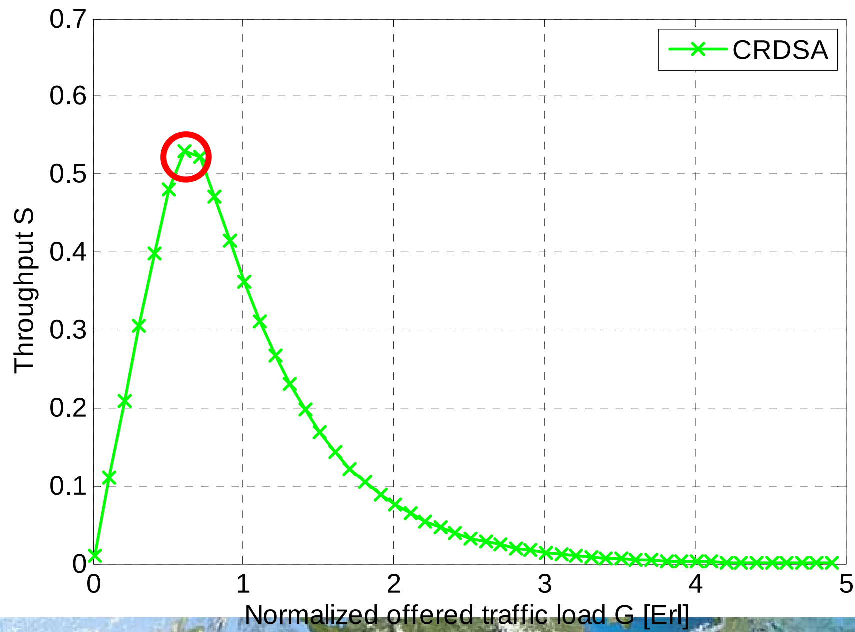
Instability

- Offered traffic fluctuates statistically
- Operation at the maximum throughput desirable
- Problem:
  - Traffic fluctuations move the operating point
  - Higher load reduces throughput asymptotically to zero
- Without retransmissions:
  - Packets lost, but channel can always return to low load → Stable



Instability (2)

- With retransmissions
    - Retransmission feedback loop keeps load high
- Avalanche effect of more and more retransmissions
- Drives the channel into total saturation
    - Low throughput, high delay
- Low chance to get out of this situation again
- Way out:
    - Reset: Drop all pending retransmissions
    - Often not acceptable (QoS)



How can we know whether there is a risk to get stuck in the low throughput region?

How do we have to set our system parameters to be sure that we never get stuck in the low throughput region?

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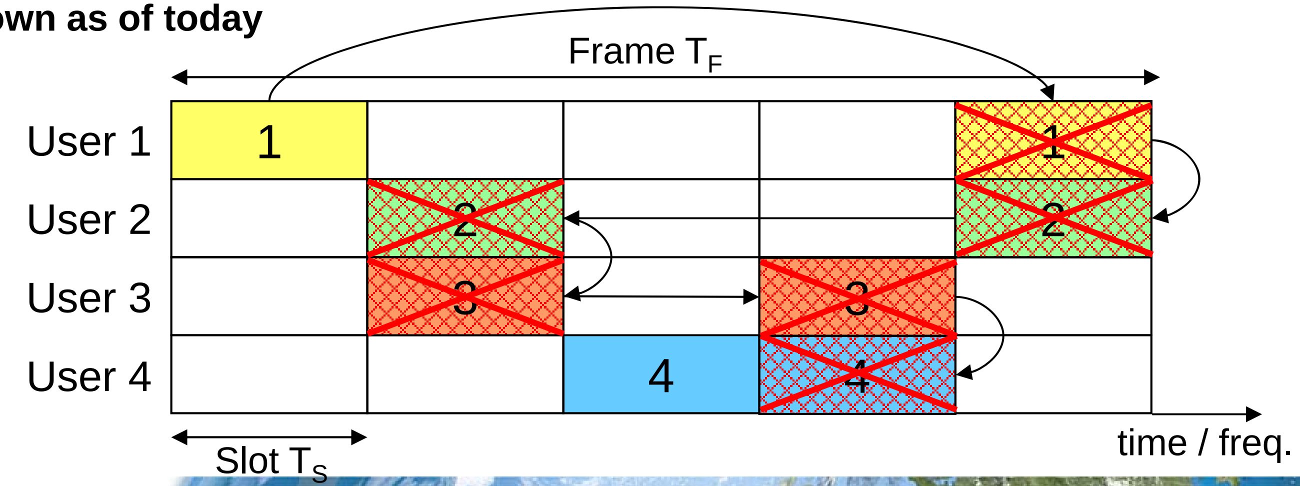
Stability Framework for SIC RA channels (1)

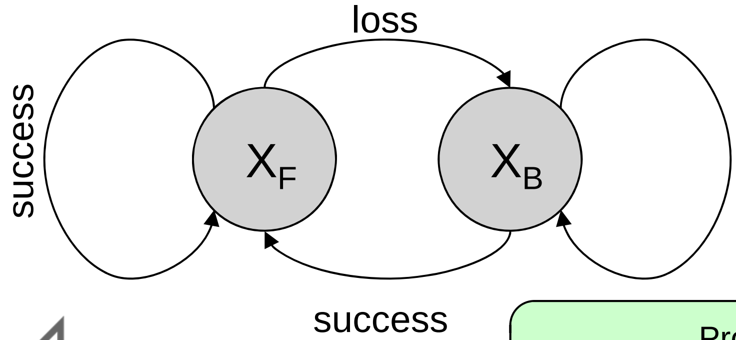
- **Stability** well **investigated** for **classical Slotted ALOHA** (SA)
- Recent **new evolutions** of Slotted ALOHA using Successive Interference Cancellation (SIC) techniques: CRDSA, IRSA (Irregular Repetition Diversity Slotted Aloha [Liva10])
- **Frame based** techniques
- Throughput can be drastically increased
- More than one user can get un-backlogged at a time
- **Methods** known **from Slotted ALOHA cannot** be **applied** anymore
- **Impact on stability on CRDSA and other SIC based RA techniques unknown as of today**



[Liva10] G. Liva, „A Slotted Aloha Scheme based on Bipartite Graph Optimization", Conference on Source and Channel Coding (SCC), 2010, pp. 1-6

Stability framework for SIC RA channels (2)

- Definition of stability
  - **Local Stability**
    - An equilibrium point is locally stable if the system, for a small (local) distortion flows back towards it
  - **Global Stability**
    - An equilibrium point is globally stable if the system is always flowing towards it
- Markov Chain model developed to describe the dynamics of the CRDSA RA channel
- Users can be in one of two states: Fresh x_F or Backlogged x_B



$$P(x'_B|x_B) = \sum_{r,t} \binom{M-x_B}{r} p_0^r (1-p_0)^{M-x_B-r} \cdot \binom{x_B}{t} p_r^t (1-p_r)^{x_B-t} \cdot q_{d,N_s,I_{max}}(r+t, r+x_B-x'_B)$$

Fresh

ReTx

User Population

Fresh traffic Generation Probability

Backlogged Users

ReTx Probability

Probability q(x,y) to succeed with x out of y transmissions

Stability framework for SIC RA channels (3)

- System can then be represented by a differential equation of first order expressing the drift d_B

$$\frac{dx_B}{dt} = E\{X_B^{l+1} - X_B^l | X_B^l\} = \ldots = (M - x_B) \cdot p_0 - N_S \cdot \overline{S}(x_B)$$

- The average throughput is dependent on the backlog state x_B

$$\bar{S}(x_B) = [(M - x_B)p_0 + x_B p_r] \cdot \bar{P}_s((M - x_B)p_0 + x_B p_r)$$

User Population

Fresh traffic Generation Probability

Backlogged Users

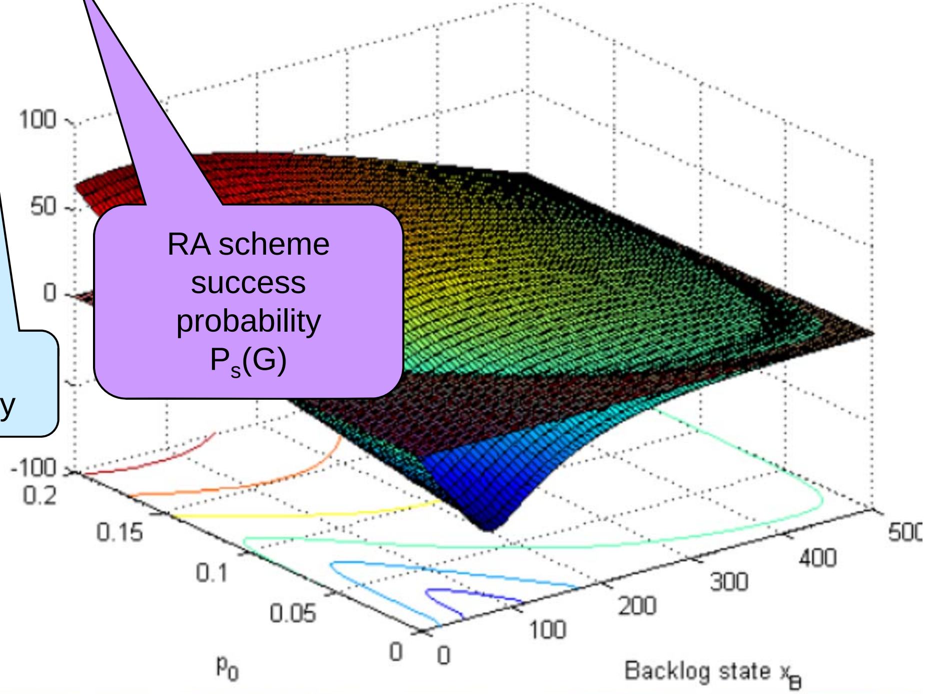
ReTx Probability

RA scheme success probability $P_s(G)$

- Fresh traffic i… generated with prob…
- Packets collide … th pr…

  1 …

- Fr… N_s slots
- Collided packets are retransmitted with probability p_r
- User population M

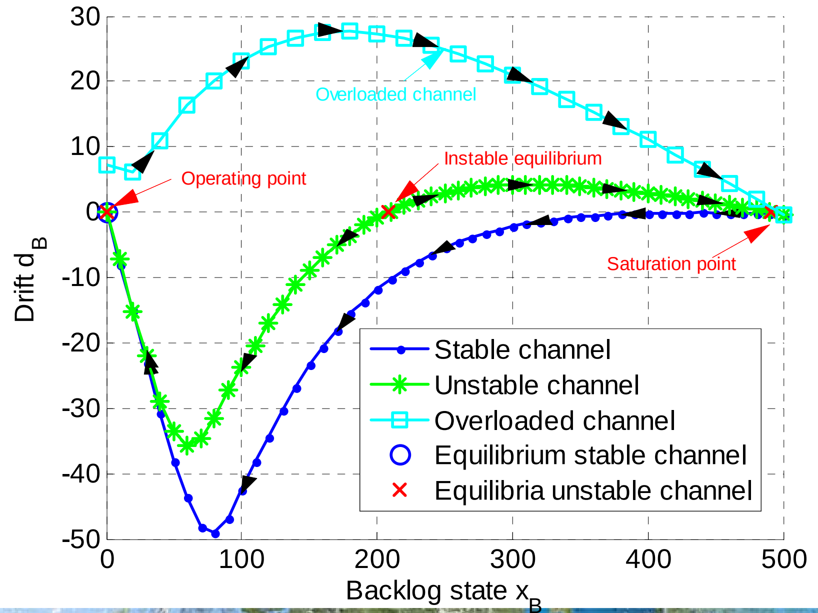


Stability Framework for SIC RA channels (4)

- $0 \leq p_0 \leq 0.01$ **Stable**: **1 Equilibrium Point**
  - Channel always tends to move back to equilibrium

- $0.01 < p_0 \leq 0.11$ **Instable: 3 Equilibrium Points**
  - The channel will remain for some time in the locally stable equilibrium in the high throughput region
  - Once positive drift, amplifying self-excitation to locally stable saturation point

- $0.11 < p_0 \leq 1.0$ **Overload:**
  - Channel moves directly into the equilibrium (saturation point)



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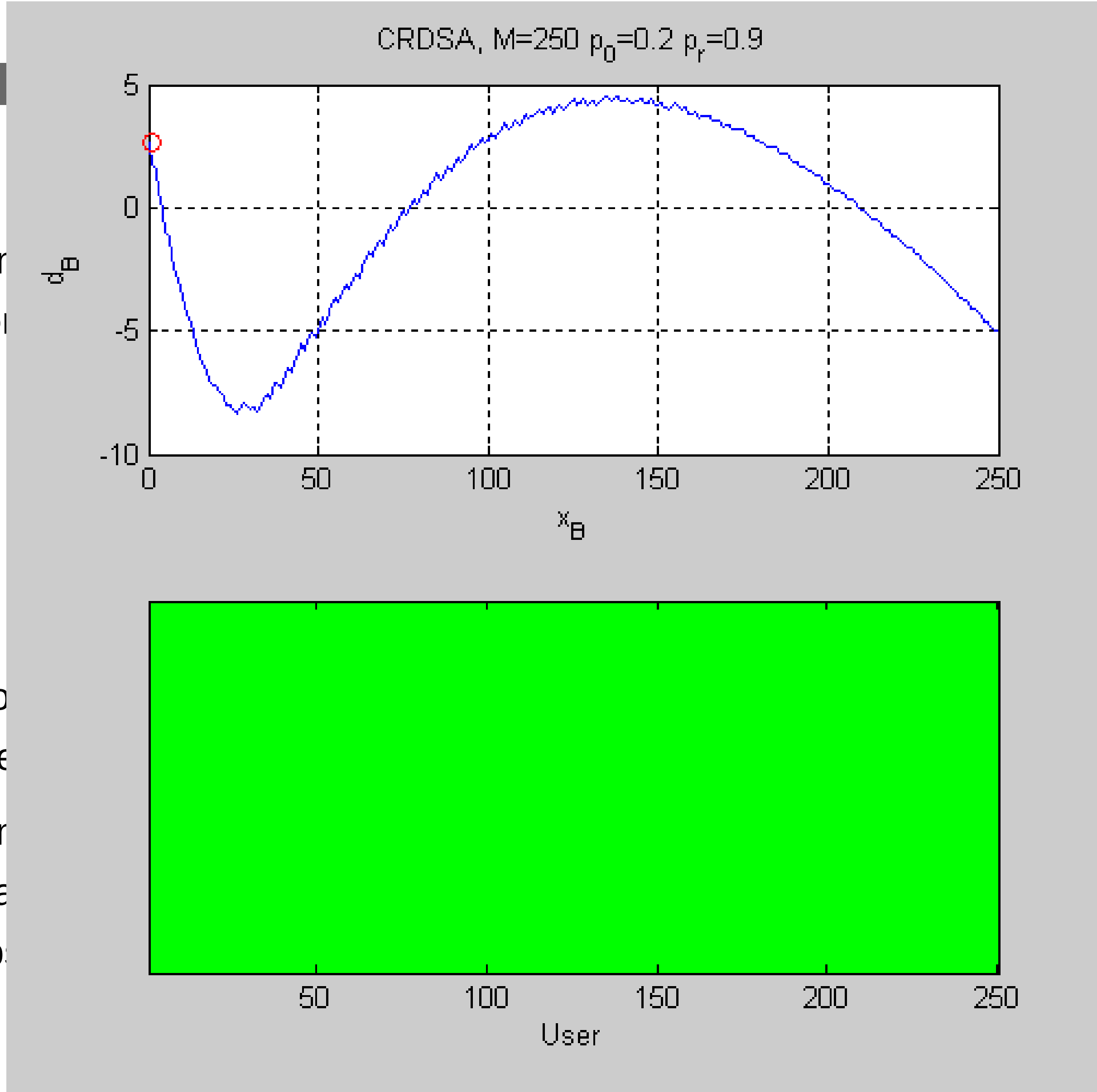

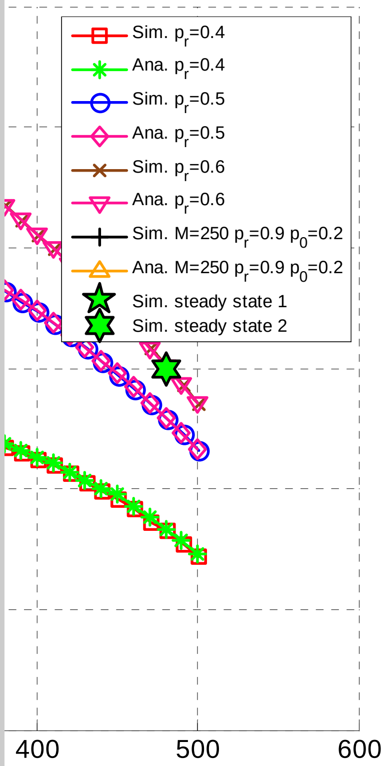

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Stability Comparison with SA

- Channel can be stabilized with parameter p_r
  - Low p_r → Better stability but also higher delay
  - High p_r → Worse stability but also lower delay

Which is the optimum p_r that minimizes the delay D_b while guaranteeing stability?

- Expected delay in equilibrium
- With Little's theorem

$$D_b = \frac{\overline{N}}{S_{out}}$$

Avg. number of packets present in the channel when in equilibrium

Avg. throughput in equilibrium

- 4 system parameters: M, p_0, p_r, D_b
- Optimization criteria: Find the optimum p_r to
  - Minimize D_b for fixed M and p_0
  - Maximize possible M for fixed p_0 and target D_b
  - Maximize possible p_0 for fixed M and target D_b

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Minimize Delay

- User population M given, e.g. $M = 400$
- Traffic generation probability p_0 given, e.g. $p_0 = 2.63 \cdot 10^{-3}$
- Optimization problem:

$$p_r^* = \underset{p_r \in [0 \ldots 1]}{\operatorname{argmin}} D_b(\Psi, p_r)$$

- Whereas

$$\Psi = \{ \quad M = 400, p_0 = 0.263, \\ d = 2, N_s = 100, I = 10\}$$

2 replicas

100 slots / frame

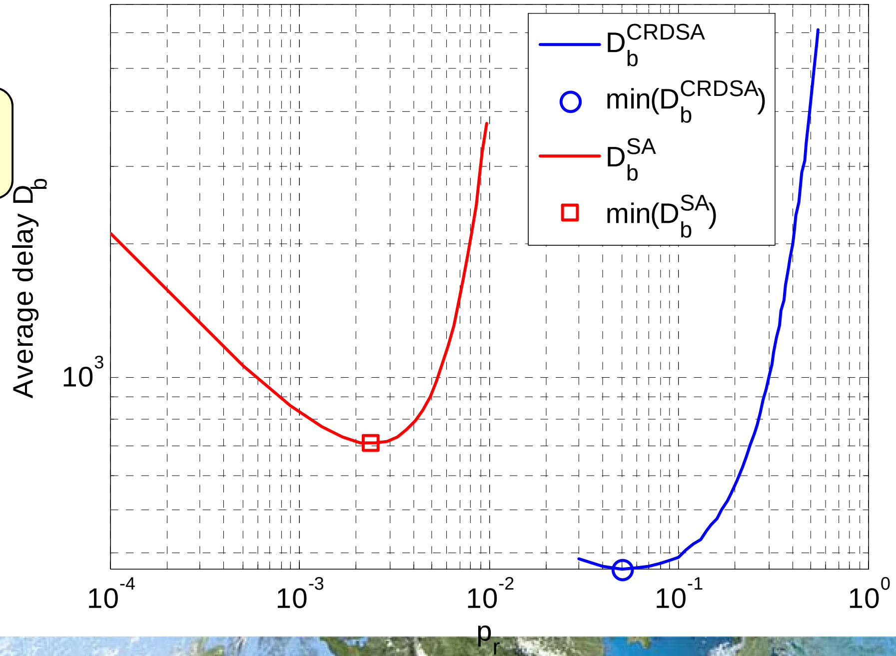
10 SIC iterations

↗ Result

$$D^{min}_{b,CRDSA'}(p_r = p^*_{r,CRDSA} \cdot 10^{-2}) = \\ 3.68\ frames \equiv 368\ slots$$

$$D^{min}_{b,SA}(p_r = p^*_{r,SA} = 2.5 \cdot 10^{-3}) = \\ 707\ slots \equiv 7.07\ frames$$

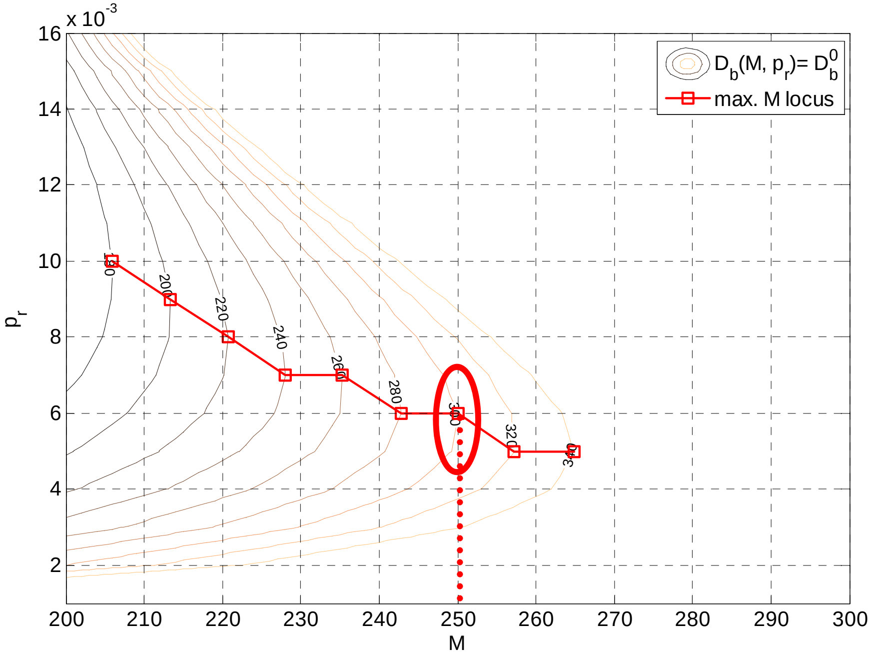
↗ CRDSA can save 48% of delay while being guaranteed stable



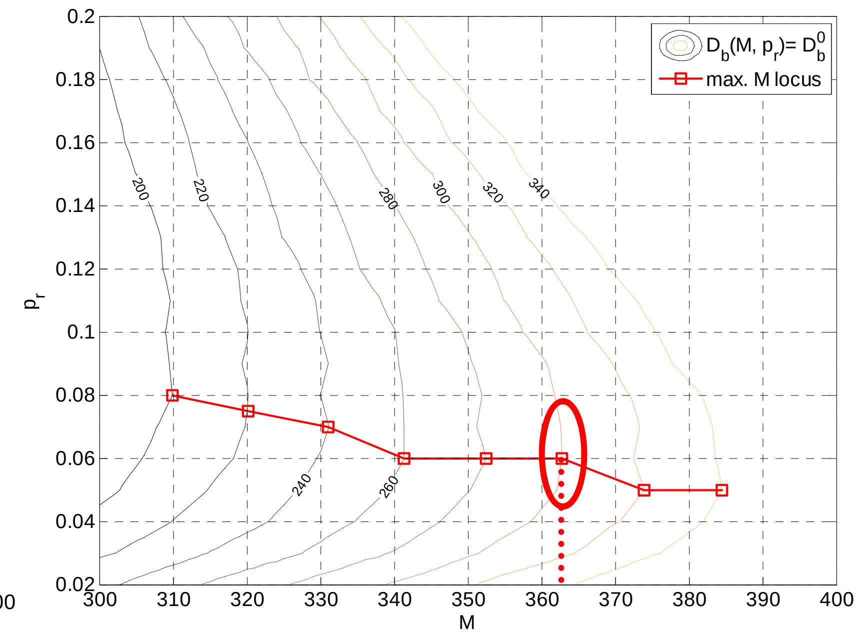
Maximize User Population M

- Find max. supported population M to achieve D_b^0 if traffic is generated with p_0
- Implicit optimization problem $g(p_r, M, p_0) = D_b(p_r, M, p_0) - D_b^0 = 0$

$$L(p_r, M, p_0, \lambda) = D_b(p_r, M, p_0) + \lambda[D_b(p_r, M, p_0) - D_b^0]$$



Slotted ALOHA



CRDSA

- CRDSA can support 45% more users than SA (363 users vs. 250 users), while achieving an average Delay of 300 slots (3 frames) and being guaranteed stable

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Conclusions Stability

- Random Access Channels show instability behaviour
- Existing tools from Slotted ALOHA not sufficient to describe new frame-based Random Access mechanisms based on Successive Interference Cancellation
- A Markov chain model was derived to describe the dynamics of the system
- The system stability can be characterized by the number of equilibrium points
  - 1 equilibrium point: Globally stable equilibrium
  - 3 equilibrium points: Instable channel, will earlier or later enter the saturation
- Model allows
  - Analysis of the stability behaviour of a channel
  - Parameterization of the retransmission probability of a channel with user population M and packet generation probability p_0 to be guaranteed stable
- Analytical model was verified by simulations
- CRDSA does not only show advantage in peak throughput but also in it stability

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Contention Resolution ALOHA (CRA)

- 2nd path of research activities ongoing
- SIC was so far only applied to synchronous (=slotted) schemes (CRDSA, IRSA)
- New Idea: Apply SIC to asynchronous scheme (ALOHA like)
    - **Contention Resolution ALOHA**
- Investigations have shown that this scheme has high potential and offers advantages in several usage scenarios