

# Implementation of a Modelica Library for Energy Management based on Economic Models

Dirk Zimmer, Daniel Schlabe  
Deutsches Zentrum für Luft- und Raumfahrt.  
Münchener Strasse 20, 82234 Weßling, Deutschland  
{dirk.zimmer, daniel.schlabe}@dlr.de

## Abstract

The use of modeling paradigms for physical systems can in some instances be stretched to reach other domains. This paper presents one such example: it describes the design of a Modelica library that implements economic models to be used for the purpose of energy management. The design principles of this library such as the use of pseudo-physical connectors are outlined and examples for managing energy sources and loads are discussed.

*Keywords: Energy management, Load management, Economic models, Object-oriented modeling.*

## 1 Introduction

This paper presents the modeling of energy management tasks by the use of economic models. In this approach, each provider of energy and each consumer is characterized by a specific cost function. A global market or a set of local markets then decide about the distribution of energy flow.

To this end, a new Modelica library has been developed. It supports the modeler in the design of his or her energy distribution system and derives an (at least partly) optimal solution for the distribution based on the provided cost functions.

The library is not coupled to any specific physical domain. All its components concern energy in its most abstract form. In fact, many energy management tasks involve multiple physical domains and therefore a domain-specific approach would be of limited value.

The library is currently split into two sub-libraries that are geared towards different application domains: source management and load management. It is still under development and currently not publicly available.

## 2 Economic Models for Energy Management

### 2.1 State of the Art

The links between models and theories used in micro-economics and typical tasks of an energy management function are very close. In both cases, there is a set of providers and a set of consumers. The consumers pay a price of a utility depending on the availability or production of the providers. The main difference is the type of the utilities. In micro-economics this is typical any kind of product, for an energy management the utility is power or energy.

The application of economic models for a power management is already demonstrated in [9]. An energy manager based on economic models for the electrical system of automobiles, especially for hybrid cars, has been studied in [1] and [6]. Additionally, available methods for energy management of aircraft electrical systems can be found in [8].

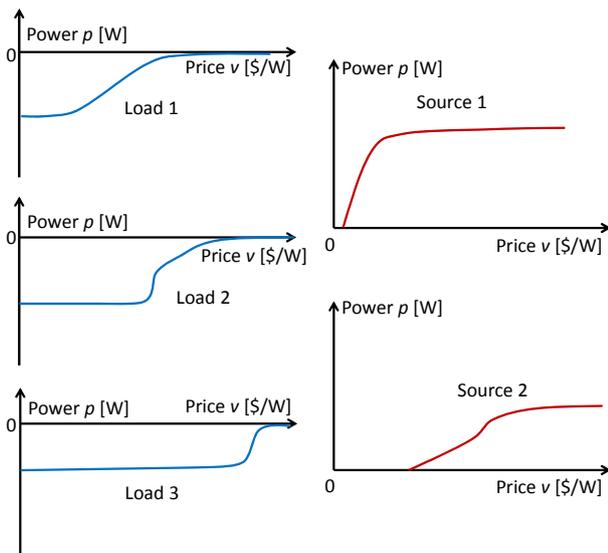
The main idea behind this market-oriented approach is the usage of power  $p$  over price  $v$  functions for each source/provider and consumer/load as illustrated in Figure 1.

$$p = f(v)$$

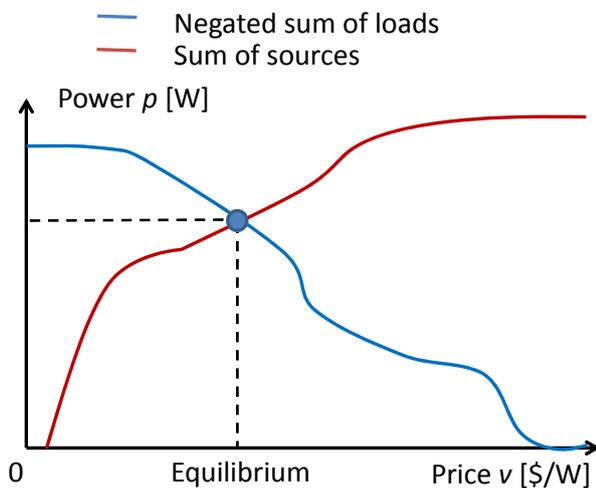
These functions describe how much price a load is able to pay for a dedicated power and how much power a source will provide for a certain price respectively. These functions could be determined by e.g. the efficiency or the priority of a component.

Since  $p$  denotes the outflowing power, the cost functions are typically positive for sources and negative for loads.

Subsequently for all sources and loads the sum-functions are calculated as shown in Figure 2. The intersection of load and source sum-functions determines the current price and thus the power of each component.



**Figure 1:** Cost-functions of single loads and sources.



**Figure 2:** Sum-functions and equilibrium.

The advantage of such an approach is the integration of different relevant aspects like efficiency of the sources or availability of the consumers for an energy manager in one single characteristic cost function. Furthermore, this enables the modeling of sources and consumers in an object-oriented way and thus an easy set-up of an energy management function of a dedicated system within an early stage of design.

## 2.2 Limitations

To guarantee the existence of a unique intersection of load and source cost-functions, these have to be monotone and continuous. If this restriction is not maintained, one has to guarantee with other means that a stable intersection can be found in either case.

In addition, economic models are best suited for finding an optimal solution at one specific time in-

stant, but not for optimizing the energy consumption predictively regarding dynamic influences. For this case, further means are needed that have to be integrated to these models.

## 2.3 Scientific Contributions of this paper

Based on the described state of the art, this paper demonstrates the implementation of a market-oriented energy management library in Modelica. Therefore the library including its components and the working principles are outlined in the following sections.

New concepts for dealing with non-monotone cost functions of sources are introduced. For this task, several rounds of negotiation are being used. Multiple negotiation rounds are also used for dealing with switchable and continuous loads in one system to reach a maximum availability of loads.

The modeling of energy systems is not confined to models for sources and loads. Hence also further components like limiters or transformers are considered that modify the cost-functions in a dedicated way.

## 3 Fundamental design of the library

The goal of this paper is to describe how such economical models for energy management can be modeled in a truly object-oriented way. The idea is that energy distribution systems can be assembled from basic components such as producers and consumers. Also the modeler shall not be directly concerned with the cost functions. Instead the cost-functions should be derived by parameters such as efficiency or priority levels.

To this end, a Modelica library has been developed. In this section, we present its common interface and the most basic components.

### 3.1 Connector design

The connector of the energy management library is a so-called pseudo physical connector. This means that it mimics the characteristics of classic physical connectors without describing actual physical quantities. In concrete terms: the connector contains a pair of a potential variable and a flow variable just like a physical connector. In this way, we profit from the advanced support of physical connectors (like the check of balanced models) in Modelica.

The potential variable of the connector is the price per watt [\$/W] and the flow variable is the power outflow [W]. A positive value for the power

outflow is typical for a source. Consequently consumers have negative values of their flow variable. Similar pairs have already been suggested during the 1970s in [2] and [3] and enable a more natural modeling than sheer System Dynamics for Modelica [5].

The product of the potential variable and the flow results in the amount of money that is transmitted through the connector (negative values represent costs, positive values represent income). The money is of course virtual and not related to any real currency.

A connection between a set of connectors thus represents an ideal market where all participants pay or receive an equivalent price for an equivalent product.

**Listing 1:** Code of the power socket.

```
connector Socket

  parameter Integer n=1;

  PricePerWatt price[n];
  flow SI.Power power[n];

end Socket;
```

Listing 1 presents the Modelica code of the connector. Evidently, price and power represent not scalars but vectors of a parameterized size  $n$ . The reason for this is explained in section 4.6. For the moment, let us continue by pretending these are scalars. We simply assume:  $n=1$ .

### 3.2 Icons

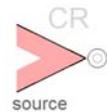
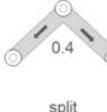
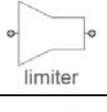
A component of the library may represent a source of energy, a consumer, a transformer of energy or redistributors.

These are all components that also occur in many physical domains such as electric systems. However, since this library shall be domain independent, no symbols of such libraries shall be used.

There are only a few domain neutral symbol languages. One of them is bond graphs. For our purpose bond graphs [4] are however too low-level and too technical. For instance there is no distinction between a source and a sink in bond graphs.

Another set of icons has been developed for the Energy Systems Language developed in the field of ecology by Howard T. Odum [7]. It is also not directly usable for our purpose, but at least the abstract forms used in this language inspired the design of our set of symbols that is listed in Table 1.

**Table 1:** Icons used for energy management.

|   |                        |
|---|------------------------|
|    | Source / Producer      |
|    | Sink / Consumer / Load |
|    | Waste                  |
|    | Transformer            |
|    | Split                  |
|   | Limiter                |
|  | One-way                |

A source can represent a source of fuel or an energy producer such as a power plant. The sink is its counterpart element. It mostly represents a consumer. The waste element is a special case for the sink that enables the system to waste energy.

Energy can be transformed into other forms by imposing further costs using a transformer. The split element can be used to distribute energy into different branches. For instance in a combined heat and power plant 40% of the power is electricity and the remaining 60% are available as heating power.

The components one-way and limiter are explained in section 4.4 and section 5.2 respectively.

### 3.3 Example

Given the set of components, it is now possible to compose an energy distribution system. Figure 3 shows the model diagram of an example system. Here, two sources are available: one for heating and one for electricity. Two consumers model the respective demand. In addition there is the possibility to use electricity for heating. A waste element ensures that energy can be dumped in the unlikely case that the electricity demand may fall below the idle power output of the electricity generation plant.

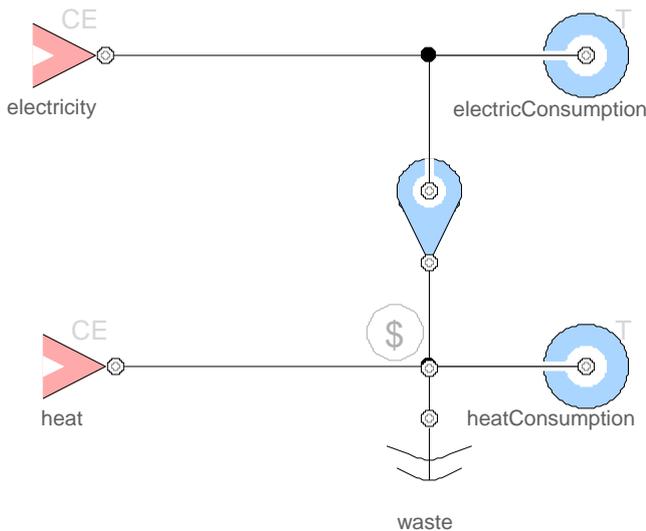


Figure 3: The model diagram of an example market.

### 3.4 Solving the non-linear systems of equations

All component models contain a description of their cost function that expresses the price as function of the power. The connection of this components leads then to (typically) non-linear equation systems. If all cost functions are strictly monotonic increasing or decreasing, there will be a unique solution.

Depending on the cost-function and the specific connection structure, a simulation software such as Dymola might be able to solve this non-linear equation system, but in some practical examples this turns out not to be the case.

Hence we have developed an auxiliary controller unit that regulates the price  $v$  on the market by a simple differential equation. The controller may compensate for any lack or excess of power  $p$ . It increases the market price in case of a power outflow ( $p > 0$ ) due to a lack of power and decreases the price in case of a power inflow ( $p < 0$ ) due to excess of power.

$$dv/dt \cdot T = p$$

where  $T$  is an arbitrary time constant.

This controller is typically applied to a connection set. In the diagram of Figure 3, it is depicted as grey "\$" placed in a circle. With this element, it is possible to find the solution in robust way by approaching steady state. The drawback of such a controller is that it makes the system potentially stiff and requires implicit solvers such as DASSL for the efficient simulation of the system.

The application of such a controller could probably be avoided if there exists a Modelica language construct to suggest suitable tearing variables.

## 4 Application Domain: Source Management.

### 4.1 Motivation

In this application domain, we want to fulfill a given consumer demand by using the most efficient combination of sources available. Hence the cost functions take into account the efficiency of sources and subsequent processes of energy transformation.

### 4.2 Derivation of cost functions

In this scenario, the consumer demand is regarded as a given that is required to be fulfilled at any cost. Hence modeling the cost function of a consumer is very simple: A consumer is the equivalent to an ideal flow sink. Prescribing the flow variable for any potential price per watt while leaving the price to be determined by other parts of the system:

$$p = -\text{demand}$$

The waste element is a special case of a consumer. An ideal waste element is similar to an ideal diode. It is a sink of zero flow for prices above zero and consumes arbitrary amounts of energy at a price of zero. A price below zero means that the producers would have to pay for their energy to be consumed. Although this actually occurs in real markets, the waste element can be used to prevent such cases.

$$s = \text{if } (s > 0) \text{ then } p \text{ else } v;$$

$$0 = \text{if } (s > 0) \text{ then } v \text{ else } p;$$

where  $s$  is a curve parameter

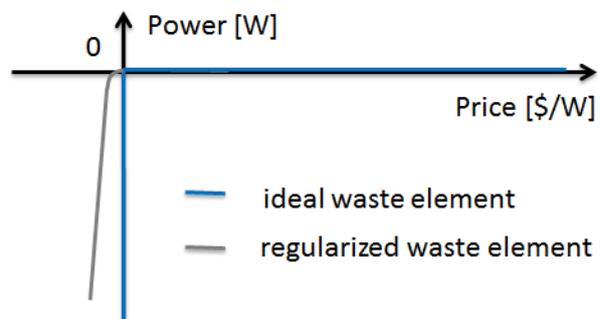


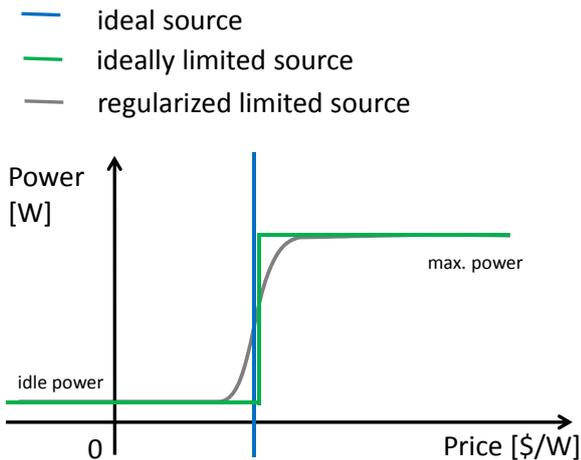
Figure 4: Cost function of a waste element.

Modeling sources is a little more difficult. The price shall reflect the efficiency of energy use. The simplest case is a source of constant efficiency. In the ideal case, this source stipulates the price for any

arbitrary power output to be the inverse of the efficiency:

$$v = 1/\text{efficiency}$$

No real source of energy is unbounded. All sources have a maximum capacity and many of them have an idle power output beyond which their production cannot decrease. These limitations can be modeled by a step function.



**Figure 5:** Cost function of different source models.

Finally, a split element can be used to model the separation of power into distinct branches by a fixed fraction. It distributes the power inflow  $p_{in}$  into two power outflows  $p_{out1}$  and  $p_{out2}$  by a given fraction  $R$ . The split element is connected to markets with a different price per watt. The price per watt of the power inflow  $v_{in}$  is then the weighted mean of the two outflow prices:  $v_{out1}$  and  $v_{out2}$ . Here are the corresponding equations to relate the three connectors:

$$\begin{aligned} p_{in} + p_{out1} + p_{out2} &= 0; \\ p_{out1} \cdot (1-R) &= p_{out2} \cdot R; \\ v_{in} &= v_{out1} \cdot R + v_{out2} \cdot (1-R); \end{aligned}$$

### 4.3 Regularizing the cost functions

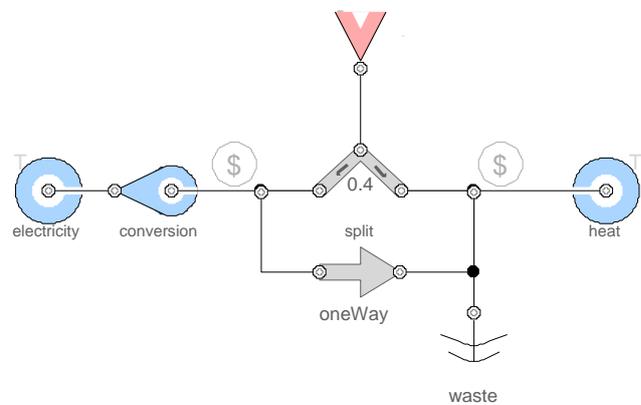
For the numerical solution, it is advantageous if all cost functions are continuous and strictly monotonic functions. Then a unique solution is guaranteed in case the total demand can be met. But the curves for the ideal limiter or the ideal waste element substantially differ from this requirement. They represent multi-valued functions that are also strictly monotonic increasing or decreasing. Indeed their modeling would require the use of parametric curves such as for ideal diodes. To avoid this effort and the resulting

numerical problems, a regularization scheme is applied.

The regularization is indicated by the grey curves in Figure 4 and Figure 5. For its realization, a mixture of sigmoid and exponential functions is used. The precise realization is somewhat arbitrary and also of no particular importance and hence has been omitted here.

The regularization is of course a further potential cause of stiffness and/or implies a loss of precision. The trade-off between precision and stiffness can be set by fudge parameters. These are provided globally by an outer model so they do not have to be set of each element individually.

### 4.4 Example 1: A combined power generator



**Figure 6:** Model diagram of a combined power generator and two corresponding consumers for electricity and thermal energy (heat).

Figure 6 presents the example of a combined power generator of electricity and heat. Up to 60% of the thermal energy can be converted into electricity. This is modeled by a combination of a split element and a one-way component that acts like a diode: power can only flow in one direction.

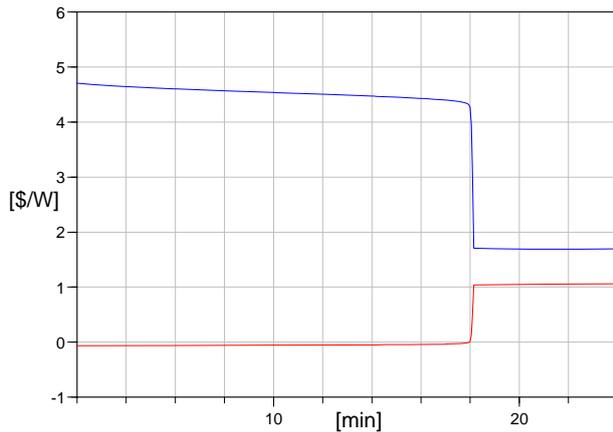
The loss in conversion between thermal and electric energy is modeled by a transformer component. Both consumer models stipulate the total power demand that is varying over time.

For the simulation, the electric consumption is constantly decreasing from 250 kW to 100 kW. The demand of thermal energy is constantly increasing from 50 kW to 500 kW. The impact on the price can be observed in Figure 7. It contains the simulation result for the price per Watt for both consumers.

Due to the initial high demand for electricity, the consumers of thermal energy do not have to pay anything at all (the price is actually even slightly below zero because of the regularization of the waste element). The generation of electric energy produces sufficient heat as side product.

During the simulation, the demand shifts towards the need for thermal energy. Then the bill needs to be split. Electric energy still remains more expensive than thermal energy because it needs to be converted (at loss) from thermal energy and the combined producer can control how much of that needs to be converted.

This example demonstrates how the cost-function of a more complex source like a combined generator can be modeled in a true object-oriented way by combining simple components.

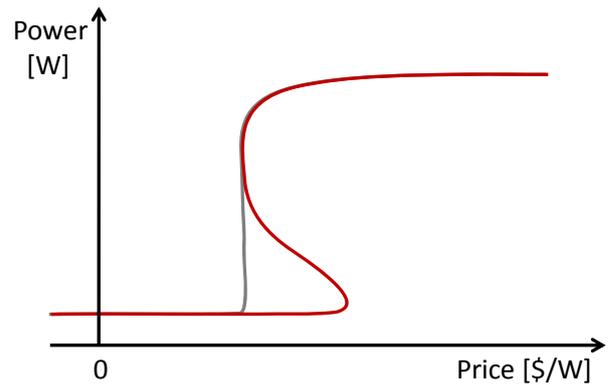


**Figure 7:** Price development of thermal energy (red) and electric energy (blue).

#### 4.5 Treatment of non-monotonic cost-functions

The presumption that the cost function is strictly monotonic increasing is not realistic for a large set of power generators. Many of them have an ideal operating range that does not start at idle power. This means that when these generators are used for low power output they can be very inefficient. The multi-valued cost-function of Figure 8 represents such a characteristic curve.

The solution of systems with such cost functions can be numerically very difficult and often there are multiple equilibriums in the market. Finding the optimal equilibrium is a very demanding optimization problem that in general cannot be handled in polynomial time. Hence a robust handling of such non-monotonic cost function requires a good solution strategy.

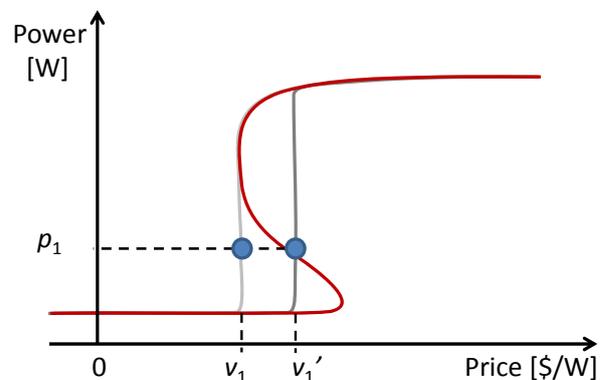


**Figure 8:** A non-monotonic, multi-valued cost function (red) and a corresponding monotonic, single-valued hull curve (grey).

In this paper, we propose a bullying strategy. It reflects a behavior that also exists in real markets. Big players, in our case large and potentially very efficient power generators, compete for a contract. They pretend to be more efficient than they actually are. When the order finally turns out to be too small to be efficiently handled by the big player, the contracts are handed over to small players by issuing sub-contracts. The final point of equilibrium is hence determined in several rounds of negotiation: first the big players then the smaller players.

In our library such a bullying strategy is implemented by creating hull curves in multiple rounds of negotiation. Figure 8 shows the effective cost-functions for our producer. However, in the first round of negotiation this curve is not used but the grey hull curve instead.

The hull curve must be monotonic increasing and must always be greater or equal than the effective cost curve. Within these constraints, it should be as low-valued as possible. In those sections where the hull curve does not coincide with the effective cost curve, the producer is hence pretending to be more efficient than he actually is.



**Figure 9:** A new hull curve is generated for the non-monotonic cost-function based on the previous market solution  $(v_1, \rho_1)$ .

Since all participants in the market use monotonic hull curves, a solution can easily be found. If the solution  $(v_1, p_1)$  is now placed in a section where the hull curve does not coincide with the effective cost curve, the correspondent producer has to “reveal” its effective costs  $(v_1', p_1)$  in the second round of negotiation.

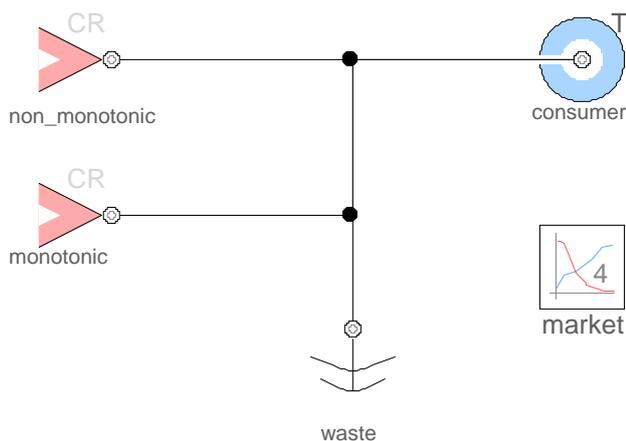
To this end, a new hull curve is generated. Again it must be monotonic increasing. But the solution of the first round now splits the hull curve in two parts:

- For  $v < v_1'$ , the curve must again be greater or equal than the part of the effective cost curve that is lower than  $p_1$  and within these constraints as low-valued as possible.
- For  $v \geq v_1'$  the curve must be greater or equal to than the effective cost curve or equal to  $p_1$ , again, as low-valued as possible.

Figure 9 illustrates such a new hull curve for a given market equilibrium. The procedure can be iterated for several rounds of negotiation. In general, this iteration scheme cannot be proven to approximate the optimal solution, but since each hull curve will be smaller valued than its predecessor the process is at least bound to converge.

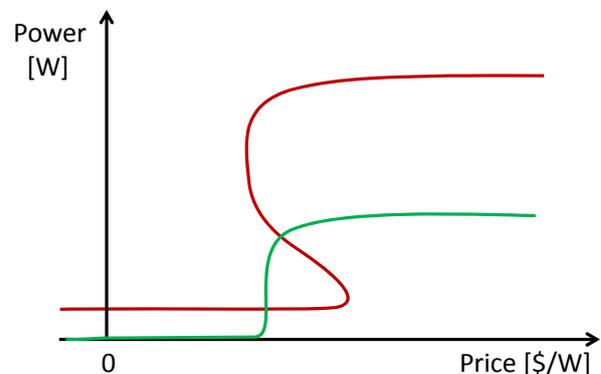
In practice, however, this iteration scheme has at least shown to work very well. Therefore let us illustrate it by an example.

#### 4.6 Example2: Non-monotonic behavior.



**Figure 10:** Two sources compete for one consumer. The consumer demand is rising at a constant rate.

In this example, two generators compete to fulfill the power demand of one source. One small generator that is rather inefficient and limited to a small capacity and a large generator that is very efficient for high-load values and very inefficient for low load values. The small generator shall thus be used to overcome the efficiency gap of the large one.

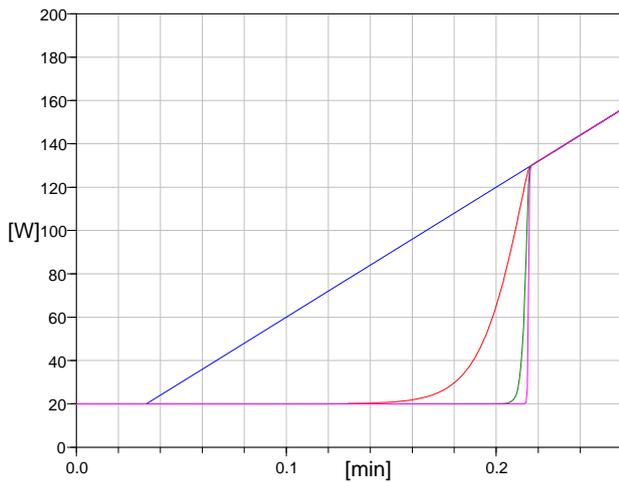


**Figure 11:** Sketch of the two cost functions for the large (red) and small (green) generator.

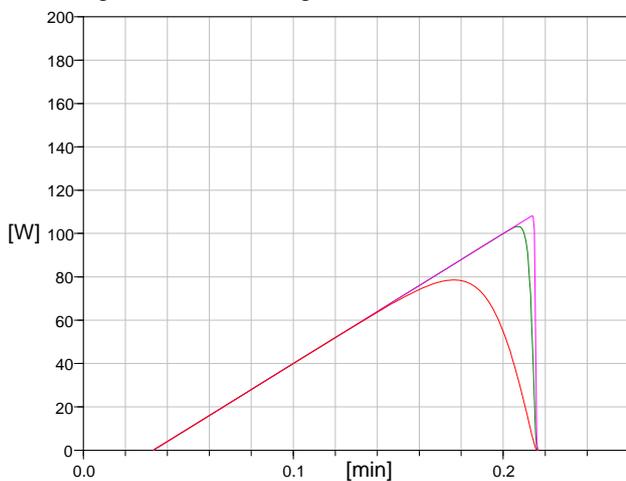
Figure 11 sketches the two cost functions and Figure 10 displays the corresponding model diagram. To enable several iterations for the final solution, the price per watt and the power have been implemented as vectors (see Section 3.1). By the parameter  $n$ , the number of iterations can be determined. In this case, we choose  $n=4$ . This means that the model contains now 4 parallel market models that each represents one round of negotiation.

During simulation the power demand is increasing with a constant rate. Figure 12 and Figure 13 presents the results of the simulation for the different rounds of negotiation. We can see the produced power of each generator.

Clearly, in the first round (blue), the large generator pushes aside its smaller counterpart. But in the following rounds of negotiation, the small generator can make its point. The resulting final behavior (magenta) almost leads to a discrete switch as soon as the large generator becomes more efficient as its smaller counterpart. The simulated results reflects an almost optimal behavior.



**Figure 12:** Power output of the large generator for different rounds of negotiation (round 1: blue, round 2: red, round 3: green, round 4: magenta).



**Figure 13:** Power output of the small generator for different rounds of negotiation (round 1: zero valued, round 2: red, round 3: green, round 4: magenta).

## 5 Application Domain: Load Management

### 5.1 Motivation

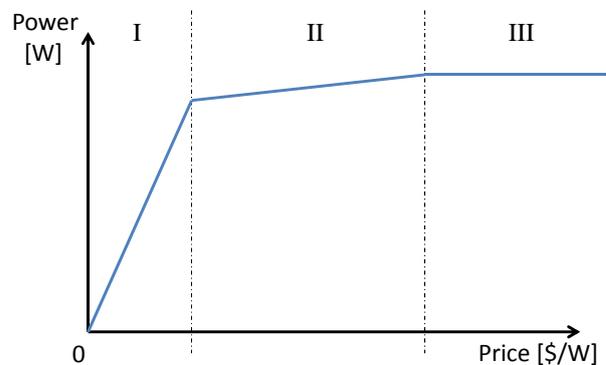
A typical load management (e.g. as applied in the electrical system of an aircraft) can cut and reconnect loads depending on its priority. The priorities can directly be translated into prices. Thus low priority loads just pay low prices for a certain amount of power whereas high priority loads pay high prices.

The goal is to get a stable, object-oriented load management function. Thus it is possible to get an implementation very quick and enable an early integration of the function into design process of system to be controlled. Furthermore, modular functionality like dealing with switchable and continuous loads in one system can easily be added.

### 5.2 Derivation of cost functions

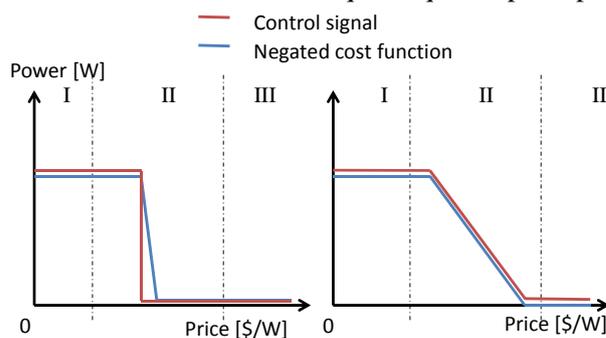
Other than source management, the model of a typical source for load management looks rather different. The focus is on maximum availability of loads and stability, not on energy efficiency. A source function as illustrated in Figure 14 is implemented having linear segments in three areas.

In area I, all loads are on. So there is no special requirement on the function rather than being monotone and continuous. Area III defines the maximum power capacity of the generator by means of a constant value. In this area all controllable loads shall be off. Within area II, cutting of switchable loads and decreasing of continuous loads take part.



**Figure 14:** Cost-function of a source having three areas: I – all loads on, II – shedding, decreasing loads with respect to its priority, III – all controllable loads off.

As shown in Figure 15 the cost-functions of switchable and continuous loads are quite equal in principle.



**Figure 15:** Negated cost functions and control signals of switchable loads (left) and continuous loads (right).

They consist of a full-power area, a linear decreasing area, and a zero-power area. The main difference is the slope of the function. The following inequation applies:

$$\text{slope}(\text{switchable loads}) \gg \text{slope}(\text{continuous loads})$$

Furthermore, the control signal is different for the two types of loads. All switchable loads receive an off-signal, if the current price is not within full-power area whereas all continuous loads receive a continuous power signal as determined in the cost-function. Since the location of the linear decreasing segment is determined via the priority of the loads and a global market model prescribes the location of the areas I, II, and III it can be guaranteed that this linear segment lies entirely in area II.

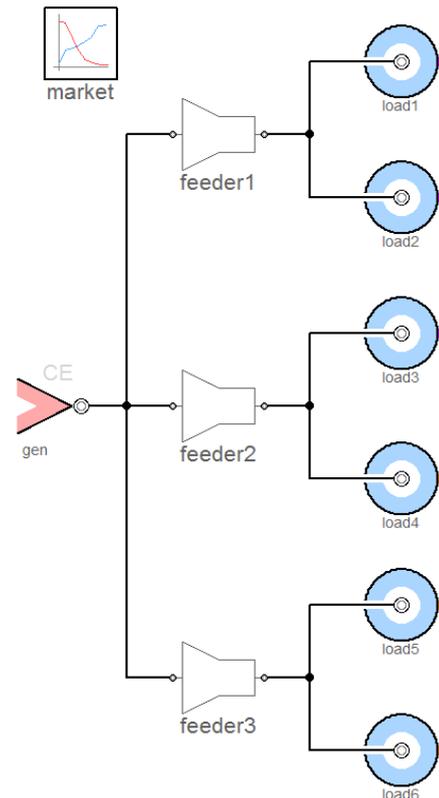
As the switchable loads are cut at the linear decreasing segment, one must avoid having two loads with the same priority. Otherwise both loads will be cut, even if not needed. Thus, each load should have its own priority.

If there are switchable and continuous loads in one system, multiple rounds of negotiations can be used to determine the power inflow for the continuous loads. This is done via setting a price in a first negotiation round using all cost-functions as described previously for calculating the control signals for the switchable loads. A second and final negotiation round for the continuous loads can then use these discrete control signals and assume all cost-functions of the switchable loads to be constant in all three areas (on or off). Thus less generator-capacity is wasted.

In typical load management systems, there are usually additional restrictions rather than the available generator capacity (e.g. a feeder that limits transmitted power or current to a set of loads). This can be modeled easily by means of a limiter as shown in Table 1. On the output plug, a price can be increased if a prescribed limit is exceeded. The preferred implementation includes qualitatively the same cost-function as for the generator (see Figure 14). At the output plug, a maximum function is applied that defines either the price at the input plug (i.e. from the price coming from the generator) or the price of the limiter. This ensures compliance with the restriction as well as an optimal availability of high priority loads.

### 5.3 Example

Figure 16 shows a simple setup of a load management model consisting of one source, three feeders (limiters) and six different loads. The model is set up in the same way like the corresponding physical electrical system



**Figure 16:** Example of a load management model having one source, 3 feeders and 6 loads (mixed continuous and switchable).

After specifying the nominal values for the source (generator) and the feeders as well as setting the priority of the loads, the load management function is ready to be used. Depending on actual power demand (input not illustrated in the figure), loads will be shed, reconnected, or reduced to comply with all restrictions of the source and limiters.

## 6 Conclusion and future work

This work represents our first approach towards a market-oriented modeling of energy-management tasks using a Modelica library. The current results look promising and demonstrate the principal functionality of the library. It can be used both for source and load management and also more difficult tasks such as non-monotonic cost functions can be reasonably well handled.

Although, we have analyzed only rather small systems so far, the simulation performance was always very good. We expect thus that the approach is also for feasible for larger systems with hundreds of generators and consumers.

One mayor advantage of having an energy management function directly implemented in Modelica is the easy coupling to the physical system it shall control. This enables an improved development pro-

cess of the system in conjunction with its control function and thus early optimization of both.

In case of source management, certain tasks need to be approached in order to create a solution that is more intuitively applicable for engineers. The import of characteristic curves (based on real data) for the efficiency of generators shall be supported by the library. In addition, the library needs to be tested at a larger set of more realistic examples. Further future potential concerns the modeling of dynamic characteristics. Power generators typically cannot increase their output power at any arbitrary rate. Also storage components like batteries have a dynamic pricing of their energy.

In case of load management, further functionality like variable cost functions shall be added to the library by allowing variable priorities. This enables a more flexible energy management, since the importance and availability of a load can change during operation. In addition, sources like generators can often be overloaded due to their heat capacity. Thus they shall also influence the cost function dynamically. Furthermore, additional elements like switches can be added to allow adaption of the management function in case of a network re-configuration.

One further major step is to combine both sub-libraries in a suitable way. This means to manage priorities of the loads as well as energy efficiency by one cost function. To this end, a more elaborated determination of price according to load priority, energy efficiency, and further restrictions is needed.

## References

- [1] Büchner, Stefan. *Energiemanagement-Strategien für elektrische Energiebordnetze in Kraftfahrzeugen*. Dresden, Germany: PhD thesis, faculty of Transportation and traffic sciences Friedrich List, TU Dresden, 2008.
- [2] Brewer, J. W. *Bond Graphs of Microeconomic Systems*. *Automatic Control Division*, ASME, New York, 1976
- [3] Brewer, J. W. and P. C. Craig. Bilinear, Dynamic Single-ports and Bond Graphs of Economic Systems. *Journal of The Franklin Institute*, No 185, 1991
- [4] Cellier, F. E., *Continuous System Modeling*. Springer Verlag New York, 1991.
- [5] Cellier, F.E., World3 in Modelica: Creating System Dynamics Models in the Modelica Framework, *Proc. 6th International Modelica Conference*, Bielefeld, Germany, Vol.2, pp. 393-400, 2008.
- [6] Engstle, Armin. *Energiemanagement in Hybridfahrzeugen*. Munich, Germany: PhD thesis, Fakultät Elektrotechnik und Informatik, TU München, 2008.
- [7] Odum, H.T, *Ecological and General Systems: An Introduction to Systems Ecology*, Colorado University Press, Colorado, USA, 1994.
- [8] Schlabe, D., and J. Lienig, "Energy Management of Aircraft Electrical Systems - State of the Art and Further Directions," *International Conference on Electrical Systems for Aircraft, Railway and Ship Propulsion (ESARS)*, Italy, October 2012.
- [9] Ygge, Fredrik, *Market-Oriented Programming and its Application to Power Load Management*. Lund, Sweden: PhD thesis, Department of Computer Science, Lund University, 1998.