1. INTRODUCTION

Highly accurate spatio-temporal traffic data (e.g. origin destination matrices, route flows and paths) can be obtained by the newly developed Dynamic Indirect Traffic Detection (DITD) approach, which was recently developed by the German Aerospace Centre [Ruppe et al. 2012]. DITD enables an efficient and powerful traffic monitoring and control system on the basis of wireless communication systems, which minimizes the number of costly stationary traffic detection infrastructure (e.g. traffic sensors, detection gantries, etc.) and thus will be superior to existing costly traffic detection systems.

Applying DITD all detections are made indirectly by traffic observers using wireless radio-based technologies (e.g. Bluetooth/Wi-Fi) while passing other traffic objects (vehicles, cyclists, pedestrians). Since many traffic participants use devices with activated Bluetooth/Wi-Fi functionality (e.g. mobile phones), a car equipped with a specific receiver (Mobile Traffic Observer Unit—MTOU) detects all traffic objects featuring Bluetooth/Wi-Fi devices and being in the detection area by their identification number. Augmented by time stamps and positions of the observer, the measured data can be processed to trajectories, travel times, etc. [Ruppe et al. 2011]. Due to the novelty of this approach a few research questions have been answered yet [Gurczik et al. 2012] while some are still in the pipeline. There we focused on the microscopic view to find out how many equipped cars are required to derive dynamic high quality traffic information on the basis of this new approach.

In this paper, the research will be taken to the next level. Hence, we focus on adapting the analytical model of the microscopic view to a macroscopic one to derive an overall detection probability of the Bluetooth/WiFi-equipped vehicles by an arbitrary number of MTOU vehicles. It will be shown, that the problem is quite complicated and still incompletely solved. It is shown that the extrapolation of the problem from one point to an area or even a single edge requires the integration of different key parameters with specific
characteristics and interdependencies. Those first have to be identified and defined to successfully put the dynamic traffic detection approach into practice for a comprehensive road network. Therefore, two approaches—an analytical and alternatively a geometrical one—are analysed to obtain a model on the macroscopic level.

The mentioned research study refers to an internal project of the German Aerospace Centre dealing with the improvement of the efficiency, safety and environmental friendliness of mobility and traffic and transportation management at different DLR sites in everyday life. In this study a concept is implemented, which includes traffic detection, simulation, communication, control and benchmark issues. Within this project the presented approach will be put in practice to display applicability in real environment.

2. ANALYTICAL MODEL

In this section the analytical model for computing the detection probability of vehicles equipped with Bluetooth/WiFi by Mobile Traffic Observer Units (MTOU) is introduced. This is done on the basis of two ways: the microscopic and the macroscopic level. Eventually, both methods are merged to result in a final model for analytically describing the detection probabilities.

2.1 Identified Influences

The equation $P_D$ defines the probability to detect a passing traffic participant. $P_D$ can be described as a function $f$ of different key parameters, which are important to put the DITD approach into practice [Gurczik et al. 2012]:

$$P_D = f(D, N, V, T, E, ...),$$

with:

- $D$ (detection related parameters, e.g. the distance between observer and vehicle, the detection technology, e.g. Bluetooth, Wi-Fi, which defines the inquiry process),
- $N$ (road network related parameters, e.g. network size and type),
- $V$ (vehicles related parameters, e.g. vehicles speeds),
- $T$ (traffic related parameters, e.g. traffic demand, traffic state, traffic control),
- $E$ (environmental parameters, e.g. rain, the construction area, multipath conditions).

Up to now only some aspects of the equation will be considered within the given analytical model approaches, the remaining ones will be neglected in this paper, since the influences and interdependencies have not been
identified yet sufficiently. Their analysis and integration in the analytical and simulation models will be the objective of our future research.

2.2 Microscopic View

To compute the detection probability $P_D$ of a vehicle detected by a MTOU it is first necessary to solve the geometrical problem shown in fig. 1, where is shown a vehicle (white), which enters the detection range of a MTOU (blue). In fact, it can be seen in fig. 1 that $P_D$ depends on road geometrical features, e.g. the distance $d$, and kinematic characteristics by the moving vehicles, i.e. the speed differences of the vehicle and the MTOU. Thus, the time the vehicle will be theoretically visible for the MTOU will be defined as $t_v$ (visibility time) and can be computed geometrically by eq. 1:

$$t_v(|\Delta v|) = \begin{cases} \frac{2\sqrt{r^2 - d^2}}{|\bar{v} - \bar{v}_{MTOU}|} & |x - x_{MTOU}| \leq r \\ 0 & |x - x_{MTOU}| > r \end{cases}$$

which contains the detection range $r$ of the MTOU, the vertical distance $d$ between the two vehicles (road geometry), the detection range of the MTOU (technical features and parameters) and the absolute values of the velocity differences $|\Delta v| = |v - v_{MTOU}|$ (kinematic characteristics).

![Figure 1. Microscopic view to compute $t_v$: (a) the way $\Delta x$ the white car will be visible for the MTOU (red line); (b) sketch of the geometrical problem](image)

It can easily be recognised that $t_v$ of the vehicle to the observer approximates zero, if $|\Delta v|$ is very high. On the other hand, the visibility time will be infinity if both vehicles follow each other at the same speed (see fig. 2). Further, $t_v$ is strongly influenced by $r$, which can vary between 10 to 100m for Bluetooth and up to 500m in the case of WiFi. The parameter $d$ can sometimes be disregarded, if the vehicles run on the same road. If $r$ grows, clearly $d$ becomes more interesting in the case of intersections, motorways, etc., which is not taken into consideration here. Further, it is clear that $P_D$ reaches the theoretical maximum if $d = 0$ and $P_D$ is minimal if $d = r$. 
Figure 2. The visibility time for a detection range \( r = 30 \text{m} \) and \( d = 2 \text{m} \), which is typical for urban areas in 30km/h zones.

To compute the detection probability, we rely on some experimental results given in [Franssens 2010], which show that it is 95% likely to detect a Bluetooth vehicle within the so called *limiting* enquiry time of \( t_{\text{lim}} = 7.68 \text{s} \). Thus, we assume the detection probability \( P_D(t_v \leq t_{\text{lim}}) \) to increase linearly to 0.95 within 7.68s, since we obtained rare enquiry detections even after 80s we assume to have a 100% detection probability at 100s (see fig. 3):

\[
P_D(t_v) = \begin{cases} 
0.95 \frac{t_v}{7.68} & t_v \leq t_{\text{lim}} \\
0.05 \frac{t_v + 87.32}{92.32} & t_{\text{lim}} < t_v \leq 100 \text{s}
\end{cases}
\]

Figure 3. The characteristics of the empirical detection probability

Inserting \( t_v(|\Delta v|) \) into \( P_D(t_v) \) we are capable of computing the detection probability \( P_D(|\Delta v|) \) in dependence on the absolute value of the velocity differences \(|\Delta v|\), which is shown in fig. 4 for \(|\Delta v| \in [0;50] \text{m/s} \):

\[
\]
\[ P_D(|\Delta \bar{v}|) = \begin{cases} 
\frac{0.95 \sqrt{r^2 - d^2}}{3.84 s |\Delta \bar{v}|} & \text{if } t_v \leq t_{\text{lim}} \cap (|x - x_{\text{MTOU}}| \leq r) \\
\frac{0.05 \sqrt{r^2 - d^2}}{46.16 s |\Delta \bar{v}|} + \frac{87.32}{92.32} & \text{if } t_{\text{lim}} < t_v \leq 100 s \cap (|x - x_{\text{MTOU}}| \leq r) \\
0 & \text{if } |x - x_{\text{MTOU}}| > r 
\end{cases} \]

It is evident that \( P_D(|\Delta v|) \) is comparably high in the case of low \(|\Delta v|\), e.g. up to 8m/s, and decreases regressively if \(|\Delta v|\) is high.

As it can be seen in fig. 4, the detection probability of a vehicle equipped with Bluetooth detected by a MTOU of a detection range of 30m results in about 60% if the vehicles pass each other at a speed difference of 12m/s, which is typically for urban areas with 30km/h zones. This is indeed not problematic if several MTOU vehicles are used in the area. It also shows that a vehicle detection is less than 15% likely in the case of speed differences increase 50m/s, e.g. on federal highways and motorways, where the speed differences can be even much higher.

2.3 Macroscopic View

So far we have found an analytical model to compute the probability to detect a Bluetooth/WiFi-equipped vehicle by a MTOU in dependence on road geometric aspects, technical MTOU parameters and the dynamics of the vehicles themselves. In this paragraph we are interested in finding an analytical model to compute the overall detection probability \( P_D \) on a macroscopic level, i.e. in dependence on a MTOU vehicle fleet of several cars detecting many vehicles in a road network.
With regard to the investigations done for longitudinal message hopping in vehicle-to-vehicle communications (V2V) according to [Thiemann et al. 2008] we assume a vehicle fleet to be spatially exponentially distributed:

\[ f(x) = \alpha \cdot \exp(-\alpha \cdot x) \]

Reminding the microscopic view, which models the detection probability \( P_D(t_v) \), or equivalently \( P_D(|v|) \), under consideration of the dynamics of the vehicles, the geometric and technical parameters, the spatial detection probability \( P_D(x) \) of a vehicle by a MTOU can be described as

\[ P_D(x) = P[|x - x_{\text{MTOU}}| \leq r \cap (t_v > 0)]. \]

Clearly, a vehicle is detected by a MTOU if the vehicle is located within the detection range radius \( r \) and if it is visible for the MTOU for a certain time \( t_v > 0 \). This happens, if a Bluetooth/WiFi-equipped vehicle and/or MTOU pass each other. Hence, \( P_D(x) \) can be modeled as

\[
\begin{align*}
P_D(x) &= \int_{0}^{\sqrt{r^2 - d^2}} f(y) \cdot P_D(x - y) \, dy \\
&= \int_{0}^{\sqrt{r^2 - d^2}} \alpha \cdot \exp(-\alpha \cdot y) \cdot P_D(x - y) \, dy
\end{align*}
\]

in which \( P_D(x) \) is implicitly defined considering the movement of the MTOU and the vehicle within the detection range under consideration of the exponentially distributed vehicle fleet. Both equations are similar to the versions of [Thiemann et al. 2008], but have slightly different meanings. The differences can be described as follows:

- The method proposed by Treiber describes the longitudinal message hopping, while we consider the detection of vehicles by a MTOU.
- The vehicle detection is considered to be in both directions and can be described microscopically by \( P_D(t_v) \) or \( P_D(|v|) \) in dependence of the visibility time or the absolute value of speed differences. Thus, the detection is non-deterministic and takes place in accordance to the time the vehicles need to pass each other at a certain speed difference.
- The MTOU density \( \alpha \) can be described as the product of the number of lanes \( m \), the average vehicle density \( \rho \) on each lane and the percentage of equipped vehicles.

The solution of the integral given the differences to the paper of Thiemann et al. seems to be complicated and cannot easily be adapted to our problem. Thus, current research questions arise:

- \( P_D(x) \) describes the spatial detection probability in dependence on a MTOU vehicle passing an equipped vehicle. How can \( P_D(x) \) be
modified to compute the overall detection probability of \( N \) vehicles and \( M \) MTOUs, with \( M, N > 0 \) in a road network of size \( R \)?

- Although it seems to be trivial, we currently do not have a suitable transformation of the microscopic detection probabilities \( P_D(t_v) \) or \( P_D(|v|) \) to \( P_D(x) \). How can the microscopic and still incomplete macroscopic models be merged?
- Thus, it seems convenient to analyse the macroscopic approach in an area-based approach, which will be introduced afterwards. Are both methods equivalent? Do they differ and how can this be quantified?
- What is the suitable value for the Penetration Rate, i.e. at which positions and times in a road network needs to be a MTOU vehicle to derive usable trajectories for traffic and transportation management or even for strategic transportation planning?

**Geometrical area-based approach**

As an alternative to the proposed, but insufficiently analysed macroscopic approach a geometrical area-based approach is proposed, which is described in the following. The key idea contains the extrapolation of the microscopic view on an area size \( A \) with \( M \) MTOU and \( N \) Bluetooth/WiFi-equipped vehicles. In fig. 5a) there is visualised a square with an edge length \( L \) and the size \( A = L^2 \), which contains \( M \) circles of the MTOU vehicles with their detection radius' \( r \) of the area size \( A_N = \pi r^2 \), that shall be the same for all MTOUs yielding an overall area of \( C = N \cdot A_N < A \). If an equipped vehicle drives through the area \( A_N \), it will be detected with the known probability \( P_D(t_v) \) or equivalently \( P_D(|v|) \).

![Figure 5.](image)

To get a first impression about the overall detection probability, we first put a single MTOU vehicle in the centre of this square, having the detection range \( r \), yielding the area \( C = \pi r^2 < A_N \) of the emerging circle. The vehicles begin to drive at randomly chosen points of one side, let's say \((x_1,y_1)\), of the square.
with a randomly chosen angle and drive through the square with a linear trajectory at constant speed. The vehicles get through the detection circle with a specific desired probability and drive straight on to another side of the square and leave the square at a point, let’s say \((x_2, y_2)\) (see fig. 5b). For reasons of simplification we assume the detection process is deterministic, i.e. the detection of the vehicles happens immediately.

To determine the unknown detection probability we have to calculate the point \((x_p, y_p)\) which is located within the circle. This can easily be done by the solution of the geometrical problem. The straight line \(g\) in fig. 5b) is defined as:

\[
g: \vec{r} = (x_1, y_1) + \lambda \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}.
\]

Using the dot product we can determine the point \((x_p, y_p)\) on the line \(g\), which is perpendicular to \(g\) and goes through the centre of the circle \((x, y)\):

\[
(x_p, y_p) = \left( \begin{array}{c} x_1 \\ y_1 \end{array} \right) + \frac{\left( \begin{array}{c} x \\ y \end{array} \right) \cdot \left( \begin{array}{c} x_2 - x_1 \\ y_2 - y_1 \end{array} \right)}{\left\| \left( \begin{array}{c} x_2 - x_1 \\ y_2 - y_1 \end{array} \right) \right\|^2} \left( \begin{array}{c} x_2 - x_1 \\ y_2 - y_1 \end{array} \right). \]

If \((x_p, y_p)\) is located within the circle, the vehicle is assumed to be detected, thus the distance between \((x_p, y_p)\) and \((x, y)\) needs to be determined by the Euclidean norm, which must be smaller than or equal to \(r\):

\[
\left( \begin{array}{c} x_p \\ y_p \end{array} \right) \in C \text{ if } \left\| \left( \begin{array}{c} x_p - x \\ y_p - y \end{array} \right) \right\|_2 \leq r.
\]

The determination of the actual detection probability of a vehicle, i.e. an arbitrary vehicle trajectory penetrates the circle can be computed by the indicator function \(I\)

\[
I \left\{ \left( \begin{array}{c} x_p \\ y_p \end{array} \right) \in C \right\},
\]

which is used for simplification of the problem for any starting and ending points of the trajectories. The parameters \(\mu\) and \(\delta\) are used for integration of all possible positions:

\[
P_D = \frac{1}{L} \int_{\mu(0,1)}^{\mu(1,0)} \int_{\delta(0,1)}^{\delta(1,0)} I \left\{ \left( \begin{array}{c} x_p \\ y_p \end{array} \right) \in C \right\} d\mu \ d\delta + \frac{1}{L} \int_{\mu(0,1)}^{\mu(1,0)} \int_{\delta(0,1)}^{\delta(1,0)} I \left\{ \left( \begin{array}{c} x_p \\ y_p \end{array} \right) \in C \right\} d\mu \ d\delta.
\]

The computation of \(P_D\) yields the detection probability of an arbitrary number of moving vehicles by a single immobile MTOU vehicle. In the next step the methods will be generalized to an arbitrary number of moving MTOU vehicles.
and will not be solved within this paper. The following research questions arise:

- How can the mobility of the MTOU vehicle be modelled in the area-based approach? Will there occur differences in the results?
- How can the proposed area-based approach with a single MTOU vehicle be adapted to an arbitrary number of moving MTOU vehicles? Which results will be obtained?
- Is it possible to transform the area-based approach to the initially proposed macroscopic model for calculating $P_D(x)$? Are these results equivalent?

3. SIMULATIONS

In this chapter we show some analytical results for the microscopic model and some simulation results for a simple generic network given in [Gurczik et al. 2012]. Since the adaption of the microscopic to the macroscopic model is still in the pipeline we will not give any results concerning the macroscopic model.

3.1 Microscopic Analytical Model

In fig. 6 there the curves of visibility times and the detection probabilities for different parameters $d$ and $r$ in dependence of the absolute value of the speed difference are visualised. It can be seen, that particularly $r$ has the greatest influence on both, $t_v$ and $P_D$, which was analysed for $r = 30m$ and $r = 100m$. If $d$ increases too, which will be the fact on wide streets like motorways, particularly the detection probability rises dramatically. Here, we analysed $d$ for the two extreme cases of $d = 2m$ and $d = 100m$. 

(a) $t_v(|\Delta v|)$

(b) $P_D(|\Delta v|)$
Figure 6. The characteristics of the visibility times and detection probability for different values for \(d\) and \(r\).

### 3.2 Generic Road Network

In fig. 6 the simulation results made for a generic network of four intersections are shown [Gurczik et al. 2012]. We computed the re-detection probabilities of two different numbers of Bluetooth/WiFi-equipped vehicles by (a) at least two MTOU vehicles and (b) generally. It can be seen, that the detection probability increases to almost 100\% of the equipped vehicles at a MTOU vehicle ratio of less than 30\%. Here, the potential of the DITD approach to derive spatio-temporal traffic data can be clearly seen. It can be stated, that already a small number of about eight MTOU vehicles is already sufficient to detect and re-detect even 80\% of the equipped vehicles.

Nevertheless, as already mentioned at the derivation of the macroscopic model, we are working on the extrapolation process of the microscopic model to the incomplete macroscopic model to support the given simulation results.

Figure 7. Re-Detection Probabilities in dependence of different MTOU ratios.

(a) 20\% equipped vehicles  
(b) 50\% equipped vehicles

### 4. CONCLUSIONS AND FUTURE PROSPECTS

In this paper some analytic investigations were done to show the potential of the Dynamic Indirect Traffic Detection (DITD) approach, which was first introduced in [Ruppe et al. 2011]. We showed microscopically, how the probabilities evolve for different speeds and for a different number of equipped Mobile Indirect Traffic Observer Units (MTOU) vehicles. To put our microscopic model onto a macroscopic level we tried to adapt investigations done for longitudinal message hopping in vehicle-to-vehicle communications.
(V2V) [Thiemann et al. 2008], which seemed to be slightly different, but in fact could not be easily and sufficiently adapted to our problem. According to the perception that a street network can be considered as an area of a specific size with a specific number of vehicles and that the microscopic view is only an extrapolation to that kind of macroscopic level we invented a second geometrical approach, the area-based approach. This one led us to further research questions which have to be answered first before going on with analytical results on base of macroscopic modelling.

5 Bibliography


SUMO (Simulation of Urban Mobility) - http://sumo.sourceforge.net/