Excitation of Lamb waves using higher order coupled field elements for smart structure applications

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ABSTRACT

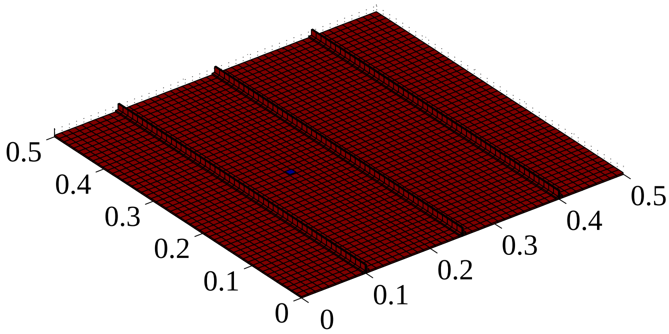
In recent years a steadily growing interest in online monitoring or structural health monitoring (SHM) of lightweight structures is seen, as SHM systems hold the promise to increase the safety and more importantly reduce maintenance costs of structures. A promising approach, in thin-walled structures, to reach the aforementioned goals is a Lamb wave based damage detection device. Currently guided waves are excited utilizing surface-bonded piezoelectric transducers. To be able to predict the wave propagation as well as the behaviour of the piezoelectric actuator/sensor accurately higher order Finite-Element-Methods (p-FEM) are am important numerical tool. Dealing with ultrasonic waves in thin-walled structures conventional linear or quadratic finite elements quickly reach their limit and are not suitable to obtain good quality results at manageable numerical costs. Additionally, even complex electrode geometries can be modelled easily using a p-FEM scheme. Thus, it is the objective of this contribution to develop different types of higher order hexahedral finite elements. They are either based on the normalized integrals of the Legendre polynomials or on non-uniform rational B-spline (NURBS). The capability of this approach is then demonstrated by computing the Lamb wave propagation in a stringer stiffened carbon fibre reinforced plastic (CFRP) plate.

From a numerical point of view the simulation of ultrasonic guided waves, especially in thin-walled structures, is a highly demanding. Deploying finite elements in the time domain not only requires a very fine spatial but also temporal discretization, due to the short wavelength and high frequency regime. The focus of the current contribution is on two alternative higher order FEM approaches:

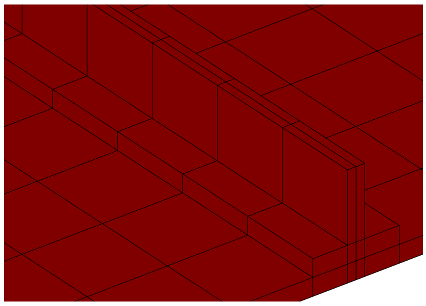
1. *Hierarchical p-FEM* based on the normalized integrals of the Legendre polynomials [2],
2. *Isogeometric FEM* based on non-uniform rational B-splines [1].

So far only the *Spectral-Element-Method* (SEM) has been deployed for high frequency wave propagation problems [3]. The capabilities of the two proposed methods the compute ultrasonic guided waves is shown subsequently. The derivation of the finite element system matrices follows [4] closely. The piezoelectric coupling is introduced by using the approach presented in [5].

As a realistic example from aeronautics a CFRP (material: T300/976, dimensions: $0.5\,m$ x $0.5\,m$ x $0.002\,m$) plate stiffened with three T10-stringer (material: T300/976, dimensions: $0.01\,m$ x $0.01\,m$ x $0.002\,m$) is studied. A piezoelectric actuator (material: PIC 151, dimensions: $0.01\,m$ x $0.01\,m$ x $0.002\,m$) is bonded to the the upper surface of the plate at point P1 ($x_1 = 0.175\,m$, $y_1 = 0.25\,m$, $z_1 = 0.002\,m$). The geometry of the plate including its finite element discretization is depicted in Figure 1a. The piezoelectric transducer was actuated using a windowed sinusoidal burst (Hann-window with three cycles) at a center frequency of $f = 150\,kHz$. A detail view, Figure 1b, of the model shows the geometry of the T-stringer. In the following snapshots of the travelling wave are presented, Figure 2. The influence of the material anisotropy introduced by the CFRP is clearly seen in the shape of the travelling wave. Instead of an circular wave front known from isotropic materials an elliptic shape develops.

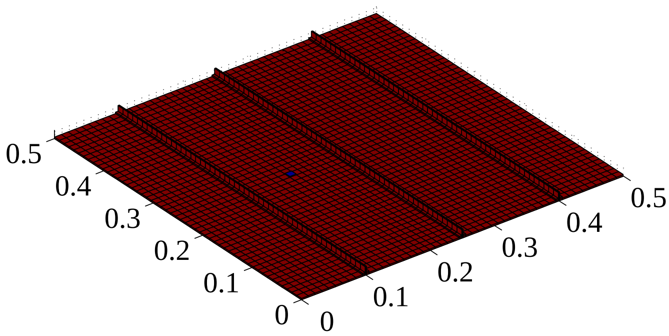


(a) Geometry and finite elements discretization of the stringer stiffened CFRP plate.

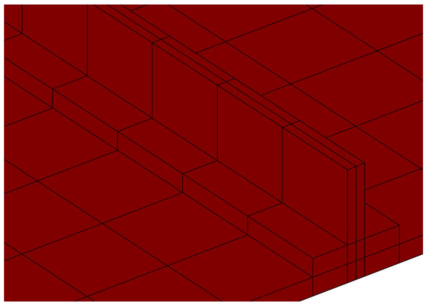


(b) Detail view of the T-stringer geomentry

Figure 1: Stringer stiffend CFRP plate.



(a) $t_1 = 1 \cdot 10^{-4}\, s$



(b) $t_2 = 2 \cdot 10^{-4}\, s$

Figure 2: Snapshots of the travelling wave at different times. Contour plots of the out-of-plane displacement component (u_z).

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