Relativistic Effects in Bistatic SAR Processing and System Synchronization

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Abstract

This paper addresses relativistic effects in bistatic and multistatic SAR systems and missions. It is shown that the use of different reference frames for bistatic SAR processing and bistatic radar synchronization is prone to notable phase and time errors. These errors are a direct consequence of the relativity of simultaneity and can be explained in good approximation within the framework of Einstein's special theory of relativity. Using the invariance of the spacetime interval, an analytic expression is derived which shows that the time and phase errors increase with increasing along-track distance between the satellites. The predicted errors are in excellent agreement with measurements from TanDEM-X and provide a satisfactory explanation for previously observed DEM height offsets that exceeded ±10 m. Consideration of the unexpected relativistic effects is essential for accurate DEM generation in TanDEM-X and has in the meantime been implemented in the operational processing chain.

1 Introduction

Bistatic and multistatic SAR systems operate with distinct transmit and receive antennas that are mounted on separate platforms. The spatial separation enables new radar imaging modes and is well suited to increase the capability, flexibility and performance of SAR systems and missions, thereby allowing for the acquisition of novel information products [1]. A prominent example is the TanDEM-X mission where a global DEM is acquired with two X-band SAR satellites flying in close formation [2]. The standard acquisition mode in TanDEM-X is the so-called bistatic mode where one satellite illuminates the scene with a sequence of radar pulses and both satellites receive the scattered signal echoes from the ground. The simultaneous reception by two receivers makes not only efficient use of the transmitted signal energy, but minimizes also the impact of temporal decorrelation.

The capabilities of bistatic and multistatic SAR missions are accompanied by new challenges regarding radar system implementation, operation and product generation. Well-known challenges are time and phase synchronization, relative position sensing, selection of appropriate transmitter and receiver trajectories, joint antenna steering, avoidance of mutual irradiation, and bistatic SAR processing. Up to now the topics bistatic radar synchronization, relative position sensing and bistatic SAR processing have been treated almost independently. By this, an important aspect has been neglected: radar time and phase synchronization are typically performed (and thought of) in a platform based reference frame, while an Earth centred Earth fixed (ECEF) reference frame is usually employed to specify the platform ephemerides and the bistatic SAR processing equations. In this paper we will show that the unreflecting mixture of different reference frames for bistatic SAR data acquisition and bistatic SAR processing may cause notable localization and phase errors in the focused bistatic SAR images. For most bistatic SAR systems, these errors can be well approximated and corrected for by considering the spacetime relations between two inertial reference frames as established in Einstein's special theory of relativity. The predicted magnitude of the phase and localization errors and their dependency on the formation geometry are in good agreement with the systematic latitude dependent DEM offsets that have been observed by evaluating a large number of bistatic TanDEM-X acquisitions.

The paper is organized as follows. Section 2 explains the difference between the reference frames used for bistatic SAR processing and bistatic radar synchronization. Section 3 forms the core of the paper and shows the peculiarities that arise if SAR processing and SAR system synchronization are performed in different reference frames. Section 4 provides examples from TanDEM-X that illustrate the time and phase errors arising from using different spacetime reference frames. The paper concludes with a short summary in Section 5.

2 Reference Frames

2.1 Bistatic Radar Synchronisation

A prerequisite for high quality bistatic and multistatic SAR imaging is an accurate synchronization between the transmitter and receiver radar systems. To this end, several techniques have been suggested, ranging from a tethered radar system over the use of ultra-stable clocks up to a direct RF link [3], [4], [5].[1] A common feature of all these synchronisation techniques is that they have the goal to establish a common time basis for the transmitter and receiver clocks. A tacit assumption, which is never made explicit, is that the clock synchronization is performed in a reference frame that is linked to the transmitter and receiver platforms.

2.2 Platform Centred Frame

For the sake of argument, we assume in the following that the transmitter and receiver platforms move with the same constant speed, so that their positions can be considered stationary in a platform centred reference frame. We also assume that the transmitter and receiver clocks are perfectly synchronized within this frame, independent of the actually employed synchronization technique.[2] Fig. 1 provides an illustration of this situation. TSX and TDX denote the transmitter and receiver satellites, which are separated by a constant baseline B. Note that while the satellites are stationary, the scene objects move relative to this frame with a velocity v. In this reference frame, the time interval between the transmission and reception of a radar pulse is denoted Δt.

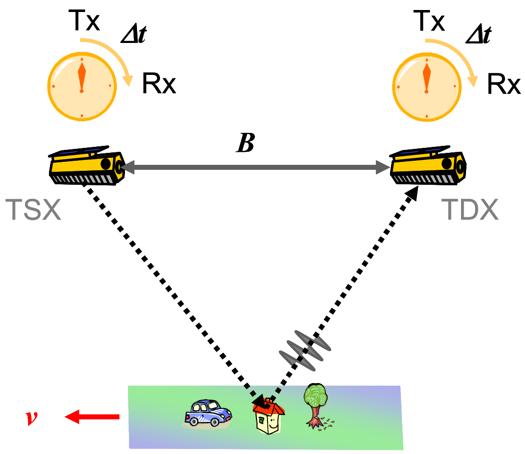

Figure 1: Bistatic SAR data acquisition as seen from a platform centred reference frame. The satellites are stationary while the scene moves as indicated by the red arrow. The Tx and Rx clocks are assumed to be perfectly synchronized in this reference frame.

[1] In TanDEM-X a special type of a direct synchronization link has been implemented where short radar pulses are periodically exchanged between the two satellites via a pair of pre-selected horn antennas [2]. An appropriate processing of the synchronisation signals allows then for the retrieval of the bistatic phase with a relative accuracy in the order of 1° in X-band, a value which has been confirmed during the bistatic commissioning phase of TanDEM-X [6]. This phase accuracy corresponds to a relative time accuracy below 1 ps.

[2] For this one may e.g. think of two highly accurate atomic clocks that are first synchronized at the same position and then slowly separated to the actual platform positions (such a procedure may pose some peculiarities in a non-inertial rotating reference frame, the discussion of which is beyond the scope of this paper).

2.3 Earth Centred Earth Fixed Frame

A different picture arises if one considers the bistatic SAR data acquisition from an Earth Centred Earth Fixed (ECEF) reference frame (Fig. 2). The platform ephemerides are supplied in this frame, where the Earth surface remains stationary. It is also common praxis to provide the geometric description and the SAR processing equations in this frame. Note that the along-track baseline between the satellites differs by $v{\cdot}(r_{bi}/c)$ from that provided in the platform centred reference frame if one compares the transmit and receive events. Here, v denotes the receiver velocity, r_{bi} the bistatic range, and c the speed of light.

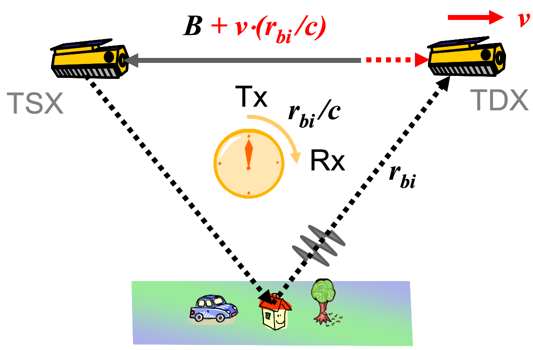

Figure 2: Bistatic SAR data acquisition as seen from a Earth Centred Earth Fixed (ECEF) reference frame where the scene is stationary and the satellites move.

3 Relativistic Effects

3.1 Relativity of Simultaneity

In 1905, Albert Einstein founded his special theory of relativity [8]. According to this theory the speed of light has always the same value, independent of the inertial reference frame one uses to describe a physical system. An immediate consequence is the so-called non-simultaneity of events. This means that two spatially separated events, which occur at the same time in one reference frame, may no longer be simultaneous in another reference frame that moves relative to the first one. This is illustrated in Fig. 3. As a result, radar transmitters and receivers, that are perfectly synchronous in the platform centred frame, are no longer synchronous in the ECEF frame.

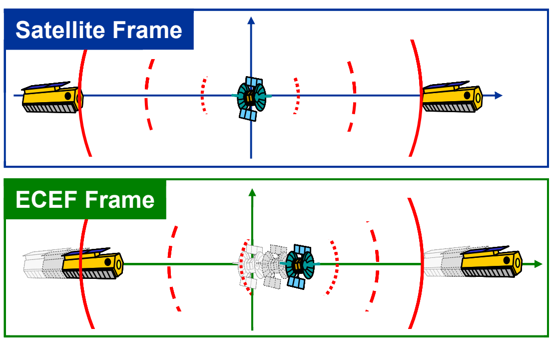

Figure 3: Illustration of non-simultaneity of events. A virtual satellite between TDX and TSX transmits pulses (red) which arrive at the same time (i.e. simultaneously) in the platform frame (top) but at different times (i.e. non-simultaneously) in the ECEF frame (bottom).

3.2 Invariance of Spacetime Interval

A central concept in the theory of relativity is the constancy of the spacetime interval that can (in case of flat spacetime geometry) be written as [9]

$$\Delta s^2 = \left(c \cdot \Delta t\right)^2 - \sum_{i=1}^{3} \Delta x_i^2 \tag{1}$$

where Δt and Δx_i denote, respectively, the time and position differences between two events as observed in a given reference frame[3]. From Fig. 1 it becomes clear that, in the platform reference frame, the spacetime interval between the transmit (Tx) and receive (Rx) events is given by

$$\Delta s^2 = \left(c \cdot \Delta t\right)^2 - \left\|\vec{B}\right\|^2 \tag{2}$$

where $\vec{B}$ is the baseline vector pointing from the transmitter satellite to the receiver satellite. Using on the other hand the ECEF frame of Fig. 2, the interval between these events is provided by

$$\Delta s^2 = \left(c \cdot \frac{r_{bi}}{c}\right)^2 - \left\|\vec{B} + \vec{v} \cdot \frac{r_{bi}}{c}\right\|^2 \tag{3}$$

By equating (2) and (3) one obtains

$$\left(c \cdot \Delta t\right)^2 - \left\|\vec{B}\right\|^2 = \left(c \cdot \frac{r_{bi}}{c}\right)^2 - \left\|\vec{B} + \vec{v} \cdot \frac{r_{bi}}{c}\right\|^2 \tag{4}$$

Solving this equation for r_{bi} yields:

$$r_{bi} = \frac{2\vec{B}\frac{\vec{v}}{c} \pm \sqrt{\left(2\vec{B}\frac{\vec{v}}{c}\right)^2 + 4\left(1 - \left(\frac{\left\|\vec{v}\right\|}{c}\right)^2\right)\left(c \cdot \Delta t\right)^2}}{2\left(1 - \left(\frac{\left\|\vec{v}\right\|}{c}\right)^2\right)} \tag{5}$$

For platform velocities that are small if compared to the speed of light, Equation (5) can be well approximated by

$$r_{bi} \approx c \cdot \Delta t + \vec{B} \cdot \frac{\vec{v}}{c} \tag{6}$$

The right hand side of this equation is composed of two terms. The first term represents the product of the velocity of light with the time difference between the Tx and Rx events as measured in the platform frame, where radar data acquisition and recording are performed. A user unaware of relativistic effects would mistake this first term as a direct measure of the bistatic range in the ECEF frame. Taking into account the spacetime structure of special relativity, the second term emerges. This term is proportional to the scalar product between the platform velocity vector $\vec{v}$ and the baseline vector $\vec{B}$, i.e. it increases with both the along-track baseline between the satellites and the satellite velocity. For a satellite tandem flying with a velocity of 7.5 km/s and an along-track baseline of 1 km, the second term amounts to a bistatic range error of 2.5 cm. This relativistic range offset may sound small, but for an interferometric X-band system with a wavelength of 3.1 cm, the phase error is with 290° already close to the ambiguity interval.

[3] For those who are familiar with the Lorentz transformations from special relativity, it is easy to show that Δs remains invariant under the Lorentz group of linear spacetime transformations.

4 Relativity in TanDEM-X

4.1 Theoretical Predictions

As an example, we investigate the predicted relativistic effects for TanDEM-X, where the two satellites fly in a close Helix formation [2]. The Helix formation provides not only suitable cross-track baselines for global DEM generation, but is also characterized by a periodic variation of the along-track separation B_{along} between the two satellites. For the present context, B_{along} can be approximated with sufficient accuracy by

$$B_{along}\left(\Phi\right) \approx A \cdot \cos\left(\Phi\right) \tag{7}$$

where Φ is the argument of latitude and A is a constant that depends on the eccentricity offset between the satellite orbits. Depending on the selected Helix formation, A has typically values between 500 m and 900 m.

Figure 4 shows the predicted relativistic range offsets for TanDEM-X as a function of latitude for $A = 600$ m. Note that the sign of the shift changes by interchanging the role of the transmitter and receiver satellites. This dependency is evident from both Equation (6) and Figure 2. The magnitude of the bistatic range error varies between +- 15 mm. While such an error may be considered small for bistatic localization and image registration, it will cause severe offsets in case of bistatic DEM generation. Such a DEM can be generated either radargrammetrically or interferometrically by combining the monostatic image from the fully active transmitter with the bistatic image recorded by the passive receiver. Assuming that the relativistic effect can be neglected for the monostatic image, a range difference of ±15 mm would translate to a height error of ±24 m for a TanDEM-X acquisition with a height of ambiguity of 50 m. From this, it becomes clear that relativistic effects cannot be neglected in the operational DEM generation chain of TanDEM-X.

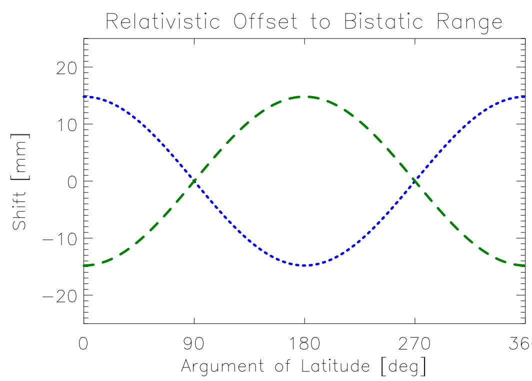

Figure 4: Predicted relativistic range offset for a typical TanDEM-X satellite formation with a maximum radial displacement of 300 m and a resulting variation of the along-track baseline between ± 600 m. The dashed green curve shows the relativistic offsets if TSX is selected for transmission, while the blue dotted curve shows the predicted offsets in case that TDX is transmitting.

4.2 Experimental Results

Figure 5 shows the estimated radargrammetric offsets obtained with an early version of the TanDEM-X processor that did not take into account relativistic effects. The offsets have been obtained by comparing TanDEM-X radargrammetric DEMs with external reference DEMs. Both the magnitude and the characteristic change of sign fit well with the prediction from Figure 4. Note that the figure contains also some other (smaller) instrument related calibration errors that have in the meantime been corrected [7].

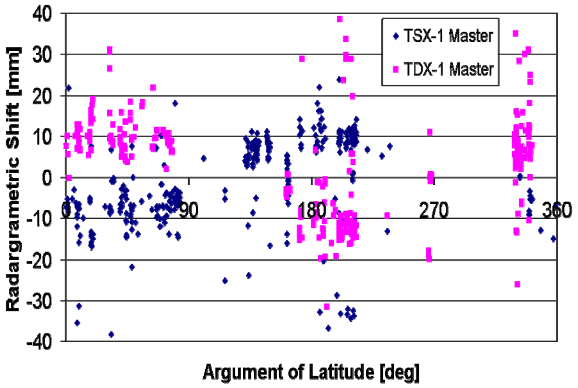

Figure 5: Measured radargrammetric shifts in TanDEM-X as a function of latitude. The shifts were obtained by comparing TanDEM-X radargrammetric DEMs to reference DEMs (this plot is from the commissioning phase and contains also some other (but smaller) errors that have in the meantime been corrected [7])

Taking into account the relativistic correction and additional calibration steps in the operational TanDEM-X processor, the accuracy of the radargrammetric shifts is now below ±5 mm where inaccuracies of the reference DEMs may be the dominant error source.

Figure 6 shows an even more clear dependency which can be explained by relativistic effects. The red and green crosses denote the interferometric phase offsets that have been obtained by comparing the interferometric TanDEM-X DEMs with reference DEMs[4]. Besides the π-ambiguity[5], which is resolved in the final processor by radargrammetry, again a clear dependency on the along-track baseline can be seen. By comparing the measured data with the relativistic prediction, which is shown by the dashed blue lines, an excellent agreement is obtained. It becomes again clear that relativistic corrections are required to avoid systematic, latitude dependent offsets in the final DEMs. Note that without the relativistic correction the interferometric phase values would be almost randomly cluttered among a complete ambiguity interval. This caused in the beginning a significant confusion within the TanDEM-X engineering team.

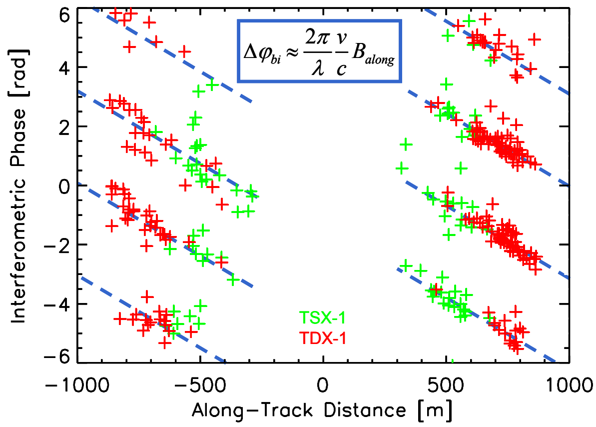

Figure 6: TanDEM-X interferometric phase offsets as a function of latitude. The relativistic prediction (dashed blue lines) agrees well with the measurements (crosses).

5 Conclusions

It has been shown that relativistic effects may cause notable errors in bistatic and multistatic SAR systems and missions. These errors caused significant confusion when the first bistatic TanDEM-X DEMs were systematically evaluated. Examples were the almost random distribution of the interferometric phase due to its dependency on the along-track baseline and the puzzling sign flip and latitude dependency of the radargrammetric shifts. Both effects could successfully be explained by using the theory of special relativity. The relativistic corrections from this paper have in the meantime been incorporated in the operational TanDEM-X processor and the overall spread of the radargrammetric shifts is now well below 10 mm.

Acknowledgment

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[4] Note that the baseline is always computed from the transmitter (master) satellite, so that the sign flip is not visible in this Figure.

[5] The π-ambiguity is a consequence of the bi-directional synchronisation technique where the average of two phase values is evaluated.