DTA2012 Symposium: Ecological User Equilibrium in Traffic Management (TM)?

Michael Behrisch, Yun-Pang Flötteröd, Daniel Krajzewicz, and Peter Wagner
Institute of Transport Systems, German Aerospace Center

michael.behrisch@dlr.de, yun-pang.floetteroed@dlr.de,
daniel.krajzewicz@dlr.de, peter.wagner@dlr.de

Abstract

With increasing environmental sustainability awareness significant attention on ecological traffic management (eco-TM) has come into the focus of researchers and practitioners. While different approaches have been applied to reach minimal pollutant production, the classic user equilibrium calculation with the pollutant production as travel costs instead of using travel times remains in the center of attention. However, the validity of such a direct transformation to find a user equilibrium is questionable. In this paper, a simplified analytical approach to examine the above aforementioned validity has been carried out, followed by a simulation approach to verify the results of the analytical approach. The result shows that the pollutant production function violates the usual assumption of a monotonous function (typically, emission has a minimum at travel speeds around 60 km/h). It also indicates that the respective algorithms to compute the user equilibrium must deal with the fact, that the equilibrium solution is not unique and is dependent on the initial solution. This means that substantial modifications to the algorithms that compute the user equilibrium have to be discussed since they do not work as intended when pollutant production is used as travel costs, especially in a transportation system with mixed speeds that cover a range around the minimum emission speed.

Introduction

With increasing environmental sustainability awareness significant attention on ecological traffic management (eco-TM) has been paid since last decades. Usually, eco-TM is performed by computing several scenarios and then selecting the one with minimal pollutant production. In fact, this does not try to minimize an objective function directly that describes the emission production as function of the traffic pattern in a given area. Another applied approach is to add a toll that takes pollutant production into account and to find a user equilibrium based on measured pollutant production. This approach can in principle be embedded in the usual formulation of the user equilibrium (UE), i. e. emissions generated by vehicles will be used as travel costs instead of using travel times, so that no users can find a route with lower emissions than the route they use. However, the validity of such a direct transformation to find a user equilibrium is questionable, since the vehicular energy consumption does not monotonously increase with descending or ascending traveling speed [1]. For each vehicle type and even for each vehicular brand, there is usually an ideal traveling speed for the optimal energy consumption.

Furthermore, in our own research we have found that our dynamic user equilibrium algorithm [6] is running into difficulties, when we try to find the dynamic equilibrium for an objective function that is not based on travel times, but on fuel consumption or pollutant production. Albeit there are several reasons imaginable for this failure, we suspect the non-monotonicity of the objective function to be the culprit, which is the reason why this paper first goes back to a simple static situation, where such an effect can be analyzed analytically. This paper investigates the validity of the classic user-equilibrium approach based on pollutant emission. In addition to this analytical approach, a dynamic micro-simulation will be used subsequently to verify the results of the analytical approach. Some remarks and perspectives will be offered at the end.

Analytical approach

Monotone validity

A classic simple example with one OD-pair and two routes is chosen here [2]. Assume that the two routes have exactly the same length with$L_1 = L_2 = 30\,\text{km}$, they have a linear travel-time function as function of demand q:

$$t_i(q_i) = T_i \left(1 + k\frac{q_i}{q_\infty}\right) \tag{1}$$

where q_∞ is a proxy of the link capacity, k is a factor that determines, how slow the travel time will be when capacity is reached, i. e. $(k+1)T_i$, and T_i is the travel time at free-flow speed ($q_i = 0$). The factor k can be link-dependent, but only one factor is used for all links here.

Pollutants, e. g. CO_2, typically have a more complicated function. A simplified form as a function of speed is adapted here with regard of analysis simplicity and shown below.

$$\tilde{e}(v) = c + dv^3 \tag{2}$$

The equation (2) is the production per unit of time. To compute the production along a link of length L_i, it has to be multiplied with the time needed to traverse the link, where this time is given by equation (1). Therefore the pollutant produced along a certain link turns out to be

$$\begin{aligned} e_i(q_i) = t_i(q_i)\left(c + dv^3\right) &= t_i(q_i)\left(c + d\frac{L_i^3}{(t_i(q_i))^3}\right) \\ &= T_i\left(c\left(1 + k\frac{q_i}{q_\infty}\right) + dV_i^3\frac{1}{\left(1 + k\frac{q_i}{q_\infty}\right)^2}\right) \end{aligned} \tag{3}$$

where V_i is the travel speed on link$i$ at free-flow speed ($q_i = 0$). An alternative form of this equation is $e(v) = cL/v + dLv^2$. In this form, the constants are easier to explain: c is clearly the pollutant production when idling, while d is a complicated constant taking into account air drag, which depends on the vehicle form, front area and so on. However, by assuming an ideal speed v_0 with minimal pollutant production, the constant d can be written as $d = c/\left(2v_0^3\right)$ which results in:

$$e_i(q_i) = cT_i \left(1 + k\frac{q_i}{q_\infty} + \frac{1}{2}\left(\frac{V_i}{v_0}\right)^3 \frac{1}{\left(1 + k\frac{q_i}{q_\infty}\right)^2} \right) \tag{4}$$

Since pollutant production is usually proportional to energy consumption (at least for the most prominent pollutant CO_2), fuel consumption can be used as a general indicator of pollutant production. In most cases, v_0 has been set to $15\,\mathrm{m/s}(54\,\mathrm{km/h})$, while $c = 1\,\mathrm{l/h}$ is a good estimate for the fuel consumption of a vehicle when idling. According to the aforementioned assumptions, the relationship between travel time, fuel consumption and the number of vehicles can be illustrated in Figure 1 on page 3. It is obvious, that the pollutant production function violates the usual assumption of a monotonous function, which also indicates that the algorithms to compute the user equilibrium can not work correctly with use of pollutant production as travel costs.

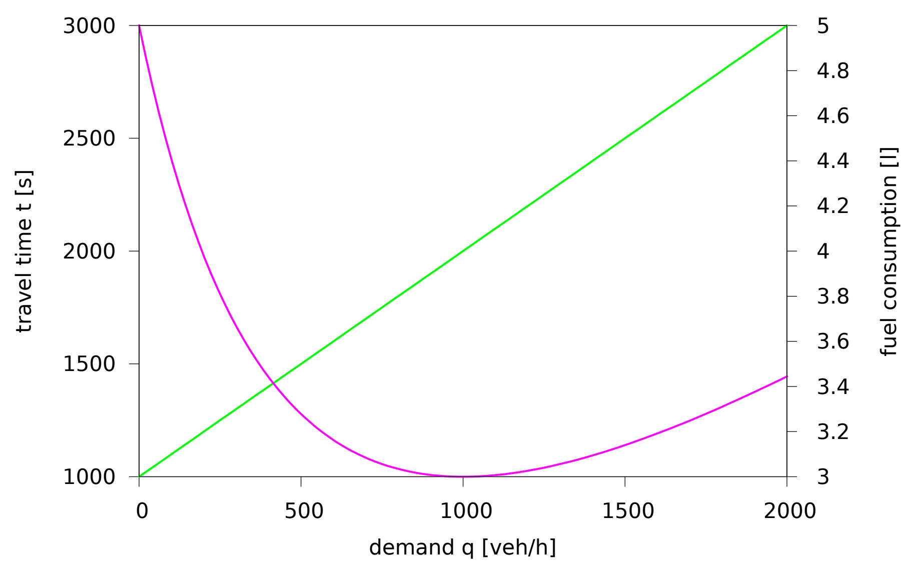


Figure 1: Relationship between travel time, fuel consumption and traffic demand. The travel-time eq. (1) and the pollutant function eq. (4) as functions of demand. The parameters have been set such that the minimum in the pollutant versus speed curve occurs at $v = 15\,\mathrm{m/s}$. The parameter settings here and for Figure 2 are: $L_1 = L_2 = 30\,\mathrm{km}$, $T_1 = T_2 = 1000\,\mathrm{s}$, c and v_0 are described in the text, the capacity on each link has been set to $q_\infty = 2000\,\mathrm{veh/h}$ and $k = 2$ has been used.

User equilibrium validity

For the travel times, the user equilibrium can be computed as usual [3], by minimizing the objective function:

$$Z(q_1, q_2) = \sum_{i=1}^{2} \int_0^{q_i} d\omega t_i(\omega), \tag{5}$$

with $t_1(q_1) = t_2(q_2)$ and $q_1 + q_2 = Q$, where Q is the total demand for travel. The same formulation can then be used with pollutant production for reaching a eco-based user equilibrium. In the two routes example, two constraints will now be

$$e_1(q_1) = e_2(q_2), \tag{6}$$

$$q_1 + q_2 = Q, \tag{7}$$

To get the solution we can either solve $e_1(q_1) = e_2(Q-q_1)$ directly or construct the complete objective function which leads to:

$$E(q) = cTq\left(1 + \frac{1}{2}k\frac{q}{q_\infty} + \frac{1}{2}\left(\frac{V}{v_0}\right)^3 \frac{1}{1 + k\frac{q}{q_\infty}}\right), \tag{8}$$

$$T(q) = Tq\left(1 + \frac{k}{2}\frac{q}{q_\infty}\right), \tag{9}$$

$$Z^{(e)}(q_1) = E(q_1) + E(Q - q_1), \tag{10}$$

$$Z^{(t)}(q_1) = T(q_1) + T(Q - q_1), \tag{11}$$

which is a one-dimensional curve, parametrized by the demand Q. Note, that the two additional solutions cannot be directly inferred from the condition $e_1(q_1) = e_2(Q-q_1)$ or$\partial E(q)/\partial q = 0$, since they stick to the boundary of the valid UE's solution region.

As shown in Figure 1, both low and high traveling speeds result in more pollutant production than a so-called ideal traveling speed with minimal pollutant production. If the demand is small, e. g. 1000 veh/h, and there is only high-speed traffic in the two routes example, the possible solutions with the aforementioned objective function can be calculated and illustrated in Figure 2 on the next page. When the demand is only 1000 veh/h, the following situation arises: start with a share of 0.5, i. e. half of the vehicles drive on route 1, and the other half drive on route 2. The condition $e_1(q_1) = e_2(q_2) = e_2(Q - q_1)$ is then fulfilled, but this is not a stable set-up and even not the optimal solution, since the fuel consumption can be further reduced when one vehicle switches to the other route. Such a route switch increases the traffic flow on this route, and then reduces the respective traveling speed. The pollutant production will also accordingly be reduced. Therefore all drivers will immediately switch to the route with more traffic. This phenomenon leads to the surprising situation that a stable eco-based UE solution in this case is given by either $p = (1, 0)$ or$p = (0, 1)$, where p is the vector of shares q_i/Q. This changes, of course, for large demand, or for links where the maximum speed is below the minimum of the pollutant curve (city traffic). Figure 2 on the facing page indicates that the minimal fuel consumption occurs with a share of 0.5 when the demand is 3000 veh/h.

Furthermore it also shows that, in the situation with a demand of 2000 veh/h, there are still the both minima at the boundaries ($p_1 = 1$ or $p_2 = 1$).

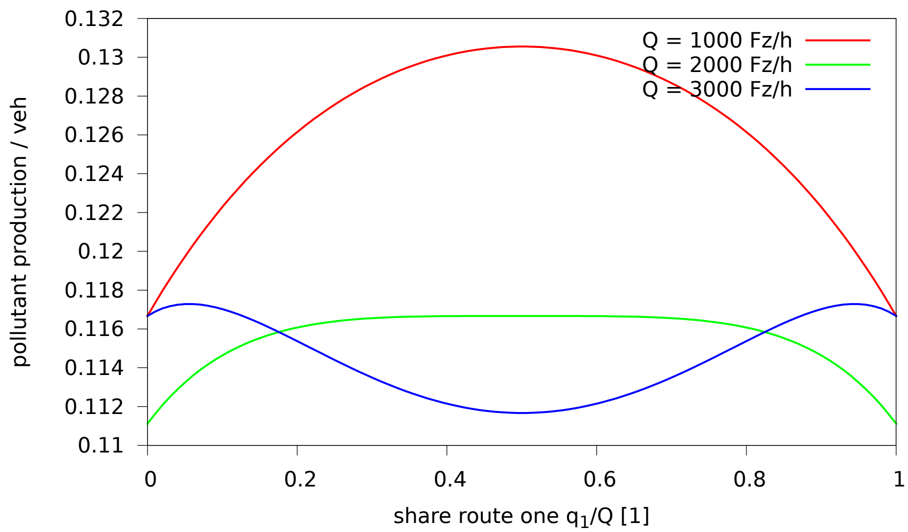


Figure 2: Pollutant production per vehicle, i. e. $Z^{(e)}(q_1)/Q$ as function of the share of vehicles using route 1.

Dynamic traffic simulation

A simulation study has been undertaken, since true emission functions are more complicated than the simple approach used here. The microscopic traffic simulation software SUMO [4] and the HBEFA-based emisson model [5], already implemented in SUMO, are used with the aforementioned two-routes example. The logit model for route choice is applied here as well. Figure 3 on the next page shows the relationship between fuel consumption and speed for a passenger car in SUMO. It is clear to see that the highest fuel consumption occurs at very low or very high speeds. The optimal speed in SUMO is around 65 km/h.

Experiment setting

Generally speaking, fuel consumption mainly depends on travel speed and acceleration in addition to travel duration. The former one is the main factor and used as standard unit when talking about fuel consumption rate. The later one occurs very often in stop-and-go traffic or at intersections, when traffic lights turn to green and vehicles try to pass the intersections as quickly as possible. In order to compare with the aforementioned analytical results, the experiment focuses on the speed and is so designed that there is no major acceleration influence on fuel consumption, i.e. no traffic lights are used in the network and road capacities are ruled by the allowed travel speed on each link. The used one-way network consists of two routes with the same length (10 km) and each route has only one lane. Both routes have a maximal travel speed of 30 m/s(108 km/h). These two routes will merge on a 2-lane exit link with an allowed travel speed of 8.3 m/s(30 km/h). No traffic weaving will occur at the merge point and

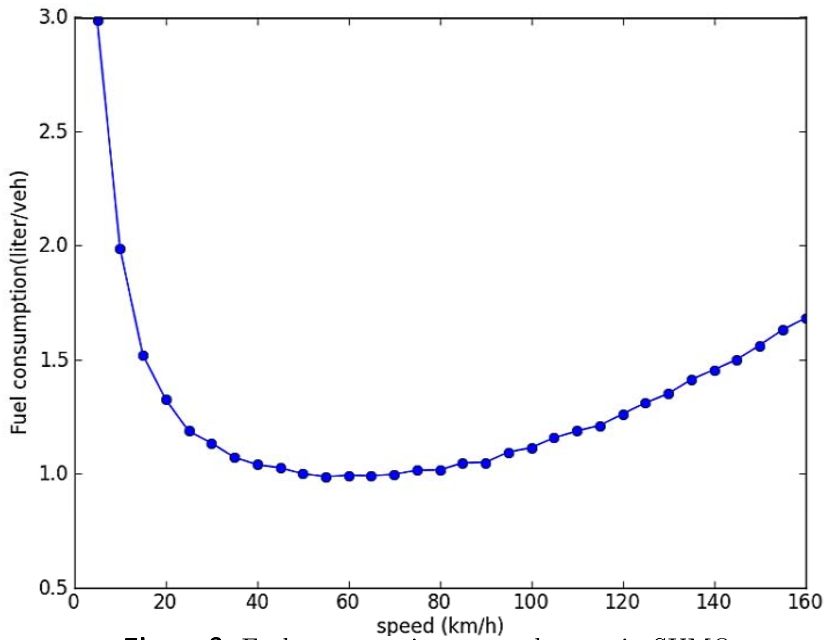


Figure 3: Fuel consumption – speed curve in SUMO

only passenger cars are applied in the experiment. Several scenarios with different demands and initial route-choice solutions are studied.

- Traffic demands: 100, 300, 500, 1300, 3000, 4000 and 5000 vehicles/hour.
- Initial route-choice solutions :
  - No route shares are given, i. e. only start and destination points given and route shares are determined by the dynamic traffic assignment.
  - Shares on Route 1: 0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, . . . , 0.9, 0.95 and 0.99: Route sets are randomly generated with the given demands and shares. For example, Given that demand is 100 and the share on Route 1 is 0.01, one vehicle with Route 1 and the other 99 vehicles with Route 2 will be defined in the respective route file.

The generated route files with different route shares is first simulated to examine the consistency and the difference between the analytical and the simulation approach. Furthermore, dynamic traffic assignments with trips and vehicular routes, used as initial solutions, are executed to investigate their influences on the ecological user equilibrium.

Results

Simulation with given route sets

The simulated results, shown in Figure 4 on page 8, support the statement made in the analytical approach. More traffic results in less fuel consumption per vehicle, since the respective travel speed decreases with the increase in traffic demand. In comparison to that, travel time is directly proportional to traffic demand as already well-known (see Figure 4 on page 8(b)).

Furthermore, like travel time, the change in fuel consumption with small traffic demands is not significant. The slight fluctuation of the fuel consumption is mainly due to the stochastic effect in the dynamic traffic simulation. When the traffic demand reaches 1000 veh/h, the fuel consumption curve begins to change. The minimal fuel consumption occurs when only one route, either route 1 or route 2, is used. A balanced route share (50/50) results in the highest fuel consumption. When the traffic demand further increases until 2000 veh/h, the fuel consumption curve turns into a bell shape form with different slopes. The above-mentioned phenomenon remains i. e. the UE solution it not unique. When the network is heavily loaded with 3000 veh/h, the shape of the fuel consumption curve becomes a flat m-shape. It shows that a local optimal solution is possible and there is no guarantee for obtaining a global optimal solution. It also implies that an initial solution has a great influence on the search of the optimal solution. In this case, an UE-algorithm may find a solution which does not use only one of the routes if the initial route-share is between 0.35 and 0.65. It is since a UE-state can be reached, not only when there is no route with lower fuel consumption, but also when two routes are with the same fuel consumption for users. The latter one is the case of the local optimal solution.

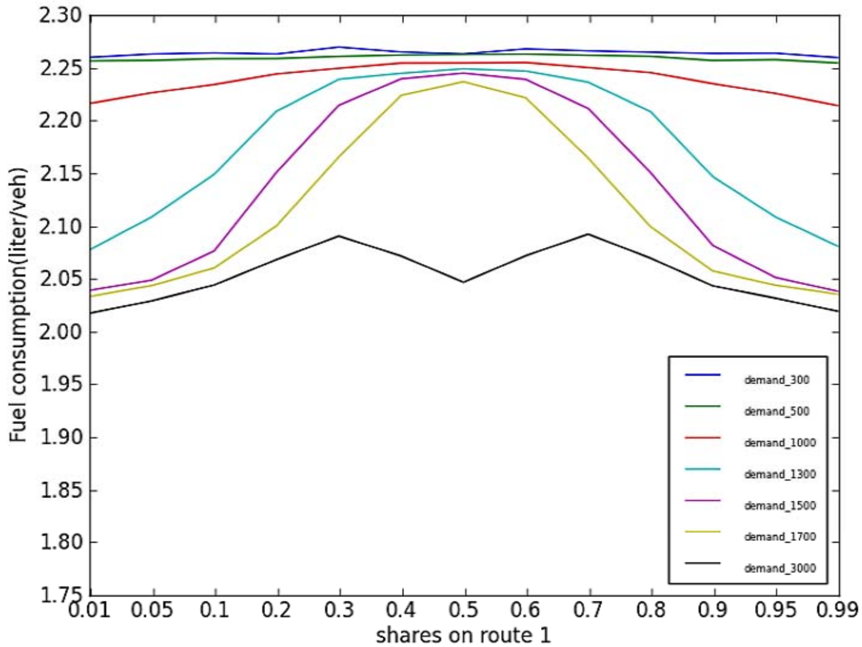
Simulation with dynamic traffic assignment

In this part, dynamic traffic assignment is adapted to find the UE solutions, based on fuel consumption, for all scenarios. The influence of the initial route share on the UE solution is examined first. Moreover, how the route shares change during the simulation iterations is investigated as well for obtaining a better overview about the solution-searching direction.

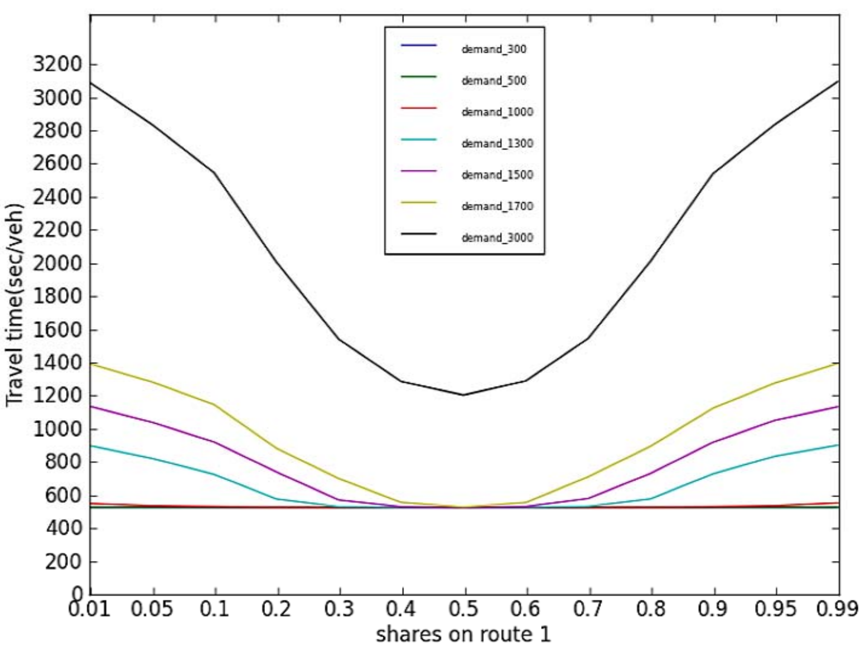
(1) Fuel-based route shares

Figure 5 (a) and (b) show that the resultant route shares on both routes are almost equal, when traffic demand is small. The initial route shares have no significant influence on the UE solution. This is due to the fact that the respective fuel consumptions for small demands are very similar, as shown in Figure 4, and the probabilities to either choose route 1 or route 2 are almost the same. With the increase in traffic demand, the travel speed has declined. Accordingly, Figure 5 (c) and (d) indicate that the route with a higher traffic load, i.e. with a lower travel speed, is preferred. However, it also shows that this preference is affected by the given initial route share. When the initial route share on route 1 is less than 0.4, the search direction to the UE solution is towards the use of route 2, while the usage of route 1 will be towards 100 % with an initial route share on route 1 greater than 0.5.

It is noticed that the local optimal solution for traffic demand 3000 is not found here, although an initial route share on route 1 is set as 0.4, 0.5 or 0.6. This is mainly because of the driver's perception, which is considered by the factor θ in the logit model: $\exp\left(-\theta C_{\text{utility},i}\right) / \sum \exp\left(-\theta C_{\text{utility}}\right)$. Here, the drivers are relatively sensible to the fuel-consumption difference. Therefore, the search direction in the first iterations goes quickly towards the optimal UE solution.



(a) fuel-consumption based

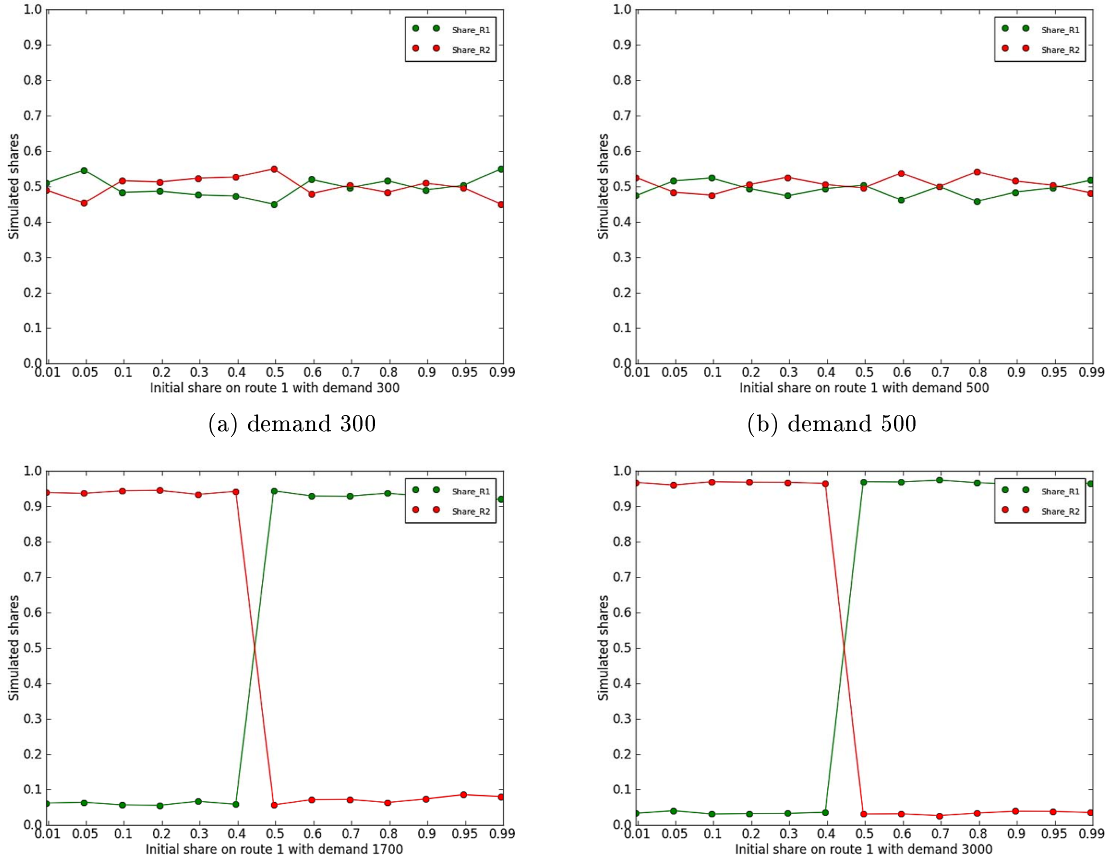


(b) travel-time based (with departure delay)

Figure 4: Simulated fuel consumptions with given route sets

(2) Iterative changes in route shares

Regarding the iterative changes in route shares, it shows that route shares with different initial values are relatively stable during simulation iterations, when demands are small (see 6 (a)



(a) demand 300

(b) demand 500

(c) demand 1700

(d) demand 3000

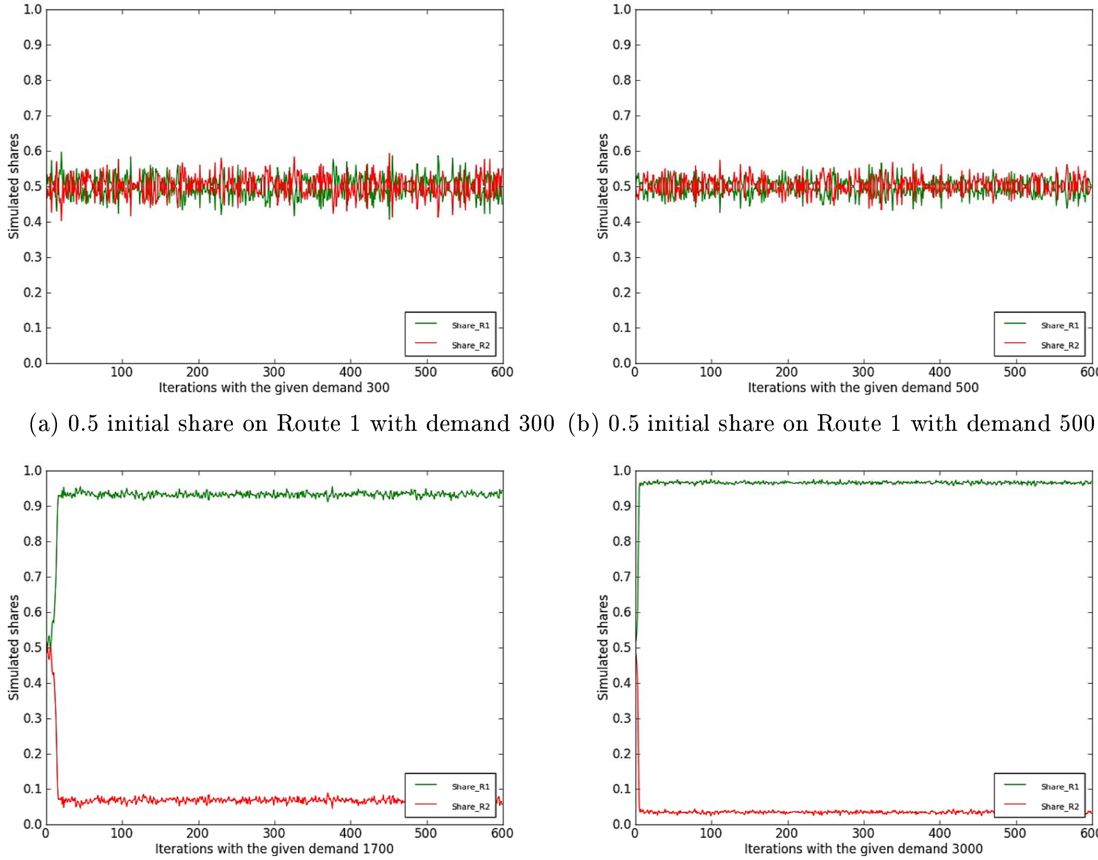
Figure 5: simulated route shares with different initial shares

and (b) as example with an initial route share of 0.5). Route shares fluctuate between 0.4 and 0.6 with traffic demand 300. In comparison to that, the respective fluctuation spectrum is slightly narrowed, when the traffic demand increases to 500 veh/h. This may be an averaging effect stemming from the larger amount of vehicles only.

When the traffic demand goes up, 6 (c) and (d), as an example, indicated that the route shares change dramatically and are towards the optimal solution within the first 10 iterations regardless the given initial route shares. Once the optimal solution is reached, only some slight fluctuation in route shares exists due to the stochastic effect in the dynamic traffic simulation. While this was to be expected for the medium demand of 1700 due to the high slopes shown in Figure4a), it is somehow unusual for the demand of 3000 and requires further studies.

Remarks and perspective

The results based on the dynamic traffic simulation approach verify the statement proposed with the analytical approach in this paper. The result with the simple two-route example shows that the pollutant production function violates the usual assumption of a monotonous



(a) 0.5 initial share on Route 1 with demand 300 (b) 0.5 initial share on Route 1 with demand 500

(c) 0.5 initial share on Route 1 with demand 1700 (d)0.5 initial share on Route 1 with demand 3000

Figure 6: Iterative changes in route shares with different traffic demands

function, which also indicates that the respective algorithms to compute the user equilibrium must now deal with the local optimum issue and the fact, that the UE solution is not unique and is dependent on the initial solutions, i.e. route shares. This non-uniqueness will have consequences for all approaches trying to seek eco-optimal solutions in large transportation systems, and it is very likely the reason to explain the convergence problems we faced with such a simulation dealing a real complex urban network.

What is even more disturbing is that the solutions that came out of such an approach are completely counter-intuitive and that it is highly unlikely that they will ever be realized in reality. Squeezing all the demand on one link to force vehicles to drive slower to achieve an eco-optimal solution is a funny idea, but nothing that is realistic. On the other side, the UE approach can, in fact, still be used for a eco-TM in a traffic system where the speed limit is smaller than the ideal speed with minimal pollutant production. In this case, only the right branch of the pollutant curve in Figure 1, i. e. the left branch of the curve in 3 in the fuel consumption case, is used and everything is still working as intended. In the next steps, the interaction between the driver's perception of fuel consumption and the solution searching will be further investigated. A further investigation with a real network will be conducted as well.

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