A NEW MODEL OF THE LITHOSPHERIC FIELD OF MARS USING MGS-MPO DATA AND L1-REGULARIZATION.

A. Morschhauser[1], M. Grott[1], V. Lesur[2], [1]*German Aerospace Center (DLR), Institute for Planetary Research, Rutherfordstrasse 2, 12489 Berlin, Germany*, [2]*Helmholtz Centre Potsdam, German Research Centre for Geosciences (GFZ), Telegrafenberg, 14473 Potsdam, Germany*.

Introduction: The nature of Mars' magnetic field was successfully investigated by the Mars Global Surveyor (MGS) mission, which operated in Mars' orbit from 1997-2006 [1,2]. The Martian magnetic field is characterized by areas of strong local magnetization of lithospheric origin in the lack of a global dipolar core-generated magnetic field [2]. This lithospheric remanent field was likely acquired from a now extinct core dynamo field and succesively altered by processes related to chemical, thermal or shock de/magnetization. Therefore, the lithospheric field of Mars contains valuable information on the thermal evolution of the planet as well as on impact and volcanic processes.

The raw satellite data of the magnetic field contains noise as well as contributions of external fields. These contributions need to be properly treated in order to build robust models of the Martian lithospheric field. Several such models have been published, which use either spherical harmonics (SH) [3,4] or equivalent source dipoles (ESD) [5,6] to describe the lithospheric field. To build these models, data from the Mapping Phase Orbit (MPO) obtained at an almost constant altitude of 400 km and global coverage as well as Areobraking (AB) data obtained at lower altitudes with non-global coverage, have been used.

Here, we will present a spherical harmonic model expanded up to degree and order 90, which is based on MGS-MPO data. We fit the model to the data by minimizing the least-squares distance between the modeled and measured values of the magnetic field data, but use a L1-norm to regularize our model in the spatial domain. This approach will support large gradients of the lithospheric field without ignoring the nature of the normally distributed errors. Furthermore, the presented model is the first which is based on the entire MPO dataset, consisting of ~54 million vector measurements obtained on the nightside of Mars.

Data: We use vector magnetic data as measured by the two fluxgate magnetometers on board the Mars Global Surveyor spacecraft, which operated for almost a decade (1997-2006) in Martian orbit. This data is provided by the Planetary Plasma Interactions Node of the UCLA Planetary Data System (PDS) and has been corrected for spacecraft-generated fields [1]. In order to minimize noise from solar wind interaction, we reject all dayside data by using the provided information of current in the solar panels. Here, we solely use Mapping Phase Orbit data, as it provides global coverage at a constant altitude. Data from the Areabraking Phase with lower altitudes could be used to subsequently improve and/or test the model.

The data was arithmetically averaged in 81838 nearly equal-area triangles with a surface of 1600-2100 km^2, as the inversion of the ~54 million datapoints would require too much computational resources. The arithmetic average minimizes the distance to the data in a least-squares sense and is compatible with the applied data inversion scheme. The typical distance between triangle centers is smaller than 100 km and therefore smaller than the mapping altitude of 400 km, thus providing sufficient resolution for data inversion. Furthermore, we reject data points which deviate more than three standard deviations (STD) from the average in their corresponding triangle. The final data set consists of ~52 million data points in 81838 triangles.

Model: We express the magnetic field in terms of a scalar potential field V, thus assuming that the data was collected in a source-free region. The vector magnetic field is then expressed as the gradient of a scalar potential V, which can itself be expanded in terms of spherical harmonics by

$$V(r,\theta,\phi) = a \sum_{l=1}^{l_{max}} \sum_{m=0}^{l} \left(\frac{a}{r}\right)^{(l+1)} \Psi_l^m(\theta,\phi) \qquad (1)$$

where

$$\Psi_l^m(\theta,\phi) = [g_l^m \cos m\phi + h_l^m \sin m\phi]\, P_l^m(\cos\theta) \qquad (2)$$

Here, a is the reference radius of the model, l, m are degree and order of the expansion series, and g_l^m, h_l^m are the expansion coefficients. Furthermore, P_l^m are the associated Legendre Polynomials and r, θ, ϕ are the spherical coordinates of the point at which the potential is calculated.

The magnetic potential and therefore the magnetic field components are a linear function of the gauss coefficients g_l^m and h_l^m. Therefore, the gauss coefficients can be fit to the data by solving

$$d = Am \qquad (3)$$

where d is the data vector, consisting of one vector measurement for each triangle (245514 data points). The model vector m consists of the $l(l+2)$ gauss coefficients and A is the matrix describing the SH model.

In order to avoid overfitting of noise in the data, we regularize our model by imposing an additional constraint. As the radial component of the field contains least of the noise of ionospheric origin [3], we chose to additionally minimize the surface integral of the horizontal gradient of the radial field.

We calculate the horizontal gradient of the radial field by first projecting the SH model onto a hexagonal grid with sufficient resolution in the spatial domain. Subsequently, we differentiate the obtained values with respect to θ and ϕ for each of the grid points. Formally, this can be expressed by a projection operator P and a differential operator D, which are both linear with respect to the model parameters. Therefore, we can extend Eq. 3 by

$$\begin{bmatrix} d \\ 0 \end{bmatrix} = \begin{bmatrix} A \\ DP \end{bmatrix} m \qquad (4)$$

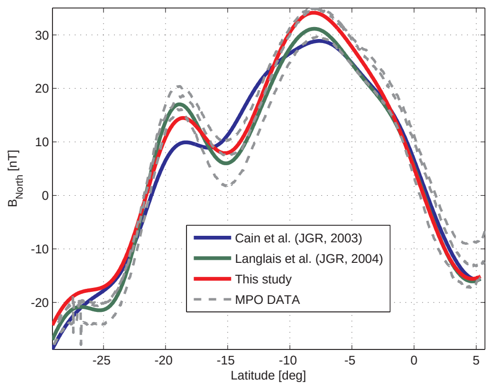

Figure 1: Comparison of several models of the lithospheric field to along-track data of three adjacent tracks from the years 1999, 2002 and 2005 (gray-dashed lines) over Terra Cimmeria. Strong gradients are well fitted by applying a L1-norm.

The Gauss coefficients will then be obtained by minimizing

$$G = (d - Am)^T w_1 (d - Am) + \Lambda (DPm)^T w_2 (DPm) \quad (5)$$

where Λ is a regularization parameter, and w_1 and w_2 are weight matrices. The damping parameter Λ was chosen such that the fit to the data and the fit to the regularization term are both as close as possible to their respective best values. Here, w_1 is diagonal, as errors are assumed to be uncorrelated, and corresponds to the STD of the data in the respective triangle. The weight matrix w_2 is also diagonal and contains the absolute value of the misfit vector, i.e. $(w_2)_{i,i} = |d_i - \sum_i A_{i,j} m_j|$, such that the term describing the regularization will be minimized using an L1-norm by iteratively repeating the inversion. This has the advantage of more accurately representing large gradients caused by local magnetic anomalies. At the same time, the normal error distribution in the data is still considered, as the data is minimized using a L2-norm.

Results: Using the above approach, we constructed a model of spherical harmonics up to degree and order 90 which was fit to the MGS-MPO nightside data of the Martian lithospheric field. Results of the calculation are shown in Fig. 1, where three close satellite tracks above Terra Cimmeria are compared to the ESD model of Langlais et al. [6], the SH model of Cain et al. [4], which was expanded up to degree and order 90, and the model presented here. The fit to the strong anomalies over Terra Cimmeria has improved compared to the model of Cain et al. [4], as the large gradients are better resolved by the applied regularization. Another approach to model large gradients is the ESD method, as it is based on local dipoles. Hence, the model of Langlais et al. [6] is also performing better in fitting these anomalies than the model of Cain et al. [4]. However, this comes at the cost of more free parameters which have to be fitted to the data, as assumptions on the shape, location and size of the dipoles have to be made.

The component of the magnetic field pointing radially down over the volcano Apollinaris Patera is shown in Fig. 2 for the model of Langlais et al. [6] (left) and the model presented here (right) at an altitude of 400 km. The large positive anomaly northeast of Apollinaris has sharper contours in our model than in that of Langlais et al. This is also the case for the small negative anomaly southwest of Apollinaris Patera. Therefore, large gradients appear to be best represented by a model derived using a L1-norm for regularization.

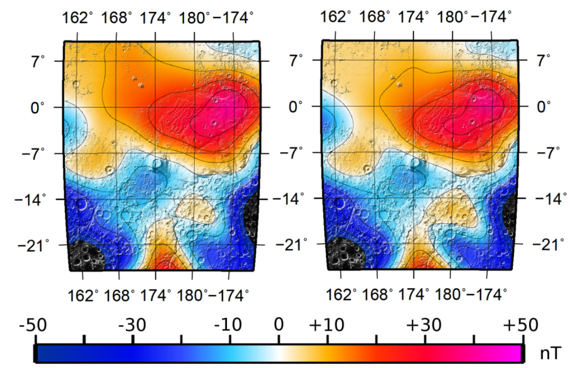

Figure 2: Magnetic field component pointing radially down as derived from the models of Langlais et al. (left) and this study (right) over the Apollinaris Patera volcano. Distance between isolines is 10 nT.

Conclusions: We have presented a model of the lithospheric field of Mars which was derived from magnetic field data obtained during the mapping phase orbit of Mars Global Surveyor. The model was regularized by minimizing the integral of the horizontal derivative of the radial component of the magnetic field over the surface using a L1-Norm. Using this approach, strong anomalies associated with large gradients, which are prevalent in the Martian lithospheric field, are well represented, while still accounting for the normal distribution of errors in the data.

References [1] Acuña et al., *J. Geophys. Res.*, 106(E10), 23403-23417, 2001. [2] Albee et al., *J. Geophys. Rss.*, 106(E10), 23291-23316, 2001. [3] Arkani-Hamed, J., *J. Geophys. Res.*, 109(E09005), doi:10.1029/2004JE002265,2004. [4] Cain et al., *J. Geophys. Res.*, 108(E2), doi:10.1029/ 2000JE 001487, 2003. [5] Purucker et al., *Geophys. Res. Let.*, 27(16), 2449-2452, 2000. [6] Langlais et al., *J. Geophys. Res.*, 109(E02008), doi:10.1029/2003JE002048, 2004.