# Wavelet-Based Compressed Sensing for SAR Tomography of Forested Areas

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## Abstract

SAR tomography is a thriving three-dimensional imaging modality that is commonly tackled by spectral estimation techniques. As a matter of fact, the backscattered power along the vertical direction can be readily obtained by computing the Fourier spectrum of a stack of multi-baseline measurements. Alternatively, recent groundbreaking work has addressed the tomographic problem from a parametric viewpoint, thus estimating effective scattering centers by means of covariance matching techniques. In this paper, we introduce a compressed sensing based covariance matching approach that allows us to retrieve the complete vertical structure of forested areas. For this purpose, we employ sparse representations in the wavelet domain and propose suitable pre-filtering techniques. Finally, we validate this approach by using fully polarimetric L-band data acquired by the E-SAR sensor of DLR.

## 1 Introduction

Three-dimensional imaging by means of SAR Tomography allows us to retrieve vertical structure parameters from measurements obtained through repeat-pass acquisitions. In fact, a common and simple approach is to employ parallel tracks, thus rendering the tomographic inversion a direction of arrival (DOA) problem. Unfortunately, the achievable resolution of conventional spectral estimators is highly dependent on the extension of the elevation aperture. Moreover, the sampling rate dictated by the celebrated Nyquist frequency imposes an additional requirement, namely regular, dense sampling.

Several successful attempts have been made in order to reduce the number of samples. For instance, the authors in [9] estimated the minimum number of tracks based on subspace methods. In addition, in [2, 13] Compressed Sensing (CS) inversion techniques for SAR tomography were successfully developed and applied. Nevertheless, the signals of interest were sparse in the space domain; a situation that is rarely true when it comes to volumetric scatterers. Alternatively, an extension of SAR interferometry from a parametric perspective was proposed in [12]. In a nutshell, this work employs covariance matching estimation techniques in order to estimate the effective scattering center of different scattering mechanisms, along with their backscattered power.

In this paper, we formulate a covariance matching methodology from a CS perspective, thus exploiting suitable sparse representations of forested areas in the wavelet domain. Further, we show how to pre-filter the covariance matrix by means of a low rank approximation. Finally, we conclude the paper by presenting promising experimental results.

## 2 Problem Formulation

We are interested in reconstructing the 3-D power distribution $p(x, r, s)$ of a complex reflectivity function $g(x, r, s)$; where $x$, $r$, and $s$ are the azimuth, range, and elevation coordinates, respectively. For a specific azimuth-range position, the corresponding discretized signals along $s$, with $1 \leq s \leq \Delta s$, will be denoted with the column vectors $g$ and $p$. Additionally, the tomographic acquisition using $m$ parallel tracks will be expressed as

$$b = \Phi g + y; \tag{1}$$

where $b$ is a stack of pixels taken from $m$ focused and coregistered SAR images. The matrix $\Phi$ is the so-called steering matrix which accounts for the phase rotations due to the distance traveled by the microwave pulses from the sensor to the targets distributed along $s$ and back to the sensor [9]. Further, $y$ is an additive noise term.
Throughout this paper, $\widetilde{k}$ will represent a vector or matrix of variables to be determined that approximate $k$.

## 3 Covariance Matching

For the purposes of this paper, we will simplify our methods by assuming that we are dealing with stationary natural scenarios where temporal decorrelation is negligible. That is to say, the main structural characteristics remain unchanged during the tomographic acquisition. As shown in section 8, this assumption has proven to be a fair approximation, since our experiments have been carried out using data acquired within five hours. Under this hypothesis, the covariance matrix of $b$ can be written out as follows

$$C = E\{b\, b^*\} = \Phi\, diag(p)\, \Phi^* + \sigma^2 I; \tag{2}$$

where $E\{\cdot\}$ is the expectation operator, $(\cdot)^*$ denotes the conjugate transpose, $diag(p)$ is a matrix whose main diagonal equals $p$ and is made up of zeros in its off-diagonal entries, and $\sigma^2$ is the unknown noise power [12].

With this in mind, we can retrieve the vertical power distribution by finding a solution that agrees with the sample covariance matrix $\widehat{C}$ as follows

$$\min_{\widetilde{p}} \left\| \Phi\, diag(\widetilde{p})\, \Phi^* - \widehat{C} \right\|_F ; \tag{3}$$

where $\|\cdot\|_F$ denotes the Frobenius norm. In accordance with the definition of $p$, the optimization has to be carried out over the set of nonnegative real numbers. However, when the number of acquisitions is small, the problem is highly ill-posed and regularization techniques are needed in order to obtain meaningful results.

# 4 Compressed Sensing

Compressed Sensing (CS) proposes measuring a signal $f$ by collecting $m$ linear measurements of the form $b = Af + y$, where $A$ is a $m$ by $n$ sensing matrix with $m$ typically smaller than $n$ by several orders of magnitude and $y$ is a noise term. The theory asserts that if $f$ is approximately sparse in a specific basis $\Psi$, it is indeed possible to recover $f$, under suitable conditions on the matrix $A$, by $L_1$ minimization

$$\min_{\widetilde{f}} \left\| \Psi \widetilde{f} \right\|_1 \text{ subject to } \left\| A\widetilde{f} - b \right\|_2 \leq \varepsilon; \tag{4}$$

where $\varepsilon$ is an upper bound on the noise level [1, 3–5]. In other words, despite the ill-posedness of the problem, CS manages to recover a signal from a reduced set of samples.

# 5 CS-Based Covariance Matching

By considering equations (3) and (4) together, we can readily recast the power distribution estimation as an instance of CS. As a matter of fact, once an appropriate sparsifying basis $\Psi$ has been chosen, we can reformulate the reconstruction of $p$ as follows

$$\min_{\widetilde{p}} \|\Psi\, \widetilde{p}\|_1 \text{ subject to } \left\| J(\widetilde{p}) - \widehat{C} \right\|_F \leq \varepsilon \tag{5}$$

with

$$J(\widetilde{p}) = \Phi\, diag(\widetilde{p})\, \Phi^*; \tag{6}$$

where $\varepsilon$ can be used to control the trade-off between sparsity in $\Psi$ and data mismatch.

In order to provide greater insight into the benefits of this CS perspective, we can use equation (2) to express each entry of the covariance matrix $C$ as

$$C_{j,k} = \langle p, \xi_{j,k} \rangle + \sigma^2 \delta_{j-k} \tag{7}$$

with

$$\xi_{j,k} = \Phi_j \odot \Phi_k^*; \tag{8}$$

where $1 \leq j, k \leq m$, $\langle \cdot, \cdot \rangle$ denotes the inner product, $\Phi_j$ represents the $j^{th}$ row of $\Phi$, and $\odot$ indicates element-wise multiplication. With this in mind, it now becomes clear that the tomographic acquisition basically samples the unknown $p$ by computing inner products with $m^2$ sensing waveforms $\xi_{j,k}$. However, due to the characteristics of the covariance matrix, these measurements may present a Toeplitz structure. In fact, in the case of regular baselines the entries of the covariance matrix become highly redundant. Consequently, this fact can be used to further augment the acquisition geometry [8].

# 6 Sparsity in the Wavelet Domain

The applicability of wavelets in combination with Fourier measurements under the framework of CS has proven to be a powerful a approach in various fields, such as in Magnetic Resonance Imaging (MRI) [7]. In the case of SAR sensors, as observed in [6,10–12], the distribution of the effective scattering over forested terrain is quite regular (see Fig. 1). This suggests that if we let $\Psi$ be a linear operator that computes the Discrete Wavelet Transform (DWT) of $p$, we will obtain a sparse expansion. Fig. 2 shows the rapid decay of the sorted magnitudes of the transform coefficients using a Daubechies Symmlet wavelet with 4 vanishing moments and 3 levels of decomposition. Additionally, Fig. 3 presents the magnitude of the inverse DWT, after zeroing out all but the largest 5 transform coefficients. Clearly, the profile is very well approximated. As a result, we can attain considerably sparse representations, thus minimizing the degrees of freedom to estimate $p$.

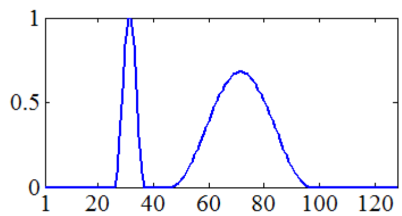


**Figure 1:** Expected vertical power distribution of the effective scattering (normalized).

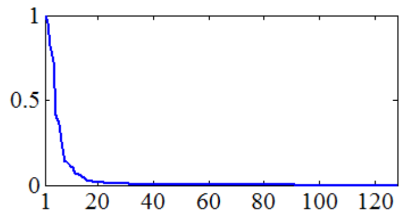


**Figure 2:** Sorted magnitudes of the transform coefficients using a Daubechies Symmlet wavelet with 4 vanishing moments and 3 levels of decomposition (normalized).

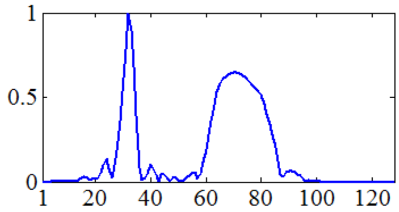


**Figure 3:** Magnitude of the inverse DWT after zeroing out all but the largest 5 coefficients (normalized).

# 7 Filtering by EVD

Although not necessary, we suggest, as a pre-processing step, that the sample covariance matrix $\widehat{C}$ in (5) be replaced by an appropriate low rank approximation. Basically, the idea is to chop off most of the noise subspace by computing an Eigenvalue Decomposition (EVD) and truncating the corresponding eigenvalues. In order to determine how to perform this truncation, we propose the following methodology. First, take the full rank covariance matrix and compute the well-known Capon filters. Then, generate tomograms by applying those filters separately on the rank one matrices corresponding to each eigenvalue. Finally, determine which rank one matrix contains forest information via visual inspection. As a rule of thumb, we have found that the forest information is generally clustered in the first 2 to 4 eigenvalues, depending on the number of tomographic acquisitions.

# 8 Experimental Results

In order to demonstrate the potential of the outlined methodology, we used a stack of 10 focused and coregistered SAR images obtained by processing fully polarimetric L-band data (see Fig. 4). This data was acquired by the E-SAR airborne sensor of DLR during a campaign near Dornstetten, Germany, in 2006. Fig. 5 shows the histogram of the corresponding baseline distribution. The center frequency used was 1.3 GHz and the nominal altitude above ground was about 3200 m. The resolution was 0.66 m and 2.07 m in azimuth and range, respectively.
In particular, we reconstructed the vertical power distribution according to (5) at a fixed range distance of 4816.30 m and for 400 contiguous azimuth positions (see yellow rectangle in Fig. 4). As a result, we obtained slices in the azimuth and elevation directions of dimensions 176 m by 40 m, respectively. The sparsifying basis we used was based on the Daubechies Symmlet wavelet with 4 vanishing moments and 3 levels of decomposition. In order to compute the sample covariance matrix, we took a 9 by 9 window.
Fig. 6 presents the span resulting from the reconstruction performed on the full rank covariance matrix for each polarimetric channel separately. In Fig. 7, we took the same approach but used the best rank 4 approximation. Evidently, Fig. 7 displays a much sharper and clearer image.

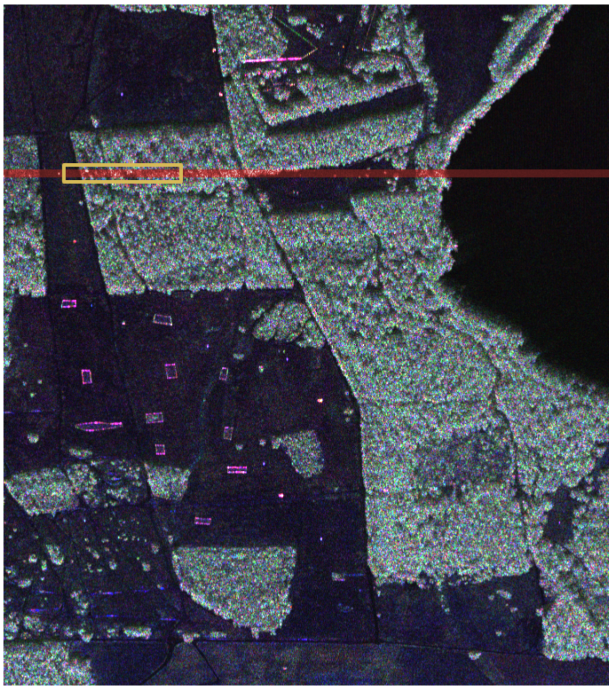

**Figure 4:** Polarimetric SAR image of the test site near Dornstetten, Germany (R: $|HH - VV|/\sqrt{2}$, G: $\sqrt{2}\,|HV|$, B: $|HH + VV|/\sqrt{2}$). The targets of interest are located within the yellow rectangle at a range distance indicated by the red line along azimuth.

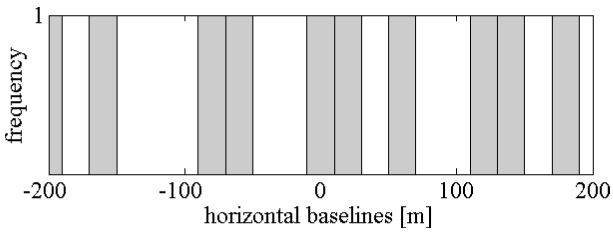


**Figure 5:** Histogram of the horizontal baseline distribution corresponding to 10 acquisitions.

# 9 Conclusions

In this paper, we have made a first attempt to recast the vertical power estimation problem in the context of covariance matching as an instance of compressed sensing. In effect, the power distribution for a specific azimuth-range position exhibits a low information rate when analyzed in the wavelet domain. Also, we have shown how we can benefit from irregular sampling by taking the redundancy of our measurements into consideration. Further, we proposed the use of low rank approximations as an efficient pre-filtering step. Future work will focus on exploiting polarimetric correlations by employing the fully polarimetric multi-baseline covariance matrix. Finally, we would like to point out that many extensions are possible, such as the combination of different bases that could be chosen to sparsify different scattering mechanisms.

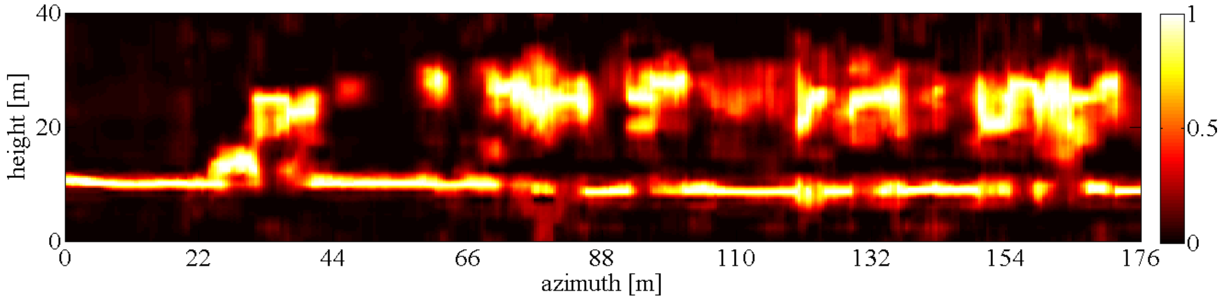


**Figure 6:** Span of the tomogram obtained by CS (176 m by 40 m) using 10 irregular passes, a 9 by 9 window, and a Daubechies Symmlet wavelet with 4 vanishing moments and 3 levels of decomposition using the full rank covariance matrix.

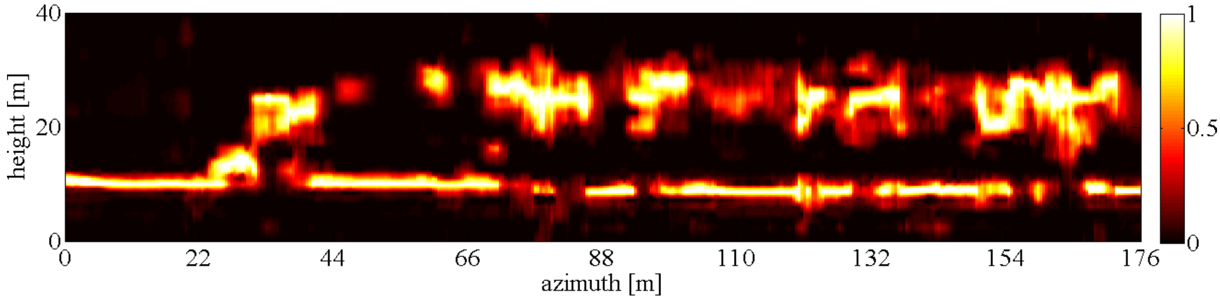


**Figure 7:** Span of the tomogram obtained by CS (176 m by 40 m) using the best rank 4 approximation of the covariance matrix.

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