Reconstruction methods of missing SAR data: analysis in the frame of TanDEM-X synchronization link

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Abstract

The reconstruction of corrupted or missing data is a conventional problem in image and audio processing, but has only raised moderate attention in the SAR community. In the frame of the TanDEM-X mission, the cooperative operation of the bistatic system requires the exchange of information between transmitter and receiver during the acquisition time in order to gather calibration and sync information. Therefore, the reception of SAR signal is periodically interrupted according to the synchronization link frequency and artifacts appear in the focused image. The data corruption is especially problematic when acquiring close to the Nyquist rate.

1 Introduction

1.1 Missing data in SAR imaging

In signal processing one is often faced with the problem of reconstructing missing samples in order to avoid or diminish the disturbing effect of the loss of information for the processed signal and its spectrum. In the case of SAR applications, the problem appears e.g. in the case of interrupted operation; new acquisitions modes with variation of the system PRF; due to calibration of monostatic spaceborne SAR, etc. [1].

The problem of missing samples is especially relevant in the bistatic SAR context. Firstly, because of uncorrelated errors on antenna pointing vectors of transmitter and receiver, gaps can be introduced on the azimuthal signal at positions corresponding to null regions in the antenna patterns. Secondly, the synchronization issues are usually in some level related with missing information in the raw data of both cooperative and non-cooperative systems.

In bistatic SAR systems, transmitter and receiver usually have different PRFs and time references. For the non-cooperative modes, when there is no exchange of information between transmitter and receiver, the lack of knowledge of the common reference can lead to bad positioning of the echo window and consequent data loss. In the case of cooperative mode, as the one existing in the TanDEM-X mission [2], there is a synchronization link between transmitter and receiver to ensure a common time reference. During the synchronization interval, usually lasting a few pulses, no SAR signal is sent or received leaving a gap in the raw data sized accordingly to the number of exchanged synchronization pulses. Furthermore, related to each sync operation, there is a second gap in the azimuthal signal positioned at the travelling distance of the transmitter sync pulses. To accurately compensate for oscillator phase errors, a few synchronization periods are required during one data acquisition. Therefore, the amount of missing data can heavily impair the data, especially when large synthetic apertures are involved or when the system operates at low oversampling rates.

This paper analyzes different approaches to reconstruct the missing samples including commonly used interpolators like the cubic interpolator, as well as a reconstruction methods based on both parametric and non-parametric spectral estimators [1,3,4,5].

2 Azimuthal gaps in SAR data

TanDEM-X [2] is a cooperative bistatic system which uses a periodic synchronization link to exchange information between transmitter and receiver in order to establish a common reference time for the system. The sync link is characterized by a certain frequency f_syn much smaller than the system PRF and a number n_syn of exchanged pulses per sync period. Each synchronization period is related with two gaps in the azimuthal signal, $s(t)$, modeled as the original signal, $s_o(t)$, modulated by a gap envelope,

$$s(t) = s_o(t) \cdot (1-g(t)) \tag{1}$$

$$g(t) = \sum_k \left(\begin{array}{c} rect\left(\frac{t-k/f\_syn}{n\_syn/PRF}\right) + \\ rect\left(\frac{t-t_{trav}-k/f\_syn}{n\_syn/PRF}\right) \end{array} \right), \tag{2}$$

where t_{trav} correspond to travelling time of the transmitter sync pulses.

The synchronization link is a duplex operation in which at least one pulse is transmitted and received by each sensor. Normally, calibration pulses are also transmitted and 3 to 4 pulses are exchange in each sync operation. After careful consideration [6], synchronization frequencies around 5Hz are chosen for TanDEM-X mission enabling the sampling of the oscillator power spectrum while compromising as little as possible the quality of the data. For the simulations carried out for this paper, frequencies between 1 and 11Hz and number of synchronization pulses between 2 and 10 were considered. Typical acquisition parameters of TanDEM-X stripmap acquisitions are considered (distance of minimum approach of 700km, sensor velocity of 7400 m/s, PRF of 3000Hz, low or no oversampling).

2.1 Effects of gap in SAR impulse response

From (1) and (2) we note that the spectrum of the gapped data is given by the convolution of the original spectrum with the spectrum of a periodic window, a combination of *sinc* signals with side lobes increasing with the number of zeros. Consequently, the resulting spectrum contains artifacts which are more or less severe according to the frequency and width of the gaps. For a point target, the corruption in the raw data spectrum leads to an increase in the side lobes energy (and therefore an increase in the Integrated Side Lobe Ratio - ISLR) and an increase in the maximum ambiguity power (maximum power of side lobes) of the normalized Impulse Response Function (IRF). No degradation is observed in the azimuthal resolution.

In order to measure the degradation in the quality parameters - Maximum Ambiguity Power and ISLR - the received SAR signal from a point target was simulated and gaps from a sync link were introduced along azimuth. The TanDEM-X parameters mentioned in section 2.2 were used and a frequency of 5Hz and 4 pulses per period were considered for the synchronization link. Figure 1 shows the original IRF (raw data without gaps) together with the one resulting from the gapped signal. For the parameters considered in the simulation, there is considerable degradation of the quality of the point target IRF there with an increase of around 7.7db in the maximum ambiguity power and of around 3.2dB in the ISLR. In Figure 2, the curves in red show the decrease in the IRF quality parameters with the variation of the synchronization frequency (n_syn = 4) and the number of pulses per period (f_syn = 5Hz).

For real SAR scenes a general decrease of the coherence is observed. This effect is more accentuated in urban areas, where the targets have a point-target like behavior and ghosts appear along azimuth, and for areas with a very poor SNR, like flooded regions. The decrease in the coherence can be relevant for interferometric applications especially when using higher synchronization frequencies or number of exchanged sync pulses. The loss in quality is accentuated for spotlight modes, given that the increase in the illumination time is associated with higher loss of information along the synthetic aperture.

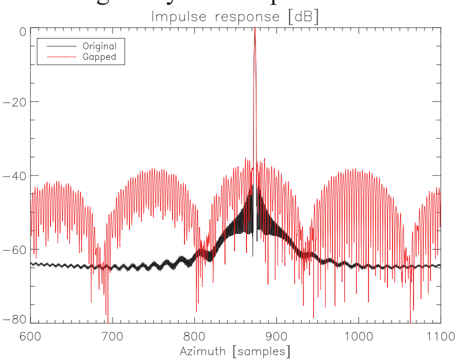

Figure 1. Impulse response of ideal point target focused with and without gaps in the SAR signal.

2.2 Reconstruction of the missing samples

The reconstruction of the missing samples is of extreme importance to ensure the quality of the focused SAR data, avoiding corruption of interferometric products, especially when dealing with point targets and urban areas. Usually TanDEM-X operates with low oversampling rates (here defined as the ration PRF/Bandwidth) and the amount of data missed per sync period is larger than the allowed by the Nyquist criterion. Therefore, low order interpolators such as nearest neighbor, linear or cubic interpolators are not able to properly reconstruct the data.

As discussed in the beginning of Section 2, the loss of information in time domain has a corruptive effect spread through all spectral components in frequency domain. Assuming that the amount of data loss is sufficiently small, which is usually the case with the considered synchronization link parameters, the spectral content of the complete data can be estimated from the spectral information of the available samples. Here, three reconstruction methods based on spectral estimation are analyzed, namely the Papoulis-Gerchberg algorithm (PG) [4], spectral estimation based on the auto-regressive model (AR-Burg) [1,3] and the amplitude and phase estimation for missing data (GAPES) [5].

Assuming that the original signal is band limited, the Papoulis-Gerchberg algorithm estimates the spectrum of the complete signal by interactively removing the high frequency components present due to the loss of information in time domain. After the low pass filter operation, only the data corresponding to the missing samples location is updated and the algorithm continues until convergence has been reached. It is proved [4] that the algorithm converges if the ratio between the number of available samples and the total number of samples is greater or equal than the ratio between null and non-null spectral components defined by the signal bandwidth. This condition limits the use of the algorithm for cases when the oversampling (ovs) is such that

$$ovs \geq \frac{PRF}{PRF - f\_syn * n\_syn} . \quad (3)$$

Out-of -band noise and the presence of more than one missing pulse per gap, as in the TanDEM-X frame, can slow considerably the convergence rates.

Alternatively, the reconstruction can be performed using a parametric model for the spectral estimation. Due to its close relation with the linear prediction scheme, the auto-regressive model (AR) is chosen [1,3]. In this case, the data is considered to be part of an auto-regressive stationary process, whose coefficients can be estimated using one of the several methods established in the literature. A viable option is the Burg algorithm, which uses the Levinson-Durbin recursion and computes the reflection coefficients based on a minimum least square criterion while ensuring the stability of the resulting model [3]. The missing data is recovered feeding the gapped signal into an IIR filter constructed with the estimated coefficients.

The reconstruction of the azimuthal signal is carried out locally in a gap to gap basis. In every step, the AR coefficients are first estimated using the anterior available segment and again using the posterior one. The two sets of reconstructed data are then combined using cross-fading windows to give the final estimation. The size of the segments used must be related with the chosen order of the model (and ultimately with the width of the gaps) and with the hypothesis of stationarity.

When oversampling is available, the reconstruction can be further improved running a few iterations of the Papoulis- Gerchberg algorithm on the resulting Doppler spectrum. This step aims the reduction of interpolation errors due to deviation of the data behavior from the assumed model and the reduction of errors associated with the window size and chosen order considered for the coefficients estimation.

The last reconstruction method based on spectral estimation considered here is the amplitude and phase estimation of gapped data (GAPES). The method was proposed in [5] for focusing SAR images using incomplete datasets and was validated with simulated data. GAPES consists in a non-parametric adaptive filter-bank approach using minimum least squares criterion to iteratively estimate the spectrum and invert the missing samples.

For each frequency of interest, the signal of length N is divided in L snapshots which are then filtered by a narrowband M-tap matched filter satisfying the LS criterion ($M = N - L + 1$). The complex amplitude of each spectral component is given by the average of the filter outputs. For the first iteration, the spectral content of the missing data is assumed to be given by the available data. According to this assumption, the missing data is reconstructed using a least square criterion ensuring that, given the estimated matched filters, the global contribution of the reconstructed samples to the complex amplitude of the complete data spectrum is minimal. The spectrum can be re-estimated using the complete dataset and the data reconstruction can be carried out iteratively. A point to be considered for implementation is that the larger the filter length the higher the spectral resolution at the cost of worse statistical stability of the estimates [5].

In analogy to the implementation of the AR based estimation discussed previously, a window is applied for the data reconstruction of each gap. Hence the estimation is performed in a forward and backward approach using only a segment from the complete azimuthal signal. The windowing process favors the assumption that the spectral content of the missing samples can be estimated by the available ones. However, it can also introduce distortions in the spectrum to be estimated. When oversampling is available, in correspondence with what was proposed for the AR approach, the complete resulting signal is feed into the Papoulis-Gerchberg estimator in an attempt to minimize those errors.

Considering a single iteration, both AR and GAPES estimators are more expensive than the Papoulis-Gerchberg in terms of number of operations. However, since the implementation in the TanDEM-X frame uses only segments around the missing samples, their time efficiency is not as influenced by the aperture size as the Papoulis-Gerchberg algorithm usually is. Furthermore, as will be shown in the next section, the number of iterations necessary for the convergence of the Papoulis method applied alone can make the algorithm unsuitable.

3 Experimental Results

At a first step, the chosen reconstruction schemes – Cubic, Papoulis-Gerchberg, AR-Burg and GAPES – were used to retrieve missing samples in the received signal of an ideal point target. For an oversampling rate of 1.1, the results presented in Figure 2 show that the reconstructions using cubic interpolation and Papoulis-Gerchberg algorithm with 200 iterations are not able to retrieve the quality parameters, even when low synchronization frequencies and number of sync pulses are considered. On the other hand, AR-Burg and GAPES methods provide very good estimations leading to an IRF with quality parameters similar to the original ones, independently of the considered synchronization parameters.

Considering the same simulation parameters mentioned earlier, the Papoulis-Gerchberg method requires more than 10^6 iterations to retrieve data from a point target with quality comparable to the original one, as shown in Figure 3. The high number of iterations necessary for convergence together with the oversampling requirements makes this reconstruction method less attractive for TanDEM-X data.

In order to analyze the reconstruction for a real SAR scenario, the synchronization link with *f_syn* = 5Hz and *n_syn* = 4 was simulated using a TerraSAR-X dataset. Figure 4 shows the coherence between the image focused without gaps in the raw data and the image focused with the reconstructed data, and its variation with the oversampling rate. As expected, better reconstructions are obtained with higher oversamplings, since more correlation between the missing samples and the samples in their surrounds is introduced. For lower or no oversampling, the reconstruction of distributed targets is poorer, negatively impacting the overall coherence. Figure 5 shows the coherence matrices for oversampling rate of 1.1.

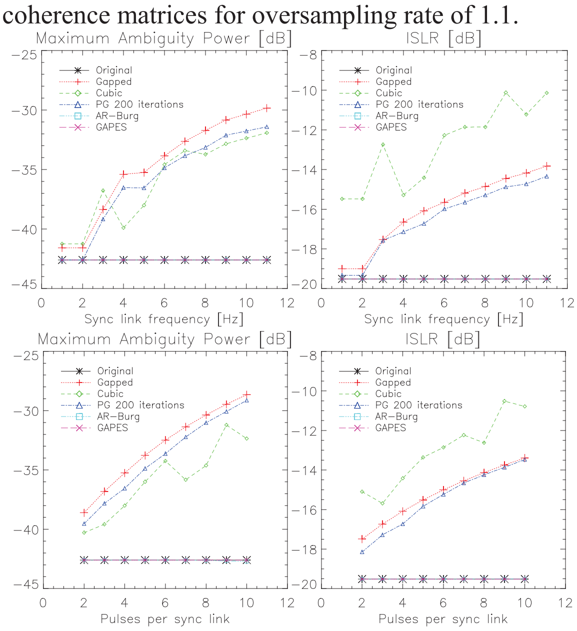

Figure 2. Variation of IRF quality parameters with increase in the synchronization frequency (*n_syn* = 4) and number of sync pulses (*f_syn* = 5Hz) using several reconstruction methods.

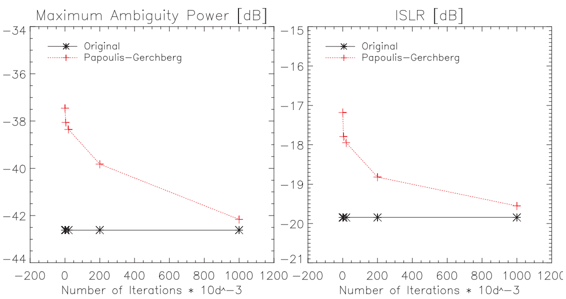

Figure 3. Papoulis-Gerchberg reconstruction variation according to the number of iterations.

Conclusions

The paper analyzed the effects of gaps introduced by the synchronization link in bistatic cooperative missions, as well as possible reconstruction methods. The methods based on AR-Burg and GAPES spectral estimators applied together with the Papoulis-Gerchberg showed the most promising results, with the latter being slightly better for distributed targets. Further investigations include sensibility analysis and the effect of the reconstruction in the interferometric results.

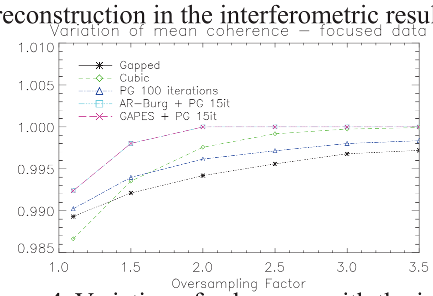

Figure 4. Variation of coherence with the increase of oversampling rate.

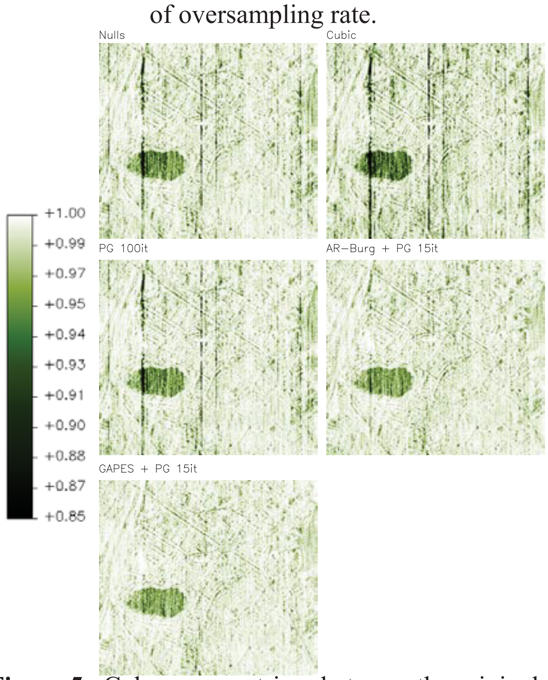

Figure 5. Coherence matrices between the original focused image and reconstructed image (oversampling rate 1.1 *f_syn* = 5Hz and *n_syn* = 4).

References

[1] Salzman, J.; Kirk, J. Interruped Synthetic Aperture Radar, in *IEEE AESS systems Magazine, 2002.*

[2] Krieger, G. et al. TanDEM-X: A Satellite Formation for High-Resolution SAR Interferometry, *in IEEE Transactions On Geoscience And Remote Sensing*, vol. 45, 2007.

[3] Marple, S.L.; *Digital Spectral Analysis with applications*, Prentice-Hall, 1987.

[4] Ferreira, P.J.S.G.; Interpolation and the discrete Papoulis-Garcheberg algortihm, *in IEEE Transactions on Signal Processing*, vol. 42, 1994.

[5] Larsson, E.G.; Stoica, P.; Jian Li; Amplitude spectrum estimation for two-dimensional gapped data, in IEEE Transactions on Signal Processing,vol. 50, 2002.

[6] Krieger, G.; Younis, M. Impact of Oscillator Noise in Bistatic and Multistatic SAR, *in IEEE Geoscience and Remote Sensing Letters*, vol. 3, no. 3, 2006.