On the Stability of Contention Resolution Diversity Slotted ALOHA (CRDSA)

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***Abstract*—The stability of Random Access protocols is of high importance to ensure an efficient usage of resources and good service perception by the users. In this paper the stability region for the recently proposed Contention Resolution Diversity Slotted ALOHA random access protocol is discussed for finite user populations. The system dynamics of a CRDSA Random Access channel are described and a mathematical model is formulated, which allows predicting the stability of the system. From the derived model quantitative approximations of the stability regions and parameter selection for stable operation are provided. Numerical results are derived by means of simulations for validating the mathematical model and the quantitative approximations of the stability region.**

I. Introduction

STARTING with the original proposal of ALOHA, one of the key problems, which need to be addressed in Random Access (RA) channels, is analyzing the protocols stability. Finding the stability region is important since the usage of resources and the service perception of the users suffers drastically in instable systems, making these systems practically unusable.

Colliding transmissions, leading to the loss of packets can naturally occur in RA channels. While lost packets could be simply discarded in principle, RA schemes usually attempt retransmission of the lost packets, either until they are successfully received or until a maximum number of retransmissions has been reached. In order to make retransmissions possible, the users need to receive feedback whether their transmission attempt was successful, e.g., by means of acknowledgements. The instantaneous throughput of the channel $S(G)$ is then dependent on the total load G, being the sum of the load due to new transmissions G_F and the load due to retransmissions G_B. In this sense the RA channel forms a feedback loop as is illustrated in Fig. 1. It is an inherent property of

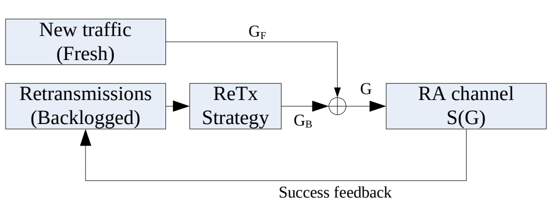

Fig. 1. Feedback loop of a RA channel

closed-loop feedback systems, that the feedback can lead to reinforcement of the signal. Here this results in an increase of the overall load due to the additional retransmissions. The well known dependency of the throughput S on the offered load G is shown in Fig. 2 for ALOHA, Slotted ALOHA (SA) [1] and Contention Resolution Diversity Slotted ALOHA (CRDSA) [2]. As can be seen, the qualitative shape of the

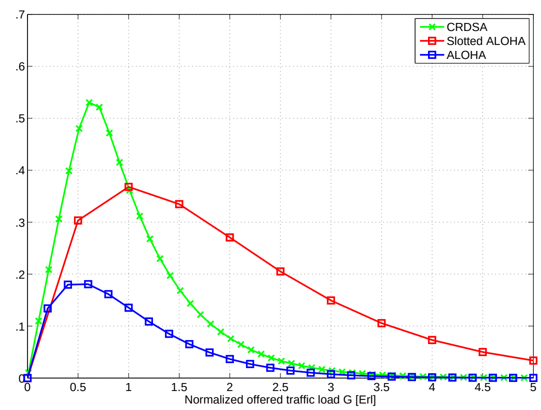

Fig. 2. Throughput in dependency of load for some ALOHA variants

throughput curves is the same for all of them. By increasing the offered load G from zero, the throughput S first increases until reaching a maximum throughput S_{max}. After this S decreases again and asymptotically approaches zero for further increasing G. If due to retransmission attempts of lost packets the total load exceeds a critical threshold, then even more packets experience a collision and get lost, resulting in an even higher retransmission load. In the end the channel is driven into total saturation in the area of having very high load and very low throughput. To avoid this amplification effect a retransmission strategy is used, which shall limit the load due to retransmissions and reduce the risk of getting more collisions (see Fig. 1). Many different retransmission strategies that try to achieve this goal are known from literature. In [3] the selection of the time of retransmission with uniform probability within a parameterizable interval $t \in [0, ..., K]$ is proposed. In [1] a strategy is described where the decision for a retransmission attempt is taken with a probability p_r in every slot (for SA) resulting in a geometric distribution. In [4] the selection of the retransmission time from an interval, which grows exponentially with every collision (Binary Exponential Backoff), is proposed. Finally so called *splitting algorithms* are known from literature (see e.g., [1], [5]), which iteratively split the set of collided users into two sets and stabilizes the system this way. Furthermore two different types of user population are distinguished, *finite* and *infinite* user populations. For a

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finite user population, every user that experienced a collision is backlogged, which means that he is not generating any new traffic until the collided packet has been successfully transmitted. The infinite user population on the other hand refers to either an infinite number of users or a finite number of users that generate new traffic independently whether another retransmission is still pending or not.[1] While some retransmission strategies assume a visibility of the channel activities by all users, here we assume that every user has no instant visibility of other users activity (as is the case in satellite system with directive links and long propagation delays) and only receives feedback about the success of his own transmission attempt from the receiving end system. The retransmission mechanisms using a uniform and geometric retransmission probability have in common that the probability of retransmission is a fixed parameter and does not change dynamically. For the binary exponential backoff and tree splitting algorithm, the actual retransmission probability may change over time dependent on the situation. In the remainder of this paper the focus is put on a geometrically distributed retransmissions mechanism, since it was shown in [6] that the channel performance of SA is mainly dependent on the average retransmission delay and largely independent of the retransmission probability distribution.

The analysis of what is the critical threshold from which on the system moves self-energizing into the low throughput saturation and for which configurations such a situation never occurs is, besides the choice of the RA scheme, naturally also dependent on the retransmission strategy. Within this paper the stability of CRDSA with a geometric distributed retransmission probability and a finite user population is addressed. For this purpose the Packet Loss Rates (PLR) properties of CRDSA are investigated and following this a mathematical model is derived, which describes the dynamics of the overall system and allows drawing conclusions about the maximum stability (section III). The results of this mathematical model are then discussed first qualitatively, followed by a quantitative analysis of the stability for representative user populations, traffic generation probabilities and retransmission probabilities. In the end the predictions of the mathematical model are validated by numerical simulations and the match of model and simulation is shown.

II. Random Access Techniques and Stability

A. Review of MF-TDMA Random Access Techniques

Over the last years the recently regained popularity of RA schemes resulted in the definition of new RA protocols. In particular a recent enhancement of the SA protocol, named CRDSA [2], [7], using Successive Interference Cancellation (SIC) techniques over a set of slots (denoted frame) to improve the throughput and PLR behaviour of SA, has been studied showing an impressive gain over SA increasing the maximum throughput from $S_{max,SA} = 0.36\frac{pkt}{slot}$ to $S_{max,CRDSA} = 0.55\frac{pkt}{slot}$. Up to now however the consequences for the system stability of this new access scheme have not been analyzed yet. The fundamental concept of CRDSA is to generate a replica burst for every transmission burst within a set of N_S slots, called frame, see also Fig. 3. While the generation of a redundant copy of a burst is similar to previous proposals such as Diversity Slotted ALOHA (DSA) [8], the fundamental difference here is that every burst contains a pointer to the location of its replica. In case a clean replica arrives, meaning that the burst could be decoded and received successfully, the channel is estimated from it and the interference that this burst introduces to other users is removed for all burst locations, including the replicas.

[1] Generating a new transmission in addition to a retransmission can be also seen as two users, one retransmitting, one transmitting new. Since the new transmissions are not bounded, this case corresponds to an infinite user population case

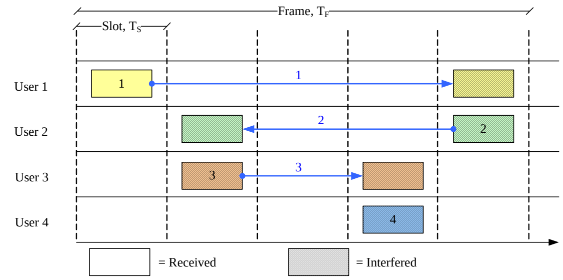

Fig. 3. SIC principle of CRDSA.

In the example in Fig. 3 the first burst of user 1 is received successfully since not interfered. As consequence of the SIC process, the interference that the replica of user 1 introduces to the second burst of user 2 is removed so that this burst of user 2 can be decoded in the next round. This process is then iteratively repeated. In the example in Fig. 3 all replicas can be recovered this way.

B. Characterization of the Packet Loss Rate in CRDSA

For SA, the necessary condition to have a successful reception is that only a single transmission must occur in a timeslot, otherwise the burst is lost. Let us denote by M the total user population of the system and p_0 the probability that a user attempts a transmission in a time slot, then the probability that a user successfully receives a packet gets $p_{Succ,SA} = p_0 \cdot (1-p_0)^{M-1}$. Increasing the overall number of users $M \rightarrow \infty$, the totally transmitted packets can be modeled as Poisson process with arrival rate λ [9]. The probability for a successful transmission then results in the well known equation $p_{Succ,SA} = e^{-\lambda \cdot T_p}$, whereas T_p denotes the slot duration. This simple closed form expression is conveniently suited to describe the throughput surfaces, which are used for the stability investigation done e.g., by Kleinrock [10]. The preconditions in CRDSA are however different due to the iterative SIC process. As was shown by Liva in [11] and [12], the SIC process can be interpreted as an erasure decoding process in a bipartite graph, such as for Low Density Parity Check Codes (LDPC) codes. For this purpose, every slot in a frame is represented as a *sum node* and every transmitted burst by a *burst node*. The edges in the graph then connect the burst nodes to the sum nodes. In [11] an expression for the average erasure probabilities for every iteration are derived for the asymptotic case of infinitely long frames, resulting in an upper bound of the achievable throughput. An expression for the exact average erasure probabilities in a non-asymptotic case with finite frame lengths however cannot be expressed accurately by these bounds. For this reason the stability analysis in this work relies on simulated CRDSA packet success probabilities and throughput (see Fig. 2) for the case of having one additional replica (degree $d = 2$), a frame consisting of $N_S = 100$ slots and a limitation of the number of SIC iterations to $I_{max} = 10$.

C. Stability Definition

The issue of stability in RA systems was already identified in the very early days of the ALOHA proposal. Abramson [13] and Roberts [14] both addressed this issue for plain ALOHA. After the evolution of ALOHA towards SA, many publications have dealt with the investigation of the stability behaviour of SA, for instance [1], [15] [9] and [10]. Stability is commonly defined as the ability of a system to maintain equilibrium or return to the initial state after experiencing a distortion. In the context of RA, the term *stability* is used in different ways in literature. In the definition given by Abramson in [13], the ALOHA channel was defined unstable if the average number of retransmissions becomes unbounded. Within [1] a channel was defined stable if the expected delay per packet is finite. Kleinrock defined in [10] a channel as stable if the SA equilibrium contour (i.e. throughput is equal to the channel input rate) is nontangentially intersected by the load line in exactly one place. In the strict mathematical definition of stability of autonomous systems, this corresponds to a sufficient condition for a global equilibrium point. In the terminology used by Kleinrock, a SA channel is instable if the load line intersects the equilibrium contour in more than one point. In the mathematical sense also then the system can have a locally stable equilibrium point, so the definition of stability by Kleinrock refers to the criterion of having a single globally stable equilibrium point. In the remainder of this work, the definitions given in [10] are followed also here, meaning that a channel is denoted as stable if it has a single globally stable equilibrium point and unstable otherwise.

III. Stability in CRDSA

Within this section, the derivation of a Markov model for a finite user population is described and the mathematical formulations for throughput and drift are derived, which form the core of the stability framework presented afterwards. This section concludes with a stability analysis for a representative CRDSA configuration.

A. Finite User Modeling Approach

Let the RA communication system under consideration be populated by a total of M users. Every user resides either in a so called *fresh (F)* state or the *backlogged (B)* state. In the beginning all M users are in state *F*. Every user in state *F* attempts a new transmission in the current frame with probability p_0. It is further assumed that all users receive feedback about the success of their transmission at the end of a frame. In case the transmission attempt was successful, the user remains in state *F*. In case a packet is lost, the user enters state *B*. A user in state *B* attempts a retransmission of the lost packet with probability p_r in the current frame. In case the retransmission is successful the user then returns to state *F*, otherwise the user remains in state *B*. Let X_α^l denote the number of users in state $\alpha \in \{F, B\}$ in frame l, then the discrete-time Markov chain can be fully described by either X_B^l or X_F^l, since both are connected by $X_F^l = M - X_B^l$. In the following X_B^l is chosen as the Markov state variable. Given the initial state $X_B^0 = 0$ and the state transition probability $P(x'|x)$, which is the probability to move within one frame from backlog state x to state x', the Markov chain is then fully described. One major difference to the SA analysis done by Kleinrock is that the backlog state X_B can decrease by more than 1 for CRDSA. While in SA at maximum a single user can get un-backlogged per slot (otherwise there would be a collision), in a CRDSA frame also more than one backlogged user can get unbacklogged. Since no closed form expression for the success probability of a user in CRDSA is known in literature, the probability $q_{d,N_s,I_{max}}(t,u)$ is introduced, which is the probability that out of t users who attempt a transmission in the frame exactly u users are successful. The success probability q is dependent on the CRDSA configuration, consisting of the repetition degree d, the number of slots in the frame N_s and the maximum number of iterations I_{max}. Here, this probability was derived numerically by simulations and averaging over the results for every offered load G. When changing state, let U be a random variable denoting the number of successful transmissions in frame l, S a random variable denoting the number of fresh transmission attempts in the frame and T the number of retransmission attempts in a frame. The change in number of users in X_F and X_B for every state transition can then be put into the following relation, which is convenient for the further description of the state transition probabilities. Let FS denote the number of fresh users, which transmit successfully in frame l. Let FU be the number of fresh users who attempted a transmission but were unsuccessful. In the same way, BS denominates all backlogged users who attempt a retransmission and were successful and BU those backlogged users, whose retransmission attempt was unsuccessful. With this the following equations (1)-(3) can be derived.

$$r_l = FS_l + FU_l \tag{1}$$

$$t_l = BS_l + BU_l \tag{2}$$

$$u_l = FS_l + BS_l \tag{3}$$

whereas (w.r.t. frame l) r equals the total number of fresh users attempting transmission, t equals the total number of back-logged users attempting transmission and u the total number of users, which transmit successfully[2]. The joint probability mass function, conditioned on state $X^l = x_B$ is then given by

[2] It should be noted that idle users have no relevance here since they neither change the size of the sets X_F and X_B nor do they generate load which impacts the transmission performance.

equation (4).

$$
\begin{aligned}
&P(r,t,u|x_B) = \\
&= \binom{x_F}{r} p_0^r (1-p_0)^{x_F-r} \cdot \binom{x_B}{t} p_r^t (1-p_r)^{x_B-t} \\
&\quad \times q_{d,N_s,I_{max}}(r+t,u) = \\
&= \binom{M-x_B}{r} p_0^r (1-p_0)^{M-x_B-r} \\
&\quad \times \binom{x_B}{t} p_r^t (1-p_r)^{x_B-t} \\
&\quad \times q_{d,N_s,I_{max}}(r+t,u).
\end{aligned} \tag{4}
$$

With equations (1)-(3) the change in number of backlogged users $\Delta x_B = x'_B - x_B = FU - BS$ can be easily reformulated into

$$
u = r + x_B - x'_B \tag{5}
$$

The state transition probability $P(x'_B|x_B)$ can then be formulated by combining (4) and (5) into (6),

$$
\begin{aligned}
&P(x'_B|x_B) = \\
&= \sum_{r,t} P(r,t,r+x_B-x'_B|x_B) = \\
&= \sum_{r,t} \binom{M-x_B}{r} p_0^r (1-p_0)^{M-x_B-r} \\
&\quad \times \binom{x_B}{t} p_r^t (1-p_r)^{x_B-t} \\
&\quad \times q_{d,N_s,I_{max}}(r+t, r+x_B-x'_B).
\end{aligned} \tag{6}
$$

With (6) in principle the entire Markov chain can be described with all its transition probabilities. In practice the computational cost of computing all transition probabilities is however enormous, mainly due to the nested summations over a large range of possible values for r and t. To avoid this computational complexity, the stability analysis in the following makes use of a drift analysis, in reminiscence of [10] and [16]. The change behaviour of the backlog state forms a differential equation, whereas the drift corresponds to the change of the state variable $d_B = \frac{dx_B}{dt}$. For the drift analysis the change in backlog x_B over time is analyzed in the following and the stability of the equilibrium points is computed by using the tools known from differential calculus. In the style of [17] and [16], the drift is here defined as the expectation value of the change of the backlog state X_B^l frame by frame as given by

$$
\begin{aligned}
d(x_B) &= E\left\{X_B^{l+1} - X_B^l \,|\, X_B^l\right\} = \\
&= \sum_{x'_B} (x'_B - x_B) \cdot P(x'_B \,|\, x_B).
\end{aligned} \tag{7}
$$

With (5) and following this with (6) this can be reformulated into

$$
\begin{aligned}
d(x_B) &= \sum_{r,t,u} (r-u) \cdot P(x'_B \,|\, x_B) = \\
&= \sum_{r,t,u} (r-u) \cdot \sum_{r,t} P(r,t,u \,|\, x_B) = \\
&= \sum_{r,t,u} (r-u) \cdot P(r,t,u \,|\, x_B) = \\
&= E\{R\} - E\{U\}.
\end{aligned} \tag{8}
$$

From (4) it is clear that R is binomial distributed so

$$
E\{R\} = (M - x_B) \cdot p_0. \tag{9}
$$

The second expectation value $E\{U\}$ is related to the throughput $S(x_B)$ of the system, i.e., the expected number of successful packets per slot in frame l as is (10)

$$
\begin{aligned}
S(x_B) &= \frac{1}{N_S} E\{U\} = \\
&= \frac{1}{N_S} \cdot \sum_{r,t,u} u \cdot P(r,t,u \,|\, x_B) = \\
&= \frac{1}{N_S} \cdot \sum_{r,t,u} \binom{M-x_B}{r} p_0^r (1-p_0)^{M-x_B-r} \\
&\times \binom{x_B}{t} p_r^t (1-p_r)^{x_B-t} \cdot q_{d,N_S,I_{max}}(r+t,u).
\end{aligned} \tag{10}
$$

With (9) and (10) the drift $d_B = d(x_B)$ gets

$$
d_B = (M - x_B) \cdot p_0 - N_S \cdot S(x_B) \tag{11}
$$

Since the exact computation of the throughput $S(x_B)$ is computationally very expensive, the notion of drift is simplified into the expected drift by averaging over S;

$$
\bar{d_B} = (M - x_B) \cdot p_0 - N_S \cdot \bar{S}(x_B), \tag{12}
$$

with

$$
\bar{S}(x_B) = [(M-x_B)p_0 + x_B p_r] \cdot \bar{P}_s((M-x_B)p_0 + x_B p_r) \tag{13}
$$

and $\bar{P}_s(x)$ the average number of successful transmissions when attempting x transmissions, as explained above. With (12) it is now possible to fully describe the stability of the CRDSA system for the case of having a user population M, a probability p_0 of fresh users generating new packets and a retransmission trial probability of p_r. Intuitively, the drift represents the tendency of the system to change over time and gives the direction of change of the backlog size. This means that for positive drifts the size of the backlog tends to increase by $\bar{d}_B$ (i.e. more users experience lost packets and get backlogged). For negative drifts, the length of backlog decreases, which means that backlogged users successfully retransmit and get fresh again. A drift of 0 corresponds to a local equilibrium point, which may be locally stable or instable. Figure 4 shows the drift-backlog surface for the scenario $M = 500$, $p_r = 0.78$ and for varying p_0. This figure nicely shows the dynamic of the system. It can be seen that for low traffic generation probabilities p_0 the drift d_B remains negative for all backlog states x_B. In other words, the system

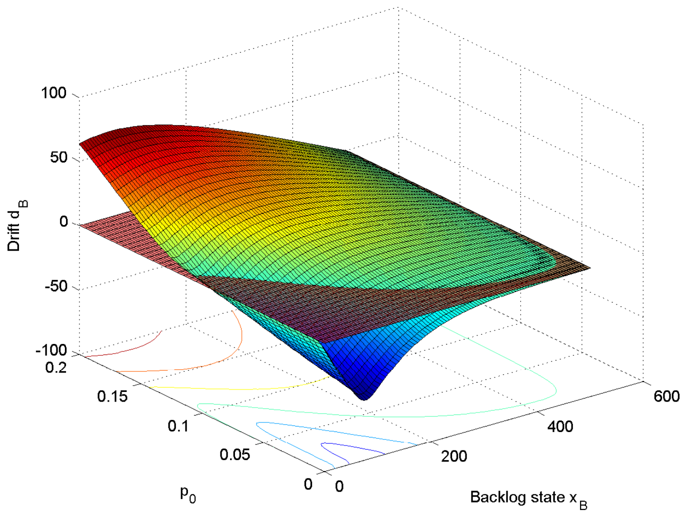

Fig. 4. Drift backlog surface for $M = 500$, $p_r = 0.78$

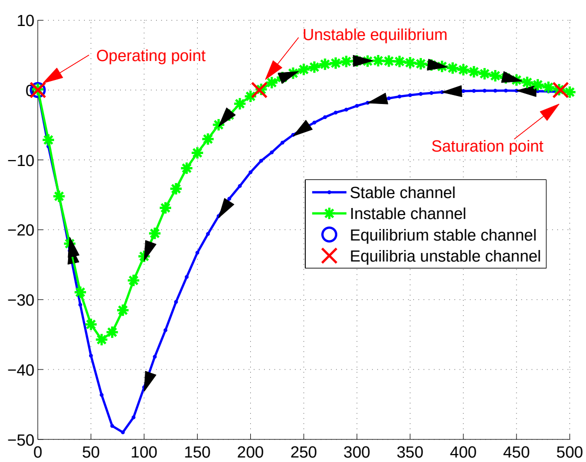

Fig. 5. Backlog drift for a stable channel with $M = 500$, $p_r = 0.78$ and $p_0 = 0.01$ and an unstable channel with $M = 500$, $p_r = 0.78$ and $p_0 = 0.04$

here shows always the tendency to return to the initial state $X_B^0 = 0$. For increasing values of p_0 (corresponding to an increase of the offered load) the equilibrium is the intersection of the drift-backlog surface with the zero plane, as shown. The meaning of this equilibrium locus is that with the given system parameters (i.e., p_r, d, N_S and I_{max}) the system is always stable up to the load p_0 where the straight line parallel to the x_B axis is for the first time tangent to the equilibrium locus. Figure 5 shows the backlog drift for two particular values of p_0, i.e. the intersection of the drift-backlog surface from Fig. 4 with the planes $p_0 = \{0.01, 0.04\}$. As can be seen here, the drift for the stable configuration is always negative independent of the backlog state x_B, which means that the system always shows the tendency to lower the current backlog state until reaching the initial state. There is thus only one equilibrium point (globally stable) close to the initial state. For the unstable configuration ($p_0 = 0.04$) it can be seen that after the initial equilibrium point (locally stable) and the following area of negative drift (up to $x_B \approx 209$) a second, locally instable equilibrium point is reached at $x_B = 209$. When reaching this point the system can either fall back into the negative drift region for $x_B < 209$ or enter the region of positive drift $x_B > 209$. In the latter case the positive drift means that any movement to a higher backlog state (which is a consequence of the positive drift) results in an accelerated increase in number of backlogged users. This behavior then persists until reaching the third and final equilibrium point (locally stable) at $x_B \approx 500 = M$. In this third equilibrium point now all M users are backlogged and the system has reached the point of maximum load and minimum throughput. From this observation the conclusion can be drawn that for a given system configuration $\Omega = \{d, N_s, I_{max}\}$ the maximum traffic generation probability p_0 for which the system is still always stable is the one resulting in a drift contour which intersects the straight line $d_B = 0$ at most once (Fig. 5), resulting in a single equilibrium point, which is locally and globally stable. For all other cases the channel is instable (e.g., unstable configuration with $p_0 = 0.04$ in Fig. 5), meaning that earlier or later the backlog will increase into the total saturation point.

IV. Numerical Results

For validation of the previously defined algebraic model of stability, simulations have been performed, which are summarized in the following section. To observe the behavior of the CRDSA system, a simulator was implemented, which implements the full CRDSA SIC mechanism. The implementation of the CRDSA-SIC assumes perfect interference cancellation and channel estimation, i.e., any clean burst allows the perfect suppression of its interference towards other bursts. The simulator allows setting up a total population of M users, whereas the system can be put into an arbitrary initial state $x_B^0 \in \{0, \ldots, M\}$. Since the defined system is a Markov chain, the simulations can be started from different initial states $x_B^0 \neq 0$, which enables a fast observation of the system drift in this state. The alternative approach of starting a simulation with $X_B^0 = 0$ can require very long simulations to reach all possible backlog states and (depending on the actual configuration) especially long to reach high backlog states x_B. In addition every state should be reached several times in order to get statistically significant results by averaging the drift for every backlog state. In the simulations shown hereafter, the system was set to every possible initial backlog state $X_B^0 = [0, \ldots, M]$ by putting x_B users in backlog state. The fresh users then attempt a transmission with probability p_0, whereas every backlogged user attempts a transmission with probability p_r. The transmission attempts and all packet replicas are then assigned random locations (uniformly) distributed within the N_s slots of the frame. Afterwards the SIC mechanism is applied, whereas only bursts with a single transmission are considered clean. Since perfect Interference Cancellation (IC) is assumed, every clean burst allows removal of the packet replica. The IC is iterated I_{max} times for every frame. The difference between the number of backlogged users before and after the frame transmission is then used to compute

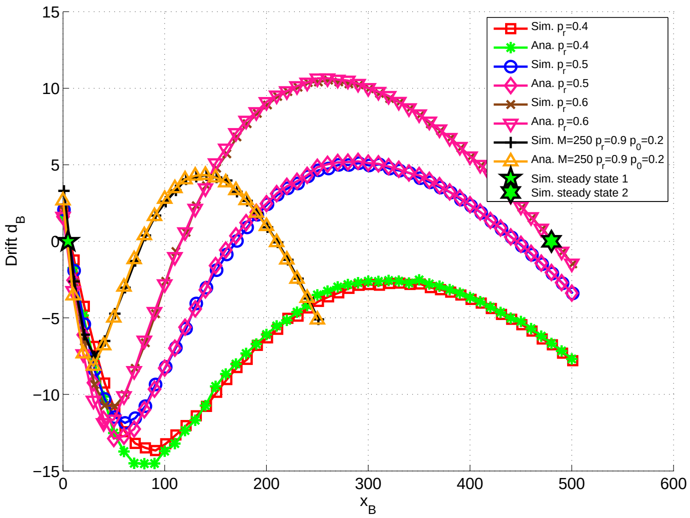

Fig. 6. Comparison of simulated (*sim*) versus analytically (*ana*) derived system drift for $\Omega = \{d = 2, N_s = 100, I_{max} = 10\}$ and $M = 500, p_0 = 0.09$

the drift. For every backlog state x_B the occurring drift was averaged over 10000 simulation runs resulting in the expected drift d_B per backlog state. Figure 6 shows the results of the expected drift as computed by the mathematical model derived in section III and the collected results of the simulations for the same scenarios $\Omega = \{d = 2,\ N_s = 100,\ I_{max} = 10\}$ and $M = 500,\ p_0 = 0.09,\ p_r \in [0.4,\ 0.5,\ 0.6]$. The retransmission probabilities p_r were chosen for getting one stable ($p_r = 0.4$, only one equilibrium point) and two instable configurations ($p_r = 0.5$ and $p_r = 0.6$). As can be seen the analytical prediction matches very well with the analytical results. To investigate the impact of the user population on the accuracy of the prediction, another simulation was performed with a reduced user population $M = 250$ and choosing $p_0 = 0.9$ and $p_r = 0.2$ to result in an instable channel. Also in this case the comparison between the analytical prediction and the simulation results show a very good match. Finally Figure 6 shows two points for the steady state that the system falls when started at zero backlog $x_B^0 = 0$ (simulated steady state 1) and when started in an arbitrary point in the unstable region, here $x_B^0 = 300$ (simulated steady state 2) for $M = 500, p_0 = 0.09, p_r = 0.5$. The two simulated steady states show the average steady state computed over 20 simulation runs each. As can be seen the reached steady state matches again very well with the theoretical predictions. Finally the initial state was set close to the instable equilibrium point at $x_B^0 = 117$. In this case the simulations have shown that the system falls in roughly 50% of the cases into steady state 1 and in 50% of the cases into steady state 2. This is also inline with the expectations and shows that the computation of the instable equilibrium matches well with the simulations.

V. Summary and Conclusions

In this paper a model for the description of stability in a CRDSA based SIC scheme was formulated. This model allows not only drawing conclusions about the stability of a system configuration but is also able to predict up to which point the system will remain stable. The derived formulation can serve as a valuable tool when designing and dimensioning a CRDSA system since it allows deriving the maximum allowable offered traffic load for a guaranteed stable system with one global equilibrium point with given retransmission probability. Alternatively the model allows for a given static offered traffic load the derivation of the maximum required retransmission probability, which ensures that the system remains stable. Finally the analytically derived drift curves have been validated by numerical simulations. The simulations, which implement the detailed CRDSA SIC mechanism and retransmission strategy have shown that the analytically predicted behavior matches very well with the observations from the simulations, validating the found model.[3]

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[3]Currently an extended journal version of this work is under preparation for submission to IEEE transactions.