Antenna Pattern Compensation in Multi-Channel Azimuth Reconstruction Algorithm

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Piotr Laskowski, Federica Bordoni, Marwan Younis

German Aerospace Center
Muenchner Str. 20, 82234 Wessling, Germany
Email: piotr.laskowski@dlr.de

ABSTRACT

In this paper the influence on azimuth performance of the antenna pattern in multi-channel SAR reconstruction algorithm is analysed and different approaches of its compensation described. Both transmit and receive antenna patterns affect the signals and hence have influence on performance of the whole system. Consideration of antenna pattern in the processing should help to achieve better performance, since more information about the signal is provided. Different approaches for antenna pattern compensation are analysed and compared with focus on azimuth performance (Azimuth-Ambiguity-to-Signal Ratio – AASR and azimuth resolution). Additionally, the influence of antenna pattern compensation on the Signal-to-Noise Ratio (SNR) scaling is examined. A novel extension of the algorithm in [1], which takes consideration in the antenna pattern directly in the multichannel reconstruction, is proposed and compared to other approaches. This new approach considers antenna pattern for each channel separately. This should help in the compensation of errors and misalignment between channels if they are known. Equations and analytical expressions for two channel case are derived and a difference between transmit and receive antenna compensation is analysed. Miscellaneous tests show the performance of new extension to the multichannel reconstruction algorithm both for extremely different antenna patterns and with more realistic patterns, where only a small error between channels is simulated. The disadvantage of the new approach is decreased SNR. To minimize it three solutions are proposed. The whole algorithm is optimised for azimuth ambiguities suppression.

INTRODUCTION

Standard one channel SAR systems cannot provide high geometric resolution and wide swath coverage at the same time. Using only one receive antenna limits the swath width by the Pulse Repetition Frequency (PRF). A lower PRF increases the imaged swath, but degrades azimuth resolution. This problem can be avoided by using multiple channels in azimuth (like by High-Resolution-Wide-Swath technique – HRWS, as presented in [1]). The basic idea is to use multiple receivers aligned in along track and combine their signals in a single channel with higher equivalent PRF. The processing is called reconstruction algorithm and is based on the theory of combining \( N \) independent representations of a signal, each sub-sampled with \( 1/N \) the Nyquist frequency. This allows for the unambiguous reconstruction of the original signal from the aliased spectra of the \( N \) representations (see [2,3,4]). The idea of the reconstruction algorithm and multi-channel modelling is shown in Fig. 1. Function \( U(f) \) characterises the scene and can be seen as a ‘mix’ of the scene reflectivity, 2-way antenna pattern and mono-aperture azimuth response function for a point target. Every signal from each channel is then filtered through system filter \( H_i(f) \), which represents the channel between the transmitter and receiver \( i \) and can be computed from the geometry of the system. All signals are sampled at PRF and filtered with reconstruction filters \( P_i(f) \). After that the signals from different channels are interleaved to one channel data with higher data rate and azimuth compressed. The reconstruction filters \( P_i(f) \) are calculated as

\[
P_i(f) = H_i^{-1}(f),
\]

(1)
i.e. they are computed by the inversion of the system matrix \( H_i \) [1].
There are several reasons for compensating both transmit and receive antenna pattern in this approach. For most multichannel systems only one transmit antenna and several antennas on receive are used. According to the transmit antenna the phase spoiling technique can be used to provide a wide azimuth pattern despite the long antenna, see [5]. However, this technique requires variation of the phase in the main lobe of the antenna pattern. This means that SAR signal suffers from an additional phase component, which is introduced by the transmit antenna pattern. If this phase shift is not corrected, a distortion of the azimuth response function occurs and the azimuth resolution degrades. Compensation of the phase spoiling would be the main aim of transmit antenna compensation.

The phase in the main pattern of the receive antennas is mostly constant, so that no pattern correction is needed at the first sight. However, since more than one channel is used, receive antennas can have misalignment between them. Since the reconstruction algorithm provides interleaving of samples from different channels into one channel data with higher sampling rate, Rx pattern correction should be performed, if the channels are not the same. This should help to achieve better performance in comparison to no pattern correction case.

In this paper, two methods for antenna pattern compensation are described and compared. The first one is a compensation of antenna pattern, which takes place after the reconstruction algorithm. The reconstructed data is multiplied by the conjugate phase of the antenna (phase compensation), conjugate complex function of the antenna (matched compensation) or is divided through the antenna function (inverse compensation). This kind of compensation will be denoted as detached approach, because it can be seen as a separate operation in the processing chain. The second possibility is to include the antenna pattern inside the reconstruction algorithm. If in each channel function \( H_i(f) \) not only the geometry, but also antenna pattern (transmit, receive or both) is additionally taken into consideration. The system function for channel \( i \) can be defined as

\[
H_i(f) = \exp\left(-j\pi \Delta x_i^2 \frac{f}{2\lambda R_0}\right) \exp\left(-j2\pi f \frac{\Delta x_i}{2v_f}\right) A_i(f),
\]

where \( \Delta x_i \) denotes the separation between phase centres of receiver \( i \) and transmitter, \( \lambda \) the wavelength, \( R_0 \) minimum slant range distance, \( f \) the azimuth (Doppler) frequency and \( v_f \) the velocity of the platform. \( A_i(f) \) is the complex antenna pattern. This last term is present only in the embedded approach. For the 2-channel case, the reconstruction filters \( P \) are given by (3).
Fig. 2. Comparison between detached and embedded approach of antenna pattern compensation.

\[ P_a(f) = \frac{\exp\left(j \frac{\pi A x_i^2}{2 \lambda R_v}\right) \exp\left(j 2 \pi f \frac{\Delta x}{2 v_j}\right)}{A\left(f + (k-1) PRF\right) \exp\left(1 - k\right) \frac{j 2 \pi PRF \Delta x}{2 v_j}} \cdot \frac{1}{1 - \exp\left(2k - 3\right) \frac{PRF}{2 v_j} \left(\Delta x_i - \Delta x_{k-1}\right) A\left(f + (2 - k) PRF\right) A_{k-1,1}\left[f + (k - 1) PRF\right] A\left(f + (k - 1) PRF\right) A_{k-1,1}\left[f + (2 - k) PRF\right]}_{i, k = 1, 2}. \]  

All antenna terms \( A() \) occur in the embedded approach only. In the P-Matrix, rows correspond to the channels and columns to the frequency bands. The second ‘antenna term’ in the denominator is a combination of different antenna patterns in different frequency bands and can be seen as a kind of energy balancing from different antennas. Without this term one could combine a strong signal from one antenna with a weak signal from the other one. With this term, both channels have similar power after the reconstruction and can be better combined/interleaved.

**REFERENCE SYSTEM FOR NUMERICAL ANALYSIS AND BASIC DEFINITIONS**

The HRWS SAR system is considered for performance analysis. Its main parameters are recalled in Table 1. All computations were performed in Stripmap single polarimetric mode.

The azimuth performance will be described here with three parameters: Azimuth Ambiguity-to-Signal Ratio (AASR), azimuth resolution and SNR scaling. Here only briefly definitions are provided, for more details see [5].

**AASR**

It is defined as the ratio between the azimuth ambiguous power \( e_x(f) \) related to the signal power \( p_s \):

\[ \text{AASR} = \frac{E\left|e_x(f)\right|^2}{p_s}, \]  

where \( E[.] \) denotes the mean value operator.

**Azimuth Resolution**

It measures the minimum distance two objects can be situated in the same range to be distinguished by the radar. It is computed as the 3dB-bandwidth of the Impulse Response Function (IRF) in azimuth.
Signal-to-Noise Scaling (SNR Scaling)

A parameter characterizing the impact of the digital beamforming network on signal and noise power. The higher it is the worse performance the system has. It can be computed as:

$$\phi_{j}(PRF) = N \sum_{j=1}^{N} E\left[P_j(f)\right],$$

(5)

where $N$ is the number of channels, $P_j(f)$ the reconstruction filter of channel $j$ by the frequency $f$ – see (1), $E[.]$ denotes the mean value operator. SNR scaling can be seen as the ratio between SNR in one channel data before the reconstruction to SNR of the interleaved data after the reconstruction. It can be computed in the whole bandwidth of the system or only in the processed bandwidth. In the first case it would give the impact of the beamforming network on the SNR due to the processing, in the second case it would be a possible measure for the image quality.

Table 1 Main parameters of a reference system.

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>UNIT</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Orbit Height</td>
<td>[km]</td>
<td>520</td>
</tr>
<tr>
<td>Platform Velocity</td>
<td>[m/s]</td>
<td>7600</td>
</tr>
<tr>
<td>Antenna Tilt Angle</td>
<td>[deg]</td>
<td>34.3</td>
</tr>
<tr>
<td>Access Area (inc. ang)</td>
<td>[deg]</td>
<td>20-50</td>
</tr>
<tr>
<td>Radar Parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RF Center Frequency</td>
<td>[GHz]</td>
<td>9.600</td>
</tr>
<tr>
<td>Duty Cycle</td>
<td>%</td>
<td>17 %</td>
</tr>
<tr>
<td>Guard Time</td>
<td>[µs]</td>
<td>4</td>
</tr>
<tr>
<td>Instrument &amp; Hardware Configuration</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tx/Rx Antenna</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Height</td>
<td>[m]</td>
<td>1.06</td>
</tr>
<tr>
<td>Length</td>
<td>[m]</td>
<td>8.750</td>
</tr>
<tr>
<td>Nr. of Tiles in elevation</td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>Nr. of Panels in azimuth</td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>Nr. of subarrays in each Tile (elev. x az.)</td>
<td>4 x 5</td>
<td></td>
</tr>
<tr>
<td>Nr. of radiators in elevation</td>
<td></td>
<td>48</td>
</tr>
<tr>
<td>Nr. of radiators in azimuth</td>
<td></td>
<td>35</td>
</tr>
<tr>
<td>Nr. of Channels in elevation (TX x RX)</td>
<td>1 x 1</td>
<td></td>
</tr>
<tr>
<td>Nr. of Channels in azimuth (TX x RX)</td>
<td>1 x 6</td>
<td></td>
</tr>
</tbody>
</table>
COMPENSATION OF TRANSMIT ANTENNA PATTERN

The magnitude and phase of the phase spoiled transmit antenna pattern used in the investigations are shown in Fig. 3.

Fig. 3. Magnitude and phase of the phase spoiled transmit antenna pattern.

Fig. 4. to Fig. 6. show the azimuth performance for the case: (a) without compensation; (b) detached; (c) embedded compensation of the transmit antenna pattern, both for the amplitude and phase compensation. Since for the compensation of the phase only of the antenna pattern both approaches are the same, these results won’t be reported here.

As regards the AASR, the best performance is achieved by the detached approach. It is up to 2 dB better than the embedded approach and about 0.5 dB better than no compensation. This fails, due to the clearly worse azimuth resolution, which is caused by the phase spoiled pattern. In case of SNR scaling, a very strong increase by the embedded approach can be observed. This is due to very strong impact of the inverse antenna ripples in (3). The magnitude of P-reconstruction filters versus the Doppler frequency, for the embedded and detached case, are shown in Fig. 7. Since the SNR Scaling is the sum of them, see (5), this strong increase is unavoidable without some extra processing.
In conclusion, if the transmit pattern is phase spoiled, it should be compensated. The critical parameter here is the azimuth resolution. If only the phase of transmit antenna pattern is corrected, there is no difference in the azimuth performance between detached and embedded approach. By the compensation of both amplitude and phase, the embedded approach fails due to increased SNR scaling. Consequently the detached approach is the most interesting approach. It has in all cases the same or better performance compared to the embedded one and doesn’t damage the SNR. It should be noted that for both approaches the pattern of the transmit antenna has to be accurately known.

COMPENSATION OF RECEIVE ANTENNA PATTERN

The magnitude and phase of receive antenna pattern used in investigations are shown in Fig. 8. The pattern is based on the uniform current distribution of all elements.
Ideal Case – Identical Rx-Antenna Patterns

Since the phase in the main lobe of the receive pattern doesn’t change, there is no need for its compensation as long patterns of all receive channels are identical. There is no significant difference between detached and embedded approach and even no compensation. For this reason no results will be reported for this case.

Different Rx-Antenna Patterns Case

The compensation of the receive pattern becomes interesting as soon as the receive channels are not identical. Since in the reconstruction algorithm data from multiple channels are interleaved to one channel, the detached approach cannot be used here. The only possibility is then to compensate the differences with the embedded approach, before the channels are combined. Differences between antennas of different channels can occur in the amplitude pattern, phase pattern, or both of them. They were simulated by applying different amplitude weightings to reference magnitude pattern or different phase shifts to reference phase pattern between channels. The reference pattern is shown in Fig. 8. In following, two results of numerous tests will be shown: “strong variation of the amplitude” and “small variation of the phase”.

Strong Variation of the Amplitude

Table 2 shows the set of weighting coefficients used for simulating strong variation of the amplitude pattern between receive antennas in the system. The weightings between antennas, which have been chosen here, are extremely different in order to check the performance of embedded approach in an extreme case.

<table>
<thead>
<tr>
<th>Channel</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coeff.</td>
<td>1</td>
<td>0.3</td>
<td>0.15</td>
<td>0.45</td>
<td>0.4</td>
<td>0.25</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Fig. 9 to Fig. 11 show azimuth performance for three cases: (a) reference; (b) no compensation; (c) embedded compensation of receive antenna pattern. Reference denotes here a case with identical receive patterns.

![Fig. 9. AASR for strong variation of Rx amplitude pattern.](image)

![Fig. 10. Azimuth resolution for strong variation of Rx amplitude pattern.](image)
Fig. 11. SNR scaling for strong variation of Rx amplitude pattern. Continuous line – computation with whole bandwidth (N×PRF), stroke line – processed bandwidth of 6.2 kHz.

From the AASR plot it becomes clear, that the compensation is in that case necessary. Combining such different channels together degrades the data. With the embedded approach, AASR is the same as by the reference case. This means that strong antenna differences can be compensated, if they are known. Azimuth resolution is for all cases the same. The reason for that is that only the levels of amplitude patterns have been changed – the shapes of the main beams of all receive antennas are identical. The only difference is a small variation of azimuth resolution as a function of the ground range for the case of no compensation. The disadvantage of embedded compensation approach is in that case strong SNR scaling. Due to multiplication of $P$ correction filters with inverse antenna function (see (3)), each filter is multiplied with the inverse coefficient from Table 2, which increases it.

The results for the strong variation of the phase pattern of Rx-antennas are very similar to the variation of the amplitude. It just has to be summarised, that also in that case the AASR for the embedded approach has the level of reference, but SNR Scaling becomes worse.

Small Variation of the Phase
The previous results with strong variation of amplitude are useful to verify the embedded approach, but are not very realistic. To simulate more realistic scenarios, small variation of phase and amplitude is deployed. Maximum amplitude derivation between channels is set to 10% and max. of 6° in the phase is allowed. Since the results of azimuth performance for magnitude and phase variation are very similar, as it is in the strong variation case, only phase variation results will be shown here. Table 3 shows the phase offsets for each channel in the simulated scenario.

Table 3 Coefficients set for simulating small variation of Rx-phase pattern.

<table>
<thead>
<tr>
<th>Channel</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coeff.</td>
<td>-2.86°</td>
<td>0.57°</td>
<td>1.72°</td>
<td>-2.29°</td>
<td>0°</td>
<td>-2.86°</td>
<td>1.15°</td>
</tr>
</tbody>
</table>

Fig. 12 to Fig. 14 show the azimuth performance for the simulated case. The differences in AASR between embedded approach and no compensation are now smaller than in previous case, but they are still up to 3 dB. However, the AASR value for the simulated system for these small phase differences between channels is already with no compensation small: between -25 and -33 dB. That would mean, small differences in the phase and amplitude are not very critical for azimuth performance and the usage of embedded approach should be traded off and will be depend on the application.

Fig. 12. AASR for small variation of Rx phase pattern.
Fig. 13. Azimuth resolution for small variation of Rx phase pattern.

Fig. 14. SNR scaling for small variation of Rx phase pattern. Continuous line – computation with whole bandwidth (N×PRF), stroke line – processed bandwidth of 6.2 kHz.

As in previous cases, the SNR scaling in the embedded approach is getting worse. It increases in the investigated scenario of about 2 dB for the whole bandwidth of the system. However, in the processed bandwidth of about 6.2 kHz the increase is minimal and achieves max. 0.5 dB which is acceptable.

POSSIBLE SOLUTIONS FOR INCREASED SNR SCALING

In all the analysed cases, when the embedded approach is used, the SNR scaling increases. This is due to multiplication of the reconstruction filters with the inverse of antenna pattern, as in (3). Since the antenna pattern is normalized to one at its maximum value, this inverse operation increases the P-filters and therefore the SNR scaling, see (5). The level of degradation depends on the processed bandwidth and is greater as the bandwidth increases, i.e. the resolution is getting better. There are some possibilities to reduce this problem. Three of them are illustrated in Fig. 15.

Fig. 15. Three possibilities of avoiding of strong SNR scaling.
The first possibility, Fig. 15 left, would be to apply a threshold for the antenna function that would reduce the ripples of the pattern leaving the main beam unchanged. However, this approach doesn’t make much sense in compensating receive pattern, since the threshold value could impact the resolution. It could be interesting for the transmit pattern, but the detached approach is better here anyhow. Another possibility would be to include in the embedded approach only the phases of all receive antennas, but for the amplitude just to take one value. This value could be for instance the mean value of the amplitude pattern (Fig. 15 middle) or the difference between patterns (Fig. 15 right). With only one value for the amplitude compensation, one could multiply the signal and the noise with the same value after or in the reconstruction and consequently the SNR wouldn’t change.

CONCLUSION

This paper deals with the antenna pattern compensation in multi-channel azimuth reconstruction algorithm. Specifically, the HRWS system is considered. Two approaches are presented and compared. First one – detached – compensates the antenna influence after the reconstruction on one channel data, whereas the second one – embedded – directly in the reconstruction algorithm. For the transmit antenna, the detached compensation is the one if the phase spoiling of the pattern is used. The embedded approach offers in that case similar performance in azimuth, but degrades the SNR. On receive, the embedded approach becomes more interesting. It can compensate the differences between Rx-channels, so that the azimuth performance becomes the same as if antennas would be identical. The disadvantage is the increase of the SNR, which depends very strongly on the processed Doppler bandwidth. This bad influence could be, however, decreased by using the compensation with only one value in the amplitude compensation.

REFERENCES