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RECENT DEVELOPMENTS AT THE NUMERICAL SIMULATION OF LANDING GEAR DYNAMICS

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Abstract

Aircraft landing gears support the aircraft during ground operations, including take-off, landing impact, taxiing, gate handling and maintenance. Mostly for reasons of minimum mass and ground clearance, landing gears are slender structures which exhibit a considerable dynamic response to ground load excitations. As the landing gear is one of the few systems on the aircraft without redundancies, the knowledge of landing gear dynamics is crucial for aircraft design and aircraft safety.

Simulation of landing gear dynamics is a cornerstone of aircraft loads analysis, as well for vertical loads resulting from touch-down as for longitudinal and lateral loads resulting from braking, steering and towing. Another important field of interest are landing gear vibrations like gear walk and shimmy. Those phenomena can be brake induced or result from tire spin up at touch-down or simply from a coupling of dynamics of the running tire and structural mechanics of the landing gear leg. All those effects strongly depend on a number of parameters such as aircraft speed, landing gear vertical deflection, tire pressure and wear of the parts. Many of those parameters can only be estimated and might change during the operation of the aircraft.

Numerical investigation is thus a challenging task. Analysis methods exist both in the frequency domain and in the time domain. As stability analysis is straight forward in frequency domain methods, this approach is still often used. However, in many cases nonlinearities are dominant which lead to limit-cycle characteristics of the vibrations. Here, multibody modelling or a mixture of multibody and finite element modelling including time domain simulation is used.

In the article, a general outline is given of how vibration problems in landing gears can be treated by numerical analysis methods. The article will start with a classification of typical problems, give a short overview of classical papers, and explain typical approaches. In

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addition, alternative approaches for stability analysis and for the detection of limit-cycle oscillations as well as state-of-the-art modelling approaches will be presented.

1 INTRODUCTION

1.1 Motivation and Background

The aircraft landing gear is one of the basic systems that have significant effects on aircraft performance. The tasks of the landing gear are complex and lead to a number of - sometimes contradictory - requirements. At landing, the landing gear absorbs the vertical energy of the aircraft via the shock absorber and the horizontal kinetic energy by means of brakes. At taxiing, the landing gear has to carry the aircraft over taxiways and runways of varying quality, a requirement that is mirrored in its British name, "undercarriage".

The dynamics of the landing gear depend on the design of the gear structure and the attachment to the aircraft (e.g. strut design, attachment stiffness) as well as on the dynamics of the components which form a part of the system, i.e. the shock absorber, shimmy damper and, of course, the tire. Furthermore, most main landing gears on conventional aircraft are equipped with a brake (nose landing gears usually are not), and anti-skid systems are state-of-the-art since the 1950ies. Two important phenomena of landing gear oscillations can be seen in Figure 1. One important phenomenon is the so-called “Shimmy”, a summarising term for self-induced landing gear oscillations where lateral bending and torsion around the vertical axis of the landing gear leg couple. Shimmy can occur under all taxiing conditions, as well as at take-off and landing. The reasons for such unstable oscillations are found in the elasticity of the frame and tires (in combination with nonlinear effects such as friction and free-play in the bearings of the king pin) which lead to limit cycle oscillations and uncomfortable vibrations causing mechanical wear and the danger of landing gear failures. Another phenomenon is the so-called “Gear Walk”, a fore/aft motion of the elastic gear structure induced either by the landing impact or from an application of the brake. In many cases, shimmy and gear walk can couple, i.e. a shimmy-type vibration can be induced by applying the brake. Thus, the dynamic properties of the gear structure and the brake have to be seen as a coupled, feed-back system.

1.2 Available Literature

There are a number of books and articles of landing gear design which must be mentioned here. The books of Conway [1], Currey [2], Pazmany [3], and Roskam [4] are standard textbooks which cover the whole conventional landing gear design process from questions of landing gear location, suspension layout and the selection of tires. Books on global aircraft design like the standard by Raymer [5] usually dedicate a few pages on global landing gear parameters like placement and overall mass, without going into detail on landing gear design and dynamics. A number of publications concerning landing gear by members of the "SAE Committee A-5 for Aerospace Landing Gear Systems" have been selected by Tanner and published in [6] and [7]. Further collections of articles have been published by the AGARD

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(Advisory Group for Aerospace Research and Development) in their conference proceedings CP-484 [8], "Landing Gear Design Loads". No more recent overviews on landing gear design are known to the authors.

Only a few specific publications exist with respect to the simulation of aircraft ground dynamics. An overview of computer simulation of aircraft and landing gear is published in another AGARD volume [9], which has its main emphasis on the simulation of shimmy. Two publications of the IAVSD (International Association for Vehicle System Dynamics), Hitch in 1981 [10] and Krüger et al. in 1997 [11] are state-of-the-art overviews of aircraft ground simulation, the latter article also discussing different modelling approaches and tools. Pritchard [12], 1999, is another - and, to the authors' knowledge the most recent - overview of landing gear dynamics.

A number of recent publications concern selected aspects of landing gear dynamics. Works on shimmy prediction and brake modelling include the publication of Denti and Fanteria [13] who discuss the effect of different tire models and brake on the longitudinal dynamics of aircraft landing gear. Khapane examines landing gear - brake interaction in [14]. Besselink thoroughly investigates the influence of various parameters on shimmy prediction. His thesis also includes a comprehensive list of references concerning the topic [15]. Vibration reduction of landing gear by active shock absorbers, including dynamic modelling of the landing gear, has been investigated by several authors, among others, Krüger [16] and Sateesh [17]. Recent publications on nonlinear dynamics of landing gear, including stability and bifurcation analysis, and related to the approach presented in Section 2.4 below, come from the group of Krauskopf, see e.g. [18].

2 NUMERICAL ANALYSIS OF LANDING GEAR VIBRATIONS

2.1 Analysis Approaches

Classical Methods for shimmy analysis are linearization and linear system analysis and time simulation. In addition, amplitude-dependent linearization can be used to include the influence of dominant nonlinearities while maintaining efficiency of linear analysis. The three approaches will be shown using a landing gear model reduced to the basic relations important in basic shimmy analysis, Figure 2, taken from [19]. The nonlinear mathematical shimmy model used has been derived from a similar model in [20] and consists of the torsional dynamics of the landing gear, the forces and moments, and of approximations to describe elastic lateral qualities of the tire.

The two degrees of freedom describing the coupled motion of shimmy are the swivel of the wheel around the landing gear leg (yaw angle Ψ), and the slip angle of the tire α. Relevant forces include spring and damping moments M_1 and M_2 acting on the yaw, the tire side force F_y, as well as the tire, gyroscopic and tread width moments M_z, M_4 and M_5. Parameters for the analysis are the vertical force F_z, the taxiing velocity V, the castor length e, and the half

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contact length of the tire a. The complete set of parameters for the analyses shown in this section is given in [21].

2.2 Analysis of the Linearized System

For small amplitudes, the nonlinear dynamic system is Taylor linearized numerically. In parameter variation loops, one or two model parameters are changed systematically, and eigenvalues can be calculated. Figure 3, left, depicts the real part of the eigenvalues over speed for a variation of the castor length e. For the given configuration, an increase of the castor length increases the stability region, i.e. the region with a negative real part of the eigenvalues. By checking the eigenvalues for critical stability, a linear stability chart can be drawn, Figure 3, right. In this analysis, the torsional damping of the swivel motion is shown over the speed, again the castor length being varied. In the figure, the stability boundaries giving the minimum amount of damping to obtain a stable behaviour for a given castor length are plotted. Linear approaches are usually fast, making a large number of calculations in a given time possible. Parameter fields can be covered quickly. The drawback is that nonlinearities cannot be taken into account directly. Furthermore, oscillations limited in amplitude, the so-called limit cycle oscillations, cannot be predicted. In the analysis of landing gear stability, linear approaches are widely used. However, nonlinearities like free-play or friction play a crucial role, so additional methods have to be applied at least for a reduced number of analyses for selected cases.

2.3 Analysis of the System by Time Simulation

Another common method for analysis of landing gears is time simulation. This approach is able to capture arbitrary nonlinearities, provided a suitable model is available. All solutions, stable, unstable and limit cycles can be obtained, depending on the conditions. However, only single points in the parameter space can be examined. The example at Figure 4 shows two similar systems, differing in rotational damping only. For a given speed, in the system with greater damping (left), the lateral deflection of the wheel is quickly reduced. For less damping (right), the lateral motion of the wheel increases until it reaches a limit amplitude. For the given case, the limit is determined by nonlinear tire characteristics.

2.4 Quasi-Linear Analysis

The goal of this approach is to establish a numerical procedure for finding limit cycles, which is generally applicable to all dynamic systems (of ordinary, homogeneous differential equations) having several distinct nonlinearities of one variable, still using frequency domain methods. In landing gears, the predominant factors for limit cycle motions are nonlinear tire behaviour, free-play and large angles of yaw rotation.

For large amplitudes a quasi-linear system is generated by determining the unknown amplitudes via eigenvectors, then eigenvalues are calculated and a stability checking is performed. All these items are to be handled in an iterative loop because they are coupled, due

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to the amplitude dependency of the describing functions. To linearize the system, weak nonlinearities are linearized according to Taylor series, and for all other discrete nonlinearities a quasi-linear approximation using describing functions, e.g. employing harmonic balancing approaches, is applied.

The approach is complicated by the fact that in the case of several nonlinearities with different input signals, the describing functions of these nonlinearities depend on different, unknown amplitudes of the input oscillations. The amplitudes have to be matched (the so-called "amplitude synchronization"); thus a nonlinear system of equations for the unknown amplitudes results. Using the fact that in a linear system the ratios of all amplitudes A and eigenvectors EV are constant for each eigenvalue, these equations can be set up. Provided that proper initial estimates and step sizes for the unknown amplitudes A are selected, the system of nonlinear EV/A-equations can be solved iteratively with a nonlinear solver software, see Figure 5, left.

By parameter variation of the selected basic amplitude (e.g. of the slip angle oscillation) and an interesting model parameter (e.g. velocity V), eigenvalues can be calculated and displayed in a 3-d graph, Figure 5, right. By checking the eigenvalues for critical stability and by parameter variation with respect to a model parameter, conditions for amplitudes (of slip angle oscillation) are found, where limit cycles can occur. The results are best shown in a bifurcation diagram. It displays the regions in amplitude versus a model parameter, where stable and unstable behaviour occurs, separated by stable or unstable limit cycles, Figure 6, left. In addition, the frequencies of the limit cycle eigenvalue are recorded, Figure 6, right. A more detailed description of the method can be found in [21].

3 TOOLS AND MODELLING

3.1 Multibody Simulation

Multibody simulation (MBS) has shown to be a valuable software tool for virtual system design. In aeronautics, it is the state-of-the-art approach especially in the area of landing gear design, ground manoeuvres (take-off, landing, taxiing, ground handling) and the layout of high-lift systems [22] as well as in helicopter and tilt rotor analysis [23]. Comprehensive simulation allows analysis and evaluation of performance, structural loading and dynamic behaviour of the system, as well as optimization of the design. It is becoming more and more important to perform these computations in complex, realistic scenarios. For that reason, modern multibody codes include a variety of interfaces to other tools from other engineering disciplines, for example to CAD, structural dynamics, control design tools and aerodynamic tools. Using these interfaces, the simulation environment can be included in a larger design loop [24].

Usually, forces between bodies can be represented by library elements or user-defined routines. Of prime interest for landing gear simulation is the integration of structural elasticity and of aerodynamics and flight mechanics. For the simulation of flexible bodies, the

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representation in modal form is state-of-the-art, leading to a combination of large rigid body motion and linear elastic deformations [25]. The development of reliable aerodynamic models ranges from strip-theory and lifting-line-type models to interfaces with high end CFD tools, with applications for civil and military aircraft [26], [27]. A major advantage of using multibody dynamics for loads calculations is the straightforward introduction of flight mechanics into the aeroelastic simulation. The full advantage of using a complex multibody tool for that purpose becomes most evident for systems with large rotations like combined aircraft/landing gear analysis, including optimization, helicopters or tilt rotors, and for aircraft with large elastic deflections [28].

The multibody codes used in the application examples given below are SIMPACK and MBdyn. SIMPACK is a former DLR development now developed and distributed by SIMPACK AG [29]. SIMPACK is a commercial product which is used in the development of cars, trucks and railways, and is the standard multibody tool for aeroelastic applications at the DLR Institute of Aeroelasticity [26], [30]. MBDyn is a general-purpose multibody dynamics analysis software, freely available as it is released under GNU license. The software has been developed at the Dipartimento di Ingegneria Aerospaziale of the University "Politecnico di Milano", Italy [31]. Other MBS tools frequently used in landing gear design and analysis are MSC.ADAMS [32] and LMS Virtual.Lab Motion [33].

3.2 Representation of Landing Gear Elasticity

The first step in building a multibody model is to account for the exact kinematics of the landing gear. This is easily accomplished using a rigid body, with the right mass properties, for each of the landing gear structural elements, and connecting them with ideal joints. A shock absorber model is then added to the system. A simple model like this is already able to correctly reproduce the vertical response of the landing gear. As soon as the interest shifts from vertical reaction forces to longitudinal and lateral forces, the model has to account for landing gear flexibility. As an example, a model without landing gear flexibility would not be able to predict any kind of spring back load.

The flexibility of the landing gear can be reproduced by replacing the rigid bodies with beams or with modal elements, and introducing deformable joints in place of the ideal ones. Please note that, as the landing gear compresses, the position of the contact points between main fitting and shock strut changes, thus continuously changing the resulting stiffness and, consequently, the natural frequency and mode shapes of the landing gear during compression. An example for the effect is given in Figure 7, left, [34]. Here, a set-up of two elastic beams, representative for a landing gear leg, has been analysed. Figure 7, right, shows the stiffness at the tip of the model being loaded with a constant force F. L is the length of the sliding member, x is the stroke, EJ the beam bending stiffness. The resulting stiffness is a function of model length x+L and is calculated as the ratio of tip force F to tip displacement ν, normalized by EJ/L^3. The result is given for an analytical solution and for 3 approximations discussed in [34]. This effect might be very difficult to be correctly reproduced in MBS using

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a modal approach for the elastic landing gear components. The references [34], [35] and [36] show alternative modelling approaches to deal with this question. Fortunately, many cases of interest for horizontal and lateral landing gear dynamics can be analyzed assuming constant deflection and thus a constant, representative landing gear stiffness.

Fuselage dynamics can be accounted for using a modal approach as well, where the fuselage modes of vibration, extracted from a FEM model, are inserted in a floating frame of reference which accounts for average finite rotations of the body. This kind of model can be augmented with so called "static modes", allowing the recovery of relative displacements occurring due to local stationary elasticity, for example at the gear/fuselage attachment points. Of course, this last approach is feasible only if a sufficiently detailed FEM model of the fuselage part where significant deformations are expected is available. An alternative approach is to add local attachment stiffness to the joints connecting the fuselage to the landing gear. Accounting for local elasticity can be necessary in order to reliably predict the onset of instabilities. For example, Figure 8 shows the simulation results of two models during a braking manoeuvre with the intervention of an anti-skid system. The dotted line is the braking torque time history predicted for a landing gear deformable model that does not account for the elasticity of the landing gear/fuselage attachment; the continuous line, showing the occurrence of a so-called "gear-walk" instability, is the result obtained with the same model, but including landing gear/fuselage attachments stiffness.

3.3 Shock absorber

One of the central elements of landing gear design is the shock absorber. For the simulation of the longitudinal lateral dynamics, this element is often neglected, assuming a fixed landing gear stroke. For simulations of the gear dynamics during landing, however, the introduction of the shock absorber is crucial. Furthermore, a number of landing gears are equipped with so-called shimmy dampers, which often use the same damping principle as the one described below.

For transport aircraft, the main task of vertical energy dissipation is almost exclusively taken over by an oleo-pneumatic shock absorber, often just called the "oleo". This device combines gas spring with oil damping. Damping force is provided by oil flow forced through an orifice by vertical strut motion. Often the oil flow is controlled by means of a metering pin. The gas spring is represented by a law of polytropic expansion,

$$F_f = F_0 \left(1 - \left(\frac{s}{s_m}\right)\right)^{-n \cdot c_k}$$

with spring force F_f, pre-stress force F_0, oleo stroke s, oleo gas length s_m, polytropic coefficient n ($1<n<\kappa$), and a correction factor c_k, typically between 0.9 and 1.1. The properties of the damper are determined by the laws describing the laws of viscous fluid, e.g. oil, through an orifice,

$$F_d = \pm|\dot{s}| \cdot d \cdot \dot{s}^2$$

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Typical force curves are given in Figure 9; the parameters used for the figure are those suggested for the nose landing gear of the PHOENIX lander presented in Section 4.1. Furthermore, friction in the oleo seals can play a significant part, but exact modelling is difficult and often done on the basis of experience and proprietary approximation formulae.

3.4 Free Play

Free-play is typical inside joints connecting moving mechanical parts, e.g. the members of the landing gear legs. The presence of free-play might considerably change the stability margins and be the responsible effect for limit cycle motion. Free-play is modelled as nonlinear springs, see Figure 10, left. Some deflection is possible before a force develops, and if the amplitude remains inside the free-play band, the force will remain zero. For linear approximations, free-play might be treated as a spring with equivalent stiffness, the values can be taken from harmonic balance. Grossmann [37] suggests two equations to determine an equivalent linear stiffness c_{eq} for motion outside the free-play band ($a_m > a_{fp}$):

$$c_{eq} = c \cdot \left(1 - \frac{a_{fp}}{a_m}\right)$$

$$c_{eq} = c \cdot \left(1 - \left(\frac{a_{fp}}{a_m}\right)^2\right)$$

with c the linear stiffness outside free-play band, a_m the amplitude of the motion and a_{fp} half of the free-play. Obviously the stiffness has become a function of the amplitude of the motion and will increase with this amplitude. Besselink [15] suggests that the first equation gives better correspondence with nonlinear simulations than the second, and that the equivalent stiffness might be even lower. Figure 10, right, shows the effect of free-play on lateral, damped landing gear oscillations, obtained from nonlinear time simulation.

3.5 Tires

In the field of aircraft landing performance evaluation, the effects at the tire-ground interface play a very important role. Correct representation of longitudinal tire dynamics are essential for the modelling of wheel spin-up and gear walk, whereas lateral tire dynamics play a dominant role for shimmy analysis. As both phenomena are often coupled, a comprehensive tire model is crucial for the simulation of landing gear dynamics.

Several complex tire models have become standards in automotive applications with interfaces to state-of-the-art multibody codes. Among the most widely used for dynamic simulation are the so-called Pacejka “Magic Formula” (MF-Tyre) [38], TNO-SWIFT [39], and FTire [40]. Most models are based on curve fittings and require a large number of parameters, often obtained experimentally, and are mainly aimed at the range of normal loads usual for cars or trucks. The range of aircraft tire normal loads, however, is up to fifteen times wider and starts from zero. It is obvious that the car tire curve fittings are invalid in most of the user range of the aircraft tire [41]. For that reason, other dedicated tire models, or

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sometimes reduced versions of complex tire models, are usually used for simulation of aircraft and landing gear dynamics. Those models concentrate on the most important physical effects, and parameters like stiffness, shape, peak and curvature factors are often kept constant. Examples will be given in this section.

For lateral tire dynamics used in shimmy analysis, the correct representation of phase lag for lateral motion is a crucial point. The models developed first for this purpose were based on a single contact point approach. One still popular model is the Moreland tire model, first published in 1954 [42]. It includes phase lag for lateral motion using a single additional state; for a comparison of tire time constants see Besselink [15]. In 1941, von Schlippe introduced the concept of a stretched string with a finite contact length to describe the mechanics of the rolling tire. For a detailed discussion of the stretched string models see [15] and the work of Pacejka [38]. In 1960, Smiley and Horne published data collected from numerous experiments on aircraft tires, as well as empirical formulae developed to describe aircraft tire behaviour [43].

The shimmy analyses described in Section 4.1 have been performed using a re-implementation of the standard Fiala tyre model [44], augmented with a differential equation to account for a time constant in lateral motion.

A good compromise between the model complexity and its ability to reproduce the actual tire behaviours is given be the combination of so-called rigid ring models, such as the one used by Zegelaar [45], [46]with dynamical models of the frictional interaction between tire and runaway. A rigid ring model, Figure 11, is built connecting two masses with an elastic component, so that the average deformation dynamics between the rim and the belt can be accounted for. The forces exchanged between the tire and the runaway are computed by an element that represents the tire contact patch. At least two different approaches are available for the longitudinal force component; the first one, built using a simple bush-like model, is widely used in multibody codes [38]; the second one is built averaging over the contact patch the friction coefficient, computed using a dynamical model such as the LuGre's one, cf. Section 3.6, and is more used by control system analysts [47]. Both models are available in MBDyn, and, if their parameters are tuned appropriately, both can reproduce experimental results with a good precision.

All tire models discussed above assume that the wave length of runway roughness is large with respect to the tire contact patch. Tire models exist which take the direct interaction between tire and terrain into account, e.g. running over step-shaped obstacles. These models make use of a finer local discretization of the tire [44], or work with finite elements directly [48]. However, due to their numerical complexity, these models are rarely used for aircraft dynamics analysis. Yet a different modelling approach has been used in [49] for aircraft manoeuvring on soft soil.

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3.6 Friction

Different friction models can be used in order to introduce friction in joints, to model the longitudinal forces exchanged between the tire and the runaway, and to predict the behaviour of brakes. A detailed review of friction models can be found in [50]. Among all the friction models the most widely used is the Coulomb friction model. Unfortunately, this friction model is not only very difficult to implement in a dynamic multibody code, but can also lead to ill-posed problems [51]. A wide range of techniques were adopted in the past in order to regularize the Coulomb friction model, but none proves to be completely satisfactory. A good alternative to the Coulomb friction model is given by dynamic friction models, and, one of the more successful among them is the well-known LuGre friction law. This friction formulation [52], [53] considers a single state model which decomposes the rigid body displacement x at the contact point into its elastic (reversible) and plastic (irreversible) components, ξ and x- ξ, respectively, and reads:

$$\begin{aligned} &f = \sigma_0\xi + \sigma_1\dot{\xi} + \sigma_2\dot{x} \qquad\qquad \sigma_0, \sigma_1, \sigma_2 > 0 \\ &\dot{\xi} = \dot{x}\left(1 - \frac{\sigma_0}{|f_{ss}(\dot{x})|)}\operatorname{sign}(\dot{x})\xi\right), \end{aligned}$$

where f is the friction coefficient. This approach accounts for the elastic pre-sliding relative displacement (with σ_0), for viscous friction (with σ_1), for rising static friction and for frictional memory during slip (with σ_2). The Stribeck effect can also be accounted for using the steady-state friction curve $f_{ss}(\dot{x})$ (also known as the Stribeck curve).

Friction models deal with so-called "conform" contacts, where the vertical reaction and the horizontal frictional force are uniform. When dealing with friction in joints one has to consider the actual distribution of contact forces in the joint. For example, the actual distribution of normal forces in a cylindrical joint can lead to an additional part of the resulting frictional moment that can be as high as 30 % [54].

3.7 Braking

When dealing with brake performance and braking stability the models have to be enhanced with a brake model. The simplest one is a linear relation between an applied braking force F_b and the braking moment M. This simple model can be enhanced, adding dynamic friction effects, so that the applied braking force is no more in phase with the braking torque. This can be accomplished, for example, considering an average brake disk radius R_d, and computing the average friction coefficient f as a function of the average relative velocity between the disk and the brake pads. The braking moment is then

$$M = F_b\, f\, R_d$$

Thermal effects can be significant for carbon-carbon disks, for which the static friction coefficient is a known function of temperature. For this kind of brakes the model should be enhanced not only with a dynamic friction law, but with thermal conduction equations as well, in order to predict the disks and pads temperature. The simplest mode, taking in account only the conduction trough disks and pads thickness leads to the results like that of Figure 12,

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where the temperature of a small size business aircraft carbon-carbon disks is shown as a function of time and position thought the thickness.

3.8 Control Systems

Both SIMPACK and MBDyn can simulate the dynamic of control systems using state-space realizations of their transfer functions. Moreover, both codes can interact, during the simulation, with an external SIMULINK model, which can be used in order to build controllers of arbitrary complexity. The anti-skid system shown in Figure 13 was used for the example presented in Section 4.2 and implemented in MBDyn. A similar layout has been used for reference analyses and implemented using the integrated control loop functionality of SIMPACK by Khapane [14] for his analysis of brake-gear interaction.

4 APPLICATION EXAMPLES

4.1 Shimmy Analysis of a Scaled, Unmanned Re-entry Vehicle

The PHOENIX vehicle was a one-seventh scale model of the future space transport vehicle HOPPER, developed by ASTRIUM [55]. The vehicle particularly served for acquiring real flight and landing attitude data that cannot be simulated. The flight demonstrator had a wing span of 3.90 metres and an aluminium structure with a weight of about 1,000 kg. The vehicle was successfully flight tested in an autonomous flight after being dropped from a helicopter from an altitude of 2400 m. The test took place at the test airport of Vidsel in northern Sweden in 2004. PHOENIX was equipped with one nose landing gear (NLG) and two main landing gears (MLG), see Figure 14. Due to the high speeds at landing, shimmy was a concern. DLR performed a preliminary stability analysis based on data from the landing gear design.

The analysis followed the established approaches of a frequency domain analysis for all gears at pure rolling condition for fixed strokes, and a study of the transition from fully extended gears to static closure position of the gears, performed for three weight configurations. The analysis has been performed using the MBS code SIMPACK.

For normal rolling conditions at static load, no critical points were found, neither for the nose nor for the main landing gear. However, when investigating the landing, the main landing gear was analysed for several aircraft attitude angles and landing gear strokes. One configuration was found which displayed a potential instability, for an attitude angle which the aircraft would transition through at derotation.

Figure 15 shows the example of a stability analysis in the frequency and the time domain for this configuration. For the frequency analysis, the model was linearized for various forward speeds. Free-play in the joints had no influence for this model, as the main landing gear wheel, due to its installation at an angle to the fuselage, is subject to a constant force in y-direction, putting a pre-stress in the relevant bearings. The values for natural damping vs. speed are plotted in Figure 15, left. The damping decreases for increasing speed, and crosses

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the zero-boundary approximately at 60 m/s. In Figure 15, right, a nonlinear time integration of the model is shown, indicating a return to the equilibrium position after a disturbance for a speed of 50 m/s, while for 70 m/s the system is unstable. Such time analyses have been performed for several speeds and configurations to support the results of the linear analysis. It was understood that the found instability was only valid for a point which the aircraft would transition through very quickly. A set of nonlinear time simulation has thus been performed to evaluate the system behaviour for the complete landing phase. Figure 16 shows results for three different weight configurations. While the landing impact and wheel spin-up is clearly visible, no indication for an instable behaviour is seen. It is clearly a help that the unmanned aircraft settles very quickly onto static position.

4.2 Investigation of Anti-skid Induced Landing Gear Instability

In [56], the phenomenon known as gear-walk is investigated as an example of multidisciplinary modelling and simulation. The focus is on the fore-and-aft oscillation of the main landing gear due to the coupling of the landing gear deflection with the brake anti-skid control systems characteristics. The objective of the work is the development of a modelling approach that can be used as a design tool for the anti-skid controller in order to avoid malfunctioning during the braking manoeuvre. A comprehensive multibody model of an aircraft with a tripod main landing gear is developed and used, together with a simple anti-skid model, to predict the onset of the instability.

The multibody model used in this work is implemented in the MBDyn code. Particular attention has been dedicated to the development of nonlinear models: tires, shock absorbers, brakes and the anti-skid control system. In the frame of virtual testing, special elements simulating translational accelerometers have been introduced to monitor the accelerations without having to resort to a posteriori derivations. The case study presented regards an aircraft with a tripod-type main landing gear (MLG) which is known to suffer from gear walk in normal braking conditions. A tripod landing gear is peculiar from a kinematic and dynamic standpoint, as it increases the gear track during compression, see Figure 17. A symmetrical approach is adopted under the assumption that the time scale of the aircraft yaw dynamics radically differs from that of the deformable landing gear longitudinal dynamics. A multibody semi-model of the aircraft is fitted with a single tripod MLG and a telescopic nose landing gear (NLG). Although the gear walk phenomenon actually involves only the MLG, the NLG is needed to capture the pitch oscillations that arise during braking due to the longitudinal aircraft dynamics and landing gear deflection. The deceleration applied, in fact, induces a pitch in the aircraft attitude, causing a vertical load transfer between the MLG and the NLG. Although occurring at relatively low frequencies, the vertical load variation can influence the behaviour of the anti-skid control system.

Preliminary studies lead to the conclusion that the MLG model and its fuselage attachment need a certain degree of detail to fulfil gear walk instability simulation requirements. The NLG, on the other hand, is of interest only to guarantee the correct dynamic and static

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behaviour of the aircraft semi-model; it has thus been modelled without introducing structural flexibility. The MLG multibody model includes leg deformability and fuselage attachment flexibility: the main strut, the drag brace and the retraction actuator are modelled using flexible beam and rod elements, reproducing the web-like structures, Figure 17, whilst the connecting elements, the wheel axle and the wheel are rigid. The local MLG-fuselage attachment deformability, computed using an available FE model, has been introduced in the model using flexible joint elements. The mass and inertial characteristics, including those of the brakes mounted on the MLG, have been lumped at the structural element nodes using the available manufacturer mass breakdown data sheets and assembly drawings. Internal friction has been added to all the relevant joint elements, using a realistic friction model combined with a Herzian contact force distribution model in order to estimate joint friction. The metal-on-metal friction coefficient has been chosen referring to the literature, as no experimental data was available. Free-play has not been taken into account at this stage. The multibody semi-model of the aircraft comprises 429 degrees of freedom. During the simulation, the model is run through a complete landing manoeuvre with brake application after a brief ground roll.

Examining the available manufacturer documentation, it is possible to hypothesize that the anti-skid control gains were tuned taking in account at most the landing gear structural flexibility, completely disregarding the gear-fuselage attachments. For this reason, the authors have tuned (using the Ziegler-Nichols method) a set of control parameters using the multibody model without the flexible gear-fuselage attachment. This parameter set, referred to in the following as "NoFlex", leads to an unstable system when applied to the complete multibody model, which includes the gear-fuselage flexibility. This last model was also used in order to tune a second set of control parameters, indicated as "Flex" in the following, that leads to a stable system. Figure 18 shows the effects of the two different parameter sets on the behaviour of the complete simulation model, which includes the gear-fuselage flexibility.

The work showed that difficulties are encountered in the definition of an adequate dynamic model for the simulation of landing and braking manoeuvres. The approach adopted is initially time-consuming, for the fact that the single elements composing the landing gears need to be tuned referring to the available experimental data. Once the model has been assembled, however, its versatility is undoubtedly an asset in the anti-skid controller design phase. It in fact allows to explore the system behaviour in a wide range of operational conditions, also in terms of aircraft payload distribution (an aspect for which results are not presented here) and in terms of runway surface characteristics. In its present form, the effects of brake heating, tire inflation pressure and wear have not been taken into account: this will be the object of future research.

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