# Enceladus: Evidence for librations forced by Dione

**B. Giese** (1), H. Hussmann (1), T. Roatsch (1), P. Helfenstein (2), P. C. Thomas (2), and G. Neukum (3)
(1) DLR-Institute of Planetary Research, Berlin, Germany (Bernd.Giese@dlr.de), (2) Cornell University, Ithaca, NY, USA, (3) Freie Universität Berlin, Germany

## Abstract

Based on control point calculations we confirm that Enceladus experiences physical librations as predicted in [1]. The moment of inertia ratio $(B-A)/C$ is constrained to the range 0.027-0.035 which is significantly higher than expected for a body in hydrostatic equilibrium.

## 1. Introduction

Enceladus is locked in a 2:1 mean motion resonance with Dione. Within this resonance, both exchange orbital angular momentum that raises the orbital eccentricity of Enceladus and ultimately increases its internal heating [2]. Another predicted consequence of the resonance is that substantial physical librations in Enceladus' longitude can be induced [1]. In this paper we show evidence from control point calculations that the predicted librations exist. In connection, we estimate limits for the ratio $(B-A)/C$, that provide clues about the interior state of Enceladus. The analysis is based on Cassini images taken over 6 years.

## 2. Libration model and parameters

The libration amplitude $\gamma$ predicted for Enceladus was specified at [1]

$$\gamma = \sum_i \frac{\omega_0^2 H_i}{\omega_0^2 - \omega_i^2} \sin\left(\omega_i t + \alpha_i\right) \quad (1),$$

where $\omega_0 = n\sqrt{3(B-A)/C}$ is the free libration frequency ($n$ is mean motion), t is Ephemeris time, and $H_i$, $\omega_i$, $\alpha_i$ are parameters given in Table I.

| i | Period $2\pi/\omega_i$ days | Magnitude $H_i$ arcsec | Phase $\alpha_i$ deg. |
|---|---|---|---|
| 1 | 1.371983 | 1954.84 | 10.69 |
| 2 | 4035.64 | 933.30 | 73.81 |
| 3 | 1418.93 | 676.54 | -43.01 |

Table I: Libration parameters used in the calculations. i = 1 relates to librations forced by Saturn, i = 2, 3 are librations forced by Dione. Free librations are assumed to be damped. We note that the phases published in [1] suffer from an error which has been corrected here (pers. communication with N. Rambaux).

Fig. 1 is a plot of $\gamma$ over the duration of the Cassini mission showing that the amplitude can reach values as large as 0.4°.
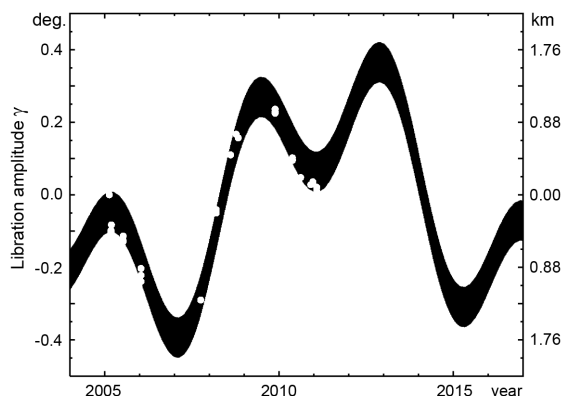


Figure 1: Libration amplitude (on the right with respect to arc lengths on the equator) calculated with parameters given in Table I and $(B-A)/C= 0.031$. The thickness of the curve traces back to librations at the orbital period (i=1). White dots mark the predicted libration amplitudes at the times when Cassini images used for our control point calculations (Sec. 4) were acquired.

## 3. Method

We use control point calculations to check for the librations. The image coordinates of control points (tiepoints) and their ground coordinates are related via the well-known collinearity equations [3]. These equations include the camera pointing angles and S/C positions at the times the images were acquired. The pointing is described by three Euler angles that rotate the ground-based body-fixed coordinate system into the camera system. The last rotation is about the body's spin axis, here denoted as $\kappa$. We apply least-

squares techniques to adjust errors in the observations (tiepoint measurements and camera pointing angles, S/C positions are fixed). The adjustment is controlled by the requirement that the multiple lines of sight to the tiepoints must all intersect. This is accomplished by adding corrections to the observations. Pointing angle corrections are those utilized in this paper. They may either be related to errors in the C-matrix (rotates the J2000 frame into the S/C frame) or to errors in the rotational model of the body. In order to detect the suggested librations in the rotations of Enceladus we therefore initially assumed that C-matrix errors are small in comparison with the libration amplitude, an assumption that proved valid after the adjustment. To test for the predicted librations we calculate

$$\lambda^2 = \sum_n (\delta\kappa_n)^2 \qquad (2),$$

where $\delta\kappa_n$ denotes the correction to $\kappa$ of the $n^{th}$ image. If Enceladus experiences librations $\chi^2$ should be significantly smaller than in case of uniform rotation.

## 4. Control points

The least-squares adjustment involved 38 Cassini images with resolutions ranging from 190 m/pxl to 1220 m/pxl, and covering a time span of 6 years. This resolution and time interval is appropriate for sensing the libration amplitudes and periods shown in Fig. 1. In total, we measured 1057 image points of 186 ground points aiming at a dense and uniform distribution across the surface (Fig. 2).
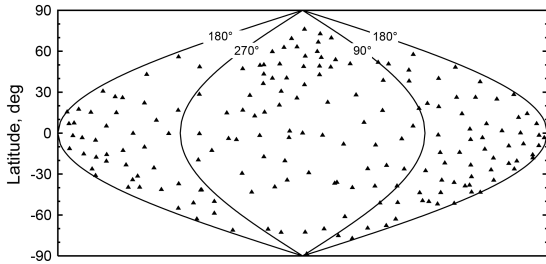


Figure 2: Sinusoidal map projection showing the control point locations resulting from adjustment of our observations.

## 5. Results and discussion

In result of the adjustment we achieved mean point precisions in (x, y, z) of (±323 m, ±292 m, ±289 m) if librations are neglected, and (±318 m, ±288 m, ±285

m) if librations are taken into account. This is a slight improvement but not the crucial point here. The more important difference, however, is that the latter result was obtained at pointing corrections only about 1/3 of those required if librations are omitted as revealed by Fig. 3. This is a strong argument in favor of the libration model, notably of the librations forced by Dione, which dominate in the model.

Even though weak, our results are also sensitive to librations at the orbital period. The adjustment showed that errors of the $\kappa_n$ are, on average, smaller than the amplitude of this libration, which is 0.056°. This allowed us to estimate the unknown parameter $(B-A)/C$, which is not constrained by the librations forced by Dione [1]. We find a minimum at 0.031 (Fig. 3), however, most likely values are in the range of 0.027-0.035. This range is clearly above the values representing a body in hydrostatic equilibrium (Fig. 3) and therefore consistent with results from shape modeling [4].
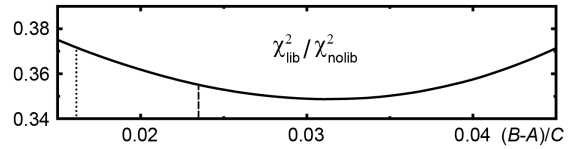


Figure 3: Variation in the $\chi^2$ (Eq. 2) ratio (librations vs. no librations) over a broad range in $(B-A)/C$. The dashed line marks a value of $(B-A)/C$ representing a homogeneous interior ($C/MR^2 = 0.4$), the dotted line corresponds to a fully differentiated body ($C/MR^2 = 0.31$).

The observed librations are too small in amplitude (i = 1) and too slow (i = 2, 3) to cause substantial heating (as compared to high heat flows measured in the south polar terrains [5]) in Enceladus, but they have impact on cartographic applications.

## References

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