Measurement and Analysis of Train Motion and Railway Track Characteristics with Inertial Sensors

Oliver Heirich and Andreas Lehner and Patrick Robertson and Thomas Strang

***Abstract*— This paper presents measurements of train motion with a low-cost inertial measurement unit (IMU) based on micro electro mechanical systems (MEMS). The measurements were recorded on-board a train during normal passenger transport service on a network with dense urban railway environment as well as a rural, regional network environment. Sensor measurements from several train runs were therefore analyzed and the data is presented with a discussion on typical characteristics, noise and dynamics. As the train motion is dependent on the track, local track characteristics are inferred from the train motion measurements. Finally, the inertial measurements are analyzed toward track feature detection for feature based localization purposes.**

I. INTRODUCTION

A special property of railways are the track constraints for trains. The train position, its motion trajectory, its attitude and the changes of the attitude are constrained by the track. In this paper the focus is on intertial measurement data, recorded on-board a train. The IMU measures acceleration and turn rates of the train and was mounted inside the passenger area at an overhead luggage rack as well as a GPS Antenna was mounted on the train roof. The sensors were used to measure the effects of actual track features on train motion while the train is moving along the railway network. The GPS measurements augment the inertial sensor information while inferring these features. The measurements were recorded during normal passenger service. Information about the train's acceleration, deceleration and its attitude can be used for position and motion estimating systems. Train localization is essential for automatic train control or safety systems like the Railway Collision Avoidance System (RCAS) [1][2] for example. Track feature information has been used in [3] to support track-selective train localization. Track features can be used for a feature based localization which has been used successfully in robotics [5] and for train localization [4].

Chapter II outlines the railway track characteristics with focus on significant inertial features depending on the position on the track. Chapter III concentrates on the theoretical expected inertial measurements of a moving train. The experimental setup (IV) contains a description about the train, sensors, track route and conditions. Following chapter V presents the recorded measurements and chapter VI analyzes the acceleration and turn rate data. The results of the track and train motion measurement are presented in VI.

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II. RAILWAY TRACK CHARACTERISTICS

A railway track has several position dependent characteristics that can be detected with inertial sensors. A track and its characteristics can be represented by a function with a single parameter s, which stands for the distance or one-dimensional track position. So, the following track position related and inertial features can be expressed as a continuous function with the parameter s.

A. Orientation and Curvature

The orientation of a track position can be described by the heading $\psi(s)$, which is the angular difference to geographic north. A track run can change its heading, which is described as the curvature of the track $\frac{d\psi(s)}{ds}$. The local curvature of a track is the reciprocal of the radius of a track segment. It causes lateral acceleration, turn rates or simply changes in orientation for a moving train. The design of tracks respect smooth curvature transitions and limits for maximum curvatures according to the design goals of train speed [8].

B. Inclination

Inclination of a track can be classified in two directions, called bank and slope. Bank $\phi(s)$ is the lateral inclination and can mainly be found in curves and is caused by one rail being higher than the other. It is used to compensate the lateral acceleration or parts of it during motion in curves. The bank angle is limited by restrictions to safety and passenger comfort. The bank of a track changes continuously by a ramp or a similar transition. This is called the bank change $\frac{d\phi(s)}{ds}$. Slope $\theta(s)$ is the inclination along a track and can be ascent or decent for changing altitude level. The slope change $\frac{d\theta(s)}{ds}$ is continuously and the slope is also limited. As an example for limits in German railroads, maximum bank angle is about $7°$ and the maximum slope angle is about $2.3°$ [8].

C. Basic track geometry

Railway track routes are mainly designed from basic geometric structures. These geometric elements are a straight line, a circular arc and transition curves like the Clothoid [8]. Straight lines have a curvature of zero, while circular arcs have a constant curvature. Transitions are necessary to avoid an instant step in the curvature which causes high jerks. A linear increase of curvature between straight and a circular arc is achieved by the Clothoid. Other transition curves have S-shaped increase of curvature. If the circular arc has a bank angle, the transition curve also changes the bank angle [8].

D. Switches

Switches have special characteristics. The detection of their existence and the estimation of the travel direction are essential for a train location based on railway infrastructure-less sensors. With focus on the turn rates and accelerations, important characteristics are the curvature and orientation. Some switches are realized by a direct straight to circular arc series. This causes a significant lateral jerk and an instant rise of a turn rate and lateral acceleration [8][3].

III. INERTIAL MEASUREMENT WITH TRAINS

Inertial measurements are realized in their sensor co-ordinate frame. For our analysis, the sensor was aligned to the train axes to measure accelerations and turn rates in the train frame. The co-ordinate frame for the train is defined here as a right-handed axis set. The translative axes and accelerations are denoted with longitudinal in the train driving direction a_x, lateral to the left side a_y and vertical pointing up a_z. The gyroscope turn rate measurements are ω_{pitch}, ω_{roll} and ω_{yaw}. A second co-ordinate frame with reference to gravity and geographic north defines the attitude of the train frame by rotation angles between the axes [6]. The reference frame has a horizontal plane which is perpendicular to gravity and rotation angles to the train frame will be θ for slope angle, ϕ for a bank angle and ψ for the heading of the train. For $\theta = \phi = \psi = 0$, the train stands perpendicular to gravity and points to north. A positive bank angle ϕ represents a bank of the train to the right, with a positive slope angle θ the train pitches downward. Positive changes of the angle ψ are caused by the train turning left.

A. Standing train

The inertial measurements of a standing train contains only the acceleration caused by gravity g. The turn rates are zero. Depending on the bank and slope of the track, the acceleration measurements of the train can be calculated with direction cosine matrices [6]:

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} -g\sin\theta \\ g\sin\phi\cos\theta \\ g\cos\phi\cos\theta \end{bmatrix} \tag{1}$$

B. Moving train

The kinematic train motion equation (2) contains the train acceleration and deceleration as a second derivative over time of the one-dimensional position vector s and the train velocity as derivative of s. The acceleration caused by traction and brakes will be measured by the IMU and effects only the longitudinal train axis,

$$s = s_0 + \dot{s}t + \frac{1}{2}\ddot{s}t^2 + [\ldots]. \tag{2}$$

Curves cause an acceleration dependent on the train speed $\dot{s}$ and the radius r or its reciprocal, the curvature c. A train turns left for positive and right for negative curvatures:

$$a_{curve} = \frac{\dot{s}^2}{r} = \frac{d\psi}{ds} \cdot \dot{s}^2 = c \cdot \dot{s}^2. \tag{3}$$

Another acceleration is caused by moving over a track with a changing slope. Similar to the motion in a curve, a vertical centripetal acceleration effects the train and the sensors. The parameter of the changing slope can be described with a vertical curvature v, which is positive for a crest and negative for a sag:

$$a_{slopechange} = \frac{d\theta}{ds} \cdot \dot{s}^2 = v \cdot \dot{s}^2. \tag{4}$$

According to the train frame definitions, the curve acceleration of a banked track affects the train by:

$$\begin{aligned} a_{y,curve} &= +c \cdot \dot{s}^2 \cos\phi, \\ a_{z,curve} &= +c \cdot \dot{s}^2 \sin\phi. \end{aligned} \tag{5}$$

The slope change acceleration affects the train on a banked track:

$$\begin{aligned} a_{y,slopechange} &= -v \cdot \dot{s}^2 \sin\phi, \\ a_{z,slopechange} &= -v \cdot \dot{s}^2 \cos\phi. \end{aligned} \tag{6}$$

Finally, the acceleration sensor output signals of an on-board IMU aligned with the train frame axis are given by:

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \ddot{s} - g\sin\theta \\ c\dot{s}^2\cos\phi + g\sin\phi\cos\theta - v\dot{s}^2\sin\phi \\ g\cos\phi\cos\theta - v\dot{s}^2\cos\phi + c\dot{s}^2\sin\phi \end{bmatrix}. \tag{7}$$

Additionally, a coriolis acceleration is caused by the train motion over the rotating earth, but it is neglected in (7) due to its small influence. Because of small bank and slope angles as well as limited train dynamics $g > \ddot{s}, a_{curve}, a_{slopechange}$, some terms can be neglected in equation (7) [8][7]. For this analysis, the measurements can be approximated by the dominant accelerations:

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \approx \begin{bmatrix} \ddot{s} - g\sin\theta \\ c \cdot \dot{s}^2 + g\sin\phi \\ g - v \cdot \dot{s}^2 \end{bmatrix}. \tag{8}$$

The bank or slope angles depend on the position of the track. Changes of bank and slope will cause roll and pitch rates dependent on the train speed $\dot{s}$:

$$\frac{d\phi}{ds}\dot{s} = \frac{d\phi}{ds}\frac{ds}{dt} = \frac{d\phi}{dt} = \dot{\phi}, \tag{9}$$

$$\frac{d\theta}{ds}\dot{s} = \dot{\theta}. \tag{10}$$

A curved track changes its orientation with the length. A moving train with a speed of $\dot{s}$ will rotate with a turn rate $\dot{\psi}$, dependent on the curvature c and the train speed:

$$d\psi \cdot r = ds, \tag{11}$$

$$c = \frac{1}{r} = \frac{d\psi}{ds}, \tag{12}$$

$$\frac{d\psi}{ds}\dot{s} = \dot{\psi}. \tag{13}$$

The gyroscopes of the on-board IMU are aligned with the train frame axes. The turn rates caused by bank or slope changes are translated in the train frame by rotation matrices [6]. Additionally a turn rate of the rotating earth will affect the measurements, but it is neglected in equation 14 due to the small amount:

$$\begin{bmatrix} \omega_{roll} \\ \omega_{pitch} \\ \omega_{yaw} \end{bmatrix} = \begin{bmatrix} \dot{\phi} - \dot{\psi}\sin\theta \\ \dot{\theta}\cos\phi + \dot{\psi}\sin\phi\cos\theta \\ \dot{\psi}\cos\theta\cos\phi - \dot{\theta}\sin\phi \end{bmatrix}. \tag{14}$$

For this analysis the gyroscope measurements will be used directly as rotation angle rates because small bank and slope angles allow a good approximation:

$$\begin{bmatrix} \omega_{roll} \\ \omega_{pitch} \\ \omega_{yaw} \end{bmatrix} \approx \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}. \tag{15}$$

The discussed measurement equations do not contain measurement errors caused by the sensor. In reality, MEMS sensors suffer from a changing bias over time and noise as can be seen in the measurements of chapter V.

IV. EXPERIMENTAL SETUP

A. Train

The train is a two direction urban and regional train of the type 'Integral' from BOB (Bayerische Oberlandbahn). The Integral is used for passenger transportation service and can carry up to 364 people. It has a length of 53 meters and can speed up to 160 km/h with its diesel hydraulic traction. The acceleration can reach 0.6 m/s^2, the deceleration is about 1.0 m/s^2. The Integral has no tilt technology [7].

B. Sensors

A low-cost MEMS IMU and a GPS were installed in the Integral. The first sensor was a 'XsensMTx' MEMS IMU from Xsens [9]. It was mounted inside the train and fixed to an overhead luggage rack near the driver cabin. The recorded data set contains 3D acceleration and 3D gyroscope turn rates. The data was sampled with 100 Hz. The GPS is used to determine the position, speed and orientation based on position changes. The data was recorded with 1Hz sample rate. The GPS receiver was a Ublox Antaris 4 device. The GPS antenna was placed on the roof above the driver cabin.

C. Track route

The Integral travels from Munich central station to Lenggries in the southern direction for about 65km. The route covers the following railway environment: Urban central station with a large shunting yard, bridges, a tunnel, several small stations, fast section with speeds up to 140km/h. The velocity profile with its 12 stops can be seen in Figure 1 with Munich central station at 0s and Lenggries station at 4000s.

D. Experiment conditions

The data was recorded on train runs with regular passenger service. The experiment was conducted without interaction of the train driver. The data represents usual conditions for passenger transportation with the Integral.

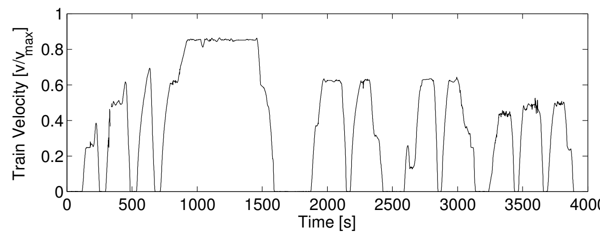

Fig. 1. Train velocity profile over time

V. MEASUREMENT

Figure 2 shows the acceleration of the longitudinal, lateral and vertical axis of the train, recorded with 100Hz sample rate. Figure 3 displays all turn rates of the train with 100Hz sample rate. The acceleration and gyroscope data is factory calibrated, but small errors in scale factors and biases are still present. The figures contain data of a full journey of about 65 km from Munich to Lenggries which took about 1:06h. The train stops at 12 stations with a slightly longer stop of 250s at Holzkirchen in the middle of the recording just after the longest and fastest section. The diesel engine was running most of the time, but just before the end of the recording ($> 3.9 \cdot 10^5$ samples) the engine was switched off.

VI. ANALYSIS

For a better understanding of inertial train measurement the sensor data is analyzed by its time signal and with power density spectrograms which display the frequency spectrum over time. To generate these plots a fast fourier transform (FFT) was performed with a windows size of 256 samples. The power was normalized to the maximum power of each sensor signal. For the dynamic situation analysis, the spectrogram visualizes significant frequencies during the train journey.

A. Acceleration signal analysis

Figure 2 shows the accelerations in time and Figure 4 shows the dominant frequency components by normalized power of each acceleration measurement. Compared with the train motion of Figure 1, the spectrograms clearly show intervals of motion and standing. Starting with a stationary train we point out that the acceleration measurements are not static. The time and spectrum plots show vibrations on all measurement axes. This changes when the engine has been switched off at the end of the measurements ($> 3.9 \cdot 10^5$ samples in Figure 2, > 3900 seconds in Figure 4). So, the source of that noise are the diesel engine vibrations from the train itself, transmitted through the railway car structure. The engine vibrations have a dominant frequency of 32Hz when the train is standing. The time and spectrum plots of the lateral axis show the strongest vibrations. As the IMU was placed near the GPS antenna at a high position in the cabin, the vibrations in longitudinal and vertical direction are damped more than in lateral direction, where vibrations can oscillate more freely. The sensor noise can be identified during the last section of the plots where the engine is

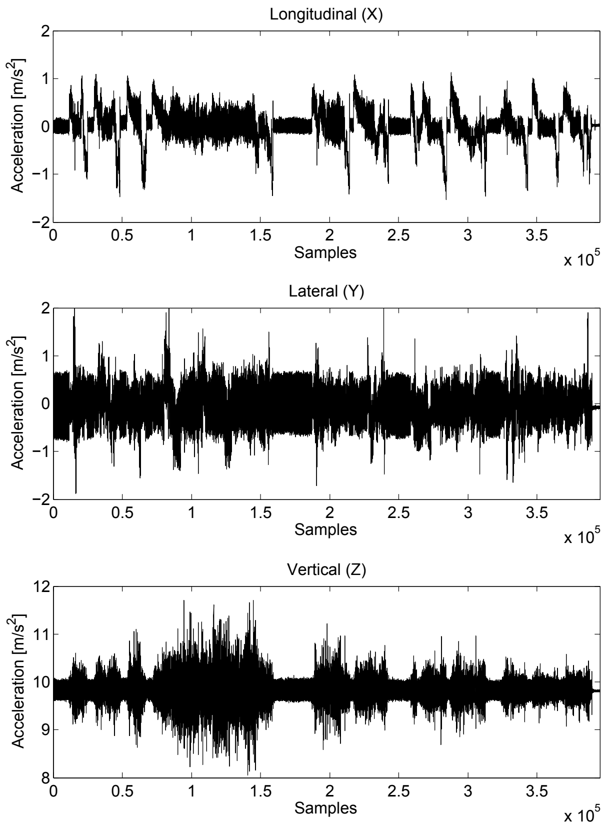

Fig. 2. Acceleration signal, 100Hz sampling rate

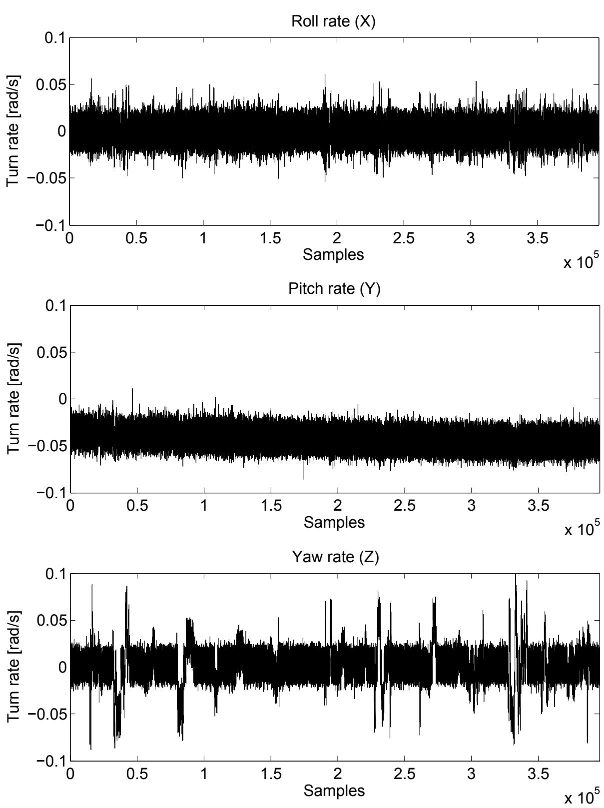

Fig. 3. Gyroscope signal, 100Hz sampling rate

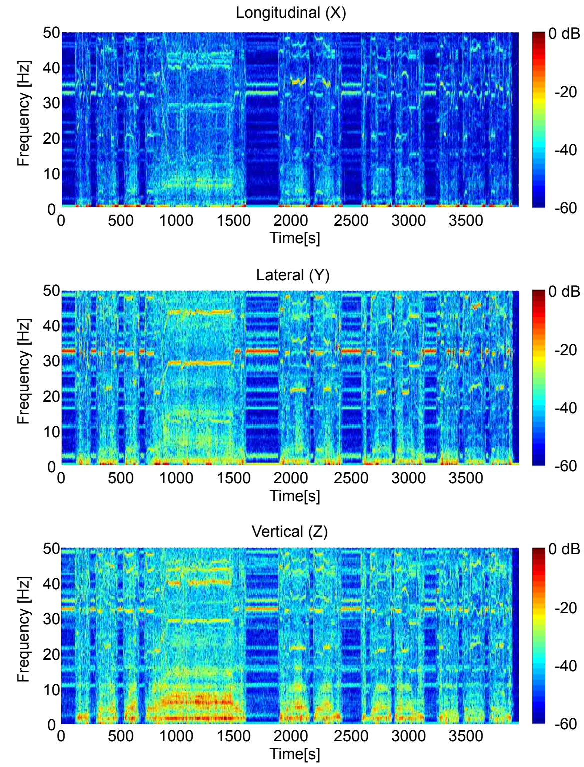

Fig. 4. Acceleration frequency spectrum

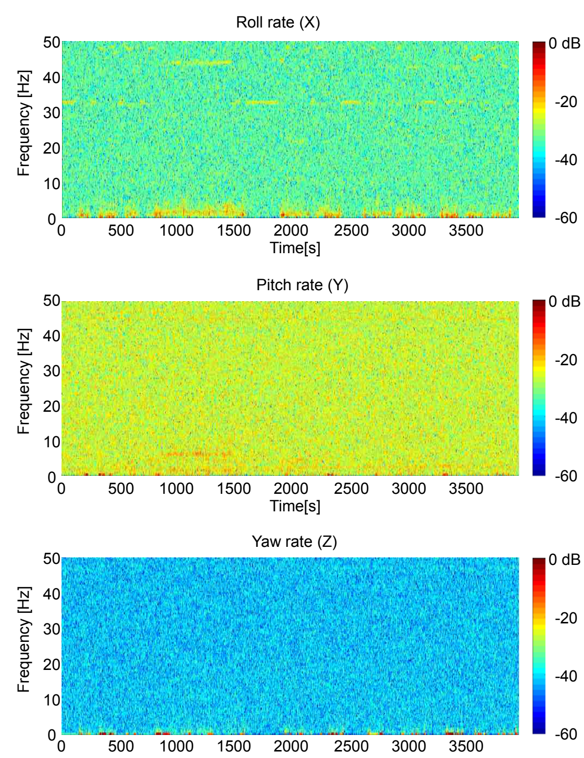

Fig. 5. Gyroscope frequency spectrum

stopped. Concentrating on the intervals in the acceleration plots where the train is moving according to Figure 1, engine vibrations and motion vibrations will occur. The source of those motion vibrations are mainly the wheels rolling on track. The vibrations are transmitted over the undercarriage to the car structure and to the IMU. The vertical acceleration time and spectrum shows most sensitivity to the motion dependent vibrations. Especially the interval with the maximum speed (750s-1600s) of the vertical axis shows significant dependencies compared to those of lateral and longitudinal. The spectrograms in Figure 4 shows that the vibrations are not Gaussian. With closer inspection toward the DC component of the longitudinal and lateral accelerations, the dominant signal parts of the measurements can be identified. That means that the acceleration or deceleration from the traction and the lateral acceleration of the train have low dynamics.

B. Turn rate analysis

Figure 3 shows the turn rate time signals of the train journey and Figure 5 displays the power density spectrum. Starting with the engine effects, only the spectrum of the roll rate shows small evidence. During a stop, the spectrum of the roll rate shows similar frequencies of about 32Hz as in the lateral acceleration spectrum. The engine causes the car to translative and rotative vibrations and the IMU was placed on a high point in the car like on an oscillating arm. Pitch and yaw rate are not affected by diesel engine vibrations because the spectrums show no evidence of this while standing, stopping or after the engine switch off toward the end. The spectrograms of Figure 5 of the gyroscope measurements show mainly uniform distribution over the frequency. The noise here has a Gaussian nature. As the spectrograms visualize the power density spectrum of the normalised power to maximum power, the spectrums visualize the signal to noise ratio by their background color. The noise of all three gyroscopes is approximately the same, but the maximum power of the signals are different. The pitch rates from the train are the smallest, the roll rate signals are in between and yaw rate measures the strongest signals. This describes how well the track characteristics can be measured. The roll rate shows some significant signals during the motion intervals below 5 Hz. These signals can be bank changes as well as a motion related shaking of the car structure. The pitch rate diagram shows mainly slowly growing negative bias in the time signal which is typical for a MEMS sensor. The pitch spectrogram analyses a detrended pitch rate and show some significant signals near DC. This will be evident as a change in the slope. The pitch spectrum show as well motion related rates in the high velocity interval (750s to 1600s). The yaw rate spectrum show very dominant signals below 1 Hz, which is the train heading change and another evidence of the low frequencies of train motion dynamics.

VII. TRACK AND TRAIN MOTION MEASUREMENT

A. Filtering

The acceleration measurements contain vibrations from motion and the engine and noise from the sensor. The gyroscope measurements are hardly affected by engine and motion distortions but more by the sensor noise. The motion dynamics of accelerating and turning of a train are low compared to the vibrations and noise. The solution for an improved signal to noise ratio is a low pass filter. The filter we used is a FIR (finite impulse response) filter with a window of 100 samples and a cutoff frequency of 2 Hz. Figure 6 expresses the accelerating and braking periods in the filtered signal. Figure 7 presents the filtered turn rate of the heading of the train. After filtering, the train motion can be interfered from the IMU. From the filtered measurements typical signal levels are extracted in order to gain knowledge of the approximate range of the acceleration and turn rates. Accelerations of the train motion occur up to 0.6 m/s^2 for an accelerating train and up to 1.2 m/s^2 during deceleration. The maximum lateral accelerations are up to 1 m/s^2 and the vertical accelerations are observed around 0.2 m/s^2 due to slope changes. The maximum roll rate signals are around 0.03 rad/sec ($1.7°/s$), the pitch rate signal limits are around 0.012 rad/sec ($0.7°/s$) and maximum yaw rate signals are around 0.07 rad/sec ($4.0°/s$). These approximate reference values for train motion measurement are small in contrast to the bias stability of $1°/s$ of the used sensor. This is a typical bias stability of todays consumer gyroscopes. Tactical grade IMUs have a bias stability of about $1°/h$. Future trends and performance goals of IMU developments include MEMS gyroscopes with lower bias stability and noise than todays tactical grade IMUs [10].

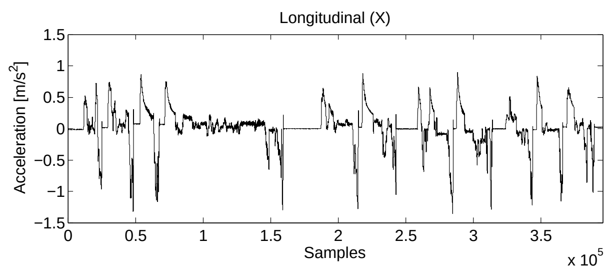

Fig. 6. Filtered longitudinal train acceleration

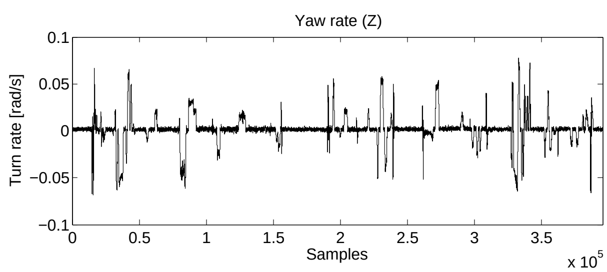

Fig. 7. Filtered yaw rate

B. Track features

Two track features are presented here, as they can easy be measured even with a low-cost sensor. The yaw rate measurement works well, as seen in Figure 7. A cause of that turn rate is the curvature of the track. The current curvature is measured within the train and is calculated by:

$$\frac{\omega_{yaw}}{\dot{s}} \approx \frac{\dot{\psi}}{\dot{s}} = \frac{d\psi}{ds} = c. \qquad (16)$$

Figure 8 shows a section of a curvature measurement over time during a train run over two right and two left turns. The basic geometric elements of the track design can be identified. Straights have zero curvatures, a circular arc has a constant non zero curvature and a Clothoid has a ramp-shaped curvature change. Methods for curvature estimation and classification has also been shown in [11]. In Figure 8, a straight track element is followed by a Clothoid to the right, followed by a circular arc to the right with a curvature of $10^{-3}\frac{1}{m}$ (radius of 1000m), etc.

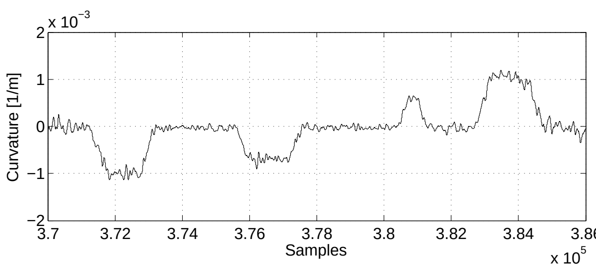

Fig. 8. Actual track curvature example of 160s (100Hz sample rate)

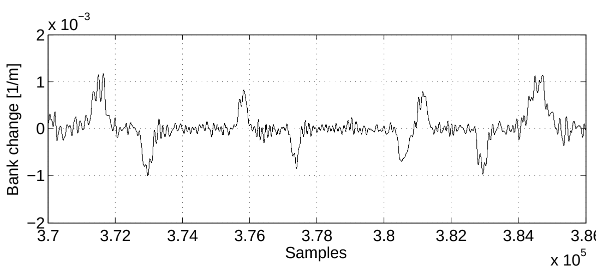

Fig. 9. Actual track bank change example of 160s (100Hz sample rate)

The bank change is measured similar to the curvature. Therefore, the roll rate is normalized by the velocity:

$$\frac{\omega_{roll}}{\dot{s}} \approx \frac{\dot{\phi}}{\dot{s}} = \frac{d\phi}{ds}. \qquad (17)$$

Tracks can be banked in curves and a transition from a non banked track to a banked track is continuously. This transition is the bank change and causes the train to roll while moving over the track. Figure 9 presents the bank changes for the same section as in Figure 8. The track in this section has four banked curves.

Another track feature is the slope change by normalizing the pitch rate by the velocity, but there is a small signal to noise ratio, as shown in the analysis of the pitch rate signal.

The track features bank, slope and relative heading of the track can be calculated from inertial measurements as well. The calculation needs drift compensation and absolute initial values [6]. The bank and slope angles are small and different sensor source for a true heading estimation is necessary.

VIII. CONCLUSIONS

The analysis of the IMU sensor data presents low dynamics for the train motion. The acceleration sensors measure train motion acceleration as well as engine and motion vibrations. The vibrations depend on motion, speed, standing, engine system, undercarriage and finally the sensor placement. The vibrations have higher dynamics than the train motions and can be filtered by a low-pass filter. The gyroscopes measure small signals compared to the sensor noise. The low dynamics of train turn rates allow a low-pass filtering to improve the signal to noise ratio. Out of the train motion measurements, track features such as bank, bank change, slope, slope change, relative heading, curvature and basic track elements can be inferred. The low-cost MEMS IMU mounted in the cabin achieves good results in traction acceleration and yaw turn rate measurements as well as inferred track features such as curvature and bank change. Analysis with data sets from different train trips show a good repeatability of the track feature measurements. The analysis of the sensor signals and dynamics provides parameters which can be used for the design of train positioning systems, train motion models or simulators. The described track features can be used for further feature based localization.

IX. ACKNOWLEDGMENTS

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