Deriving an estimate for the Fried parameter in mobile optical transmission scenarios

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Measuring the Fried parameter r_0 (atmospheric optical coherence length) in optical link scenarios is crucial to estimate a receiver's telescope performance or to dimension atmospheric mitigation techniques, such as in adaptive optics. The task of measuring r_0 is aggravated in mobile scenarios, when the receiver itself is prone to mechanical vibrations (e.g., when mounted on a moving platform) or when the receiver telescope has to track a fast-moving signal source, such as, in our case, a laser transmitter on board a satellite or aircraft. We have derived a method for estimating r_0 that avoids the influence of angle-of-arrival errors by only using short-term tilt-removed focal intensity speckle patterns.

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1. Introduction

Atmospheric index-of-refraction turbulence (IRT) distorts optical waves and leads to reduced resolution of ground-based telescopes or reduced received signal stability in optical free-space communication systems. The strength of these wavefront distortions is classically quantified in astronomy by the long-term Fried parameter r_0, which is defined according to [1]. Simplified, the IRT of the atmosphere produces optical wavefront distortions with structure size r_0, which limit the resolving power of a telescope or the signal quality of an optical receiver. While r_0 is a long-term parameter of an unbounded plane optical wave inside the atmospheric propagation medium, it can practically only be measured with limited-size telescopes. This implies that the statistics of the remaining overall tilt of the optical wavefront over the telescope aperture needs to be regarded, especially when the ratio between telescope aperture diameter D and r_0 is small.

When application scenarios are expanded from celestial observations toward mobile laser links like satellite downlinks to optical ground stations (OGSs) or optical aircraft links, the classical principle of a long-term (several seconds) observation of the received wavefront quality is no longer applicable due to the following reasons:

- Because of the fast lateral movement of the mobile link partner(s), the atmospheric volume traversed by the optical signal changes during the required long-term observation time. As this atmospheric volume cannot be regarded as isotropic, no valid long-term statistical parameter can be derived from such a dynamic measurement. Instead, one needs to reduce to intermediate-term statistical evaluations.
- The relative angular movements of the link partners requires some kind of agile tracking of the partner's signal. This tracking could have either a nonnegligible remaining tracking error that might be much larger than the angle-of-arrival caused by the atmosphere (aAoA), or it could also be of such high quality that it perfectly tracks any incoming angle-of-arrival (AoA), thereby also removing the aAoA. But this aAoA needs to be taken into account for the r_0 determination. In both situations, no distinction between mechanical and aAoA effects can be made, again changing the statistics of an r_0 measurement.

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These two facts require an adapted method for deriving an estimate for r_0 in mobile transmission scenarios.

2. Basic r_0 Estimation from a Focal Speckle Pattern

For the estimation of the Fried parameter r_0, different methods are available, e.g., the differential image motion monitor [2] or measurements with fast wavefront sensors. Another very simple and practical way is to observe the long-term focal speckle pattern produced from a point source at infinite distance onto a focal camera. This intensity distribution represents all atmospheric wavefront aberrations seen by the aperture, including the overall tilt (which causes lateral movements of the focal speckle patterns) and resembles a Gaussian distribution. The full width at half-maximum (FWHM) of this focal seeing disk (FSD) $\mathrm{FWHM}_{\mathrm{FSD}}$ can be used to calculate r_0 according to [3] (with $\mathrm{FWHM} = 1.67 * \sigma$ for the Gaussian FSD approximation), where $D \gg r_0$ must be observed (D: receive aperture diameter) to account for the error by the diffraction limited focal spot size:

$$r_0 \approx 0.98 \cdot \frac{\lambda \cdot f}{\mathrm{FWHM}_{\mathrm{FSD}}} = 0.59 \cdot \frac{\lambda \cdot f}{\sigma_{\mathrm{FSD}}}, \qquad (1)$$

where f is the effective focal length, λ is the wavelength, and σ_{FSD} is the sigma radius of the FSD. In fact, this method cannot measure r_0 larger than $0.96 \cdot D$ as the minimum (diffraction limited) focal spot size of a circular aperture already has a FWHM of $1.024 \cdot \lambda \cdot f/D$. In other words, this method, rather, estimates the *seeing resolution* of a specific telescope, where this resolution is either limited by the telescope aperture diameter D or the atmospheric seeing given by r_0, whichever is smaller (see [4] for an explanation of the effective resolution of a telescope under atmospheric turbulence).

The drawback of this simple method is the fact that a perfect geometric axial telescope alignment toward the point source must be ensured during the long-term exposure of the FSD so as to allow only atmospherically induced wavefront tilts to enlarge the FSD. The effects of nonperfect tracking of the point source, e.g., by mechanical inaccuracies of the telescope mount or through the fast angular movement of the point source (which might be emitted from a fast moving satellite or an aircraft instead of a fixed star) will lead to extra broadening of the long-term FSD and thus spoil the measurement of r_0, as already stated above.

Therefore, a method has been tested using the size of the *short-term tilt-removed* focal intensity speckle (FIS) patterns and backcalculate the values toward a standard long-term r_0. A perfect tilt removal thereby is performed through simple image processing by centering the FIS image of the focal camera around its center of gravity.

The measurements used in this paper were taken during the Kirari Optical Downlinks to Oberpfaffenhofen (KIODO) 2009 optical satellite downlink trials using the Japan Aerospace Exploration Agency (JAXA) Optical Inter-Satellite Communications Engineering Test Satellite and the DLR OGS at Oberpfaffenhofen (OGS–OP) near Munich [5]. In this paper, we use data from the downlink that took place on 28 August 2009 from 04:05:39 a.m. to 04:09:01 a.m. local time. Of course, the presented principle can be applied to any optical free-space link scenario.

3. Introduction of Tilt-Removed and Short-Term Parameters

Only for illustration of the method do we introduce the parameters "short-term r_0" $r_{0\text{-st}}$, and "tilt-removed short-term r_0" $r_{0\text{-tr-st}}$ (they have no real physical meaning). Tilt removal from the impinging optical wavefront can be performed either by perfectly tracking the AoA (e.g., by an ideal tip–tilt mirror) or —more conveniently—by calculating the individual AoA from the measured (tilt-included) FIS and accordingly cutting out the centered focal speckle pattern by image processing.

By shortening the exposure time of the focal camera down to well below the atmospheric coherence time (typically 1 ms or shorter in extreme scenarios), the focal speckle pattern is frozen in time and does not move laterally nor change its shape. Figure 1(a) shows a typical FIS as seen during the optical satellite downlink.

Values for the tilt-removed instantaneous speckle size $\rho_{\text{FIS-tr-st}}$ are derived from the sigma radius of the Gaussian fit to this FIS. The size of the tilt-removed FSD $\sigma_{\text{FSD-tr}}$ can either be found by a Gaussian fit over the long-term tilt-removed focal intensity distribution [which is found by superposing the N short-term tilt-removed FIS; see Fig. 2(b)] or by calculating the sliding mean of N times the single focal FIS radius—both methods yield the same result as expected. The averaging time typically should be

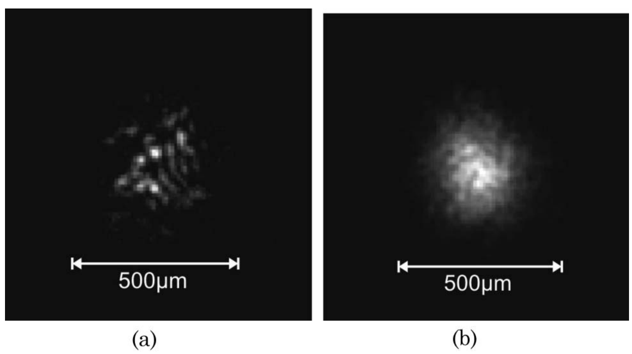
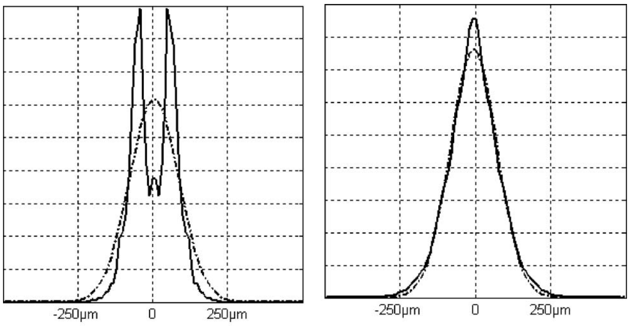


Fig. 1. Typical tilt-removed (a) short-term FIS and (b) long-term tilt-removed FSD (right, overlay of 48 samples, equivalent to 1 s exposure) measured during KIODO–2009. Both images show $990\,\mu\mathrm{m} \times 990\,\mu\mathrm{m}$ of the focal intensity image, signal wavelength $\lambda = 847$ nm, receiver aperture diameter $D = 0.4$ m, and focal length $f = 8.3$ m. These values imply a minimum diffraction-limited focal spot sigma radius of $\rho_{\mathrm{dl}} = 0.61 * \lambda * f/D = 10.8\,\mu\mathrm{m}$, which would only be seen under perfect seeing (without any atmospheric distortions). Note that all aAoA and all tracking AoA have been removed from these images.



(a) (b)

Fig. 2. Gaussian fits (dotted curves) to the (a) short- and (b) long-term FSD of Fig. 1. Instead of a simple horizontal cut through the focal intensity, a weighted circular integration of the FIS and FSD has been performed for these plots, providing a much better estimate for the sigma of the Gaussian fit. As a result of this processing, the curves are symmetric to the center axis; the ordinate is the intensity in arbitrary units.

around 1 s to have enough independent samples for a reliable calculation of statistical parameters (regarding a typical IRT-induced bandwidth of 100 Hz), while the atmospheric volume properties stay sufficiently constant during this observation time (which depends on the lateral velocity of the link partner). A shorter time should be applied for extremely fast movements The number of frames N must be chosen accordingly (regarding the focal camera frame rate).

In Fig. 3, all $\rho_{\text{FIS-tr-st}}$ samples and their 1 s sliding mean (solid curve) are plotted against the link elevation.

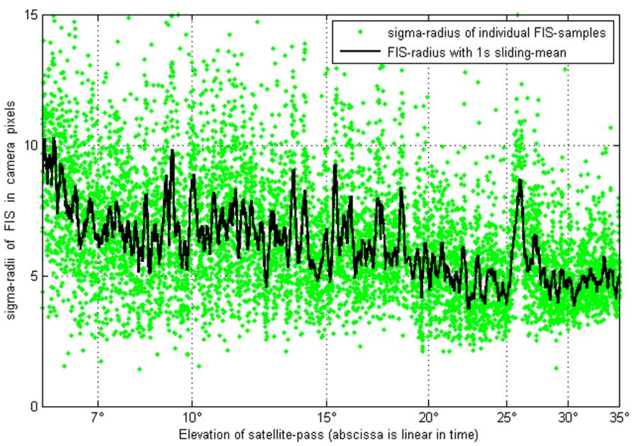


Fig. 3. (Color online) Samples of the FIS sigma radius, plotted linearly over time (different satellite elevation angles are marked on the abscissa); the ordinate is the sigma radius in camera pixels (with a 9.9 μm pixel pitch). Dots represent the short-term tilt-removed radii $\rho_{\text{FIS-tr-st}}$ (sigma of its Gaussian fit), and the black curve is their 1 s sliding mean. A lower bound of the FIS size is given by the diffraction-limited focal spot size. The FIS reduces with increasing elevation as less disturbing atmosphere is passed by the laser beam from the satellite. A total of 7775 FIS samples was evaluated for this plot, representing 163 consecutive seconds of satellite downlink measurements.

4. Relation from Short-Term Tilt-Removed Parameters to r_0

To derive a relation between the different effects of broadening the focal intensity pattern, we use the sum of the variances of the lateral focal deviations from a point image, assuming isotropic Gaussian statistics of the diverse uncorrelated deviations, which will produce, again, a Gaussian statistic of the broadening of the focal intensity spot:

$$\sigma_{\text{FSD}}^2 = \langle\rho_{\text{FIS}}^2\rangle = \langle\rho_{\text{FIS-tr-st}}^2\rangle + \langle\rho_{\text{aAoA}}^2\rangle, \tag{2}$$

with $\langle\rho_{\text{aAoA}}^2\rangle$ representing the broadening of the FSD by aAoA.

According to formula 4.58 in [6], Hardy states the variance of the aAoA β_{aAoA} over an aperture D as

$$\langle\beta_{\text{aAoA}}^2\rangle = 0.182\cdot\frac{\lambda^2}{D^{1/3}\cdot r_0^{5/3}}; \qquad r_0 \le D. \tag{3}$$

From $\langle\beta_{\text{aAoA}}^2\rangle$ the variance of the focal aAoA movement $\langle\rho_{\text{aAoA}}^2\rangle$ is derived by multiplication with f^2.

By measuring $\rho_{\text{FIS-tr-st}}$ with the short-exposure focal camera picture, we can calculate the relation between an $r_{0\text{-st}}$ and an $r_{0\text{-tr-st}} = 0.59\cdot\lambda\cdot f/\rho_{\text{FIS-tr-st}}$ by applying the relations stated in (1)–(3) to short-term parameters:

$$\left(0.59\cdot\frac{\lambda\cdot f}{r_{0\text{-st}}}\right)^2 = \left(0.59\cdot\frac{\lambda\cdot f}{r_{0\text{-tr-st}}}\right)^2 + 0.182\frac{(f\cdot\lambda)^2}{D^{1/3}\cdot r_{0\text{-st}}^{5/3}}, \tag{4}$$

where $r_0 < D$ has to be regarded to account for the limitations of the approximation formula (3). We can then express the relation between $r_{0\text{-st}}$ and its tilt-removed variant $r_{0\text{-tr-st}}$ (measured via $\rho_{\text{FIS-tr-st}}$):

$$r_{0\text{-tr-st}} = r_{0\text{-st}}\cdot\left[1 - 0.52\cdot\left(\frac{r_{0\text{-st}}}{D}\right)^{1/3}\right]^{-\frac{1}{2}}. \tag{5}$$

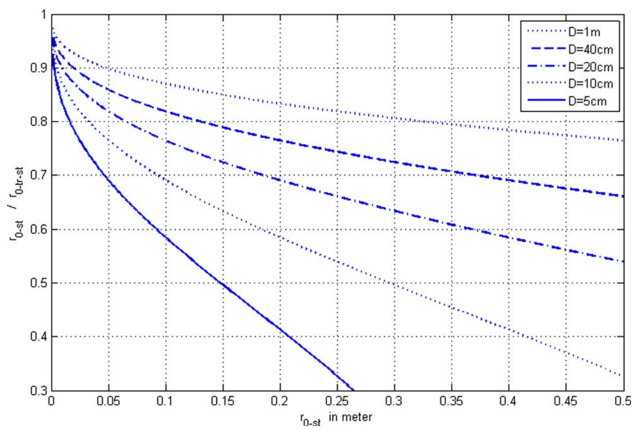


Fig. 4. (Color online) Factor between tilt-included $r_{0\text{-st}}$ and tilt-removed $r_{0\text{-tr-st}}$ as a function of $r_{0\text{-st}}$ with the receiver aperture diameter D as the parameter. The relation becomes dubious for $r_{0\text{-st}} \gg D$ and should only be applied for $r_{0\text{-st}} < D$.

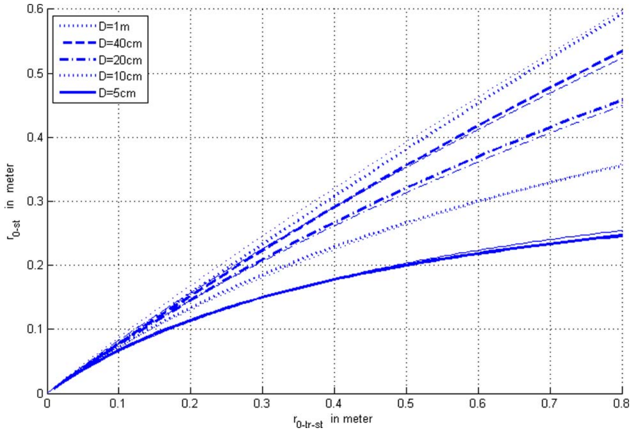


Fig. 5. (Color online) Relation between the measured $r_{0\text{-tr-st}}$ and the desired $r_{0\text{-st}}$ according to approximation (7), with the receiver aperture diameter D as the parameter. The exact numerical inversion of Eq. (5) is plotted with thin curves to show the quality of the approximation (7).

With this we can directly plot the conversion factor (Fig. 4), which is always <1:

$$\frac{r_{0\text{-st}}}{r_{0\text{-tr-st}}} = f(r_{0\text{-st}}, D) = \sqrt{1 - 0.52 \cdot \left(\frac{r_{0\text{-st}}}{D}\right)^{1/3}}. \qquad (6)$$

To directly derive $r_{0\text{-st}}$ from $r_{0\text{-tr-st}}$ would involve finding a null of a sixth-degree polynomial equation, which is generally unsolvable. Instead, by inverting function (5) numerically, we can derive the following simple approximation for $r_{0\text{-st}} = f(r_{0\text{-tr-st}}, D)$, which is usable in the region $[0 < r_{0\text{-st}} < 1\,\text{m}]$ and $[0.02\,\text{m} < D < 2\,\text{m}]$ and shown in Fig. 5:

$$r_{0\text{-st}} \approx \frac{8 \cdot D \cdot r_{0\text{-tr-st}}}{r_{0\text{-tr-st}} + 10 \cdot D}; \qquad r_{0\text{-st}} \leq D. \qquad (7)$$

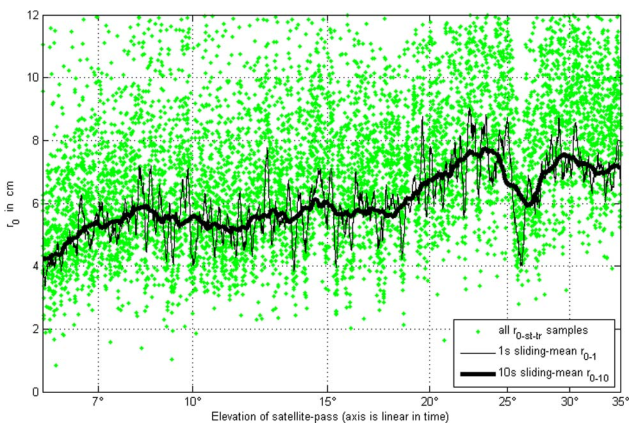


Fig. 6. (Color online) Comparison of different averaging times (1 and 10 s) for r_0 estimation in the optical satellite downlink [5] according to the above formulas. The increase of $r_{0\text{-}10}$ with elevation during the satellite's ascending path over the ground station coincides with the theory (less air mass in the link path at higher elevations); the unsteady behavior of this curve is caused by intermittent turbulent layers traversed by the link. The OGS aperture is $D = 0.4\,\text{m}$, and the wavelength is $\lambda = 847\,\text{nm}$.

Now these relations directly allow to estimate a short-term $r_{0\text{-st}}$ from the sigma radii measurements of $\rho_{\text{FIS-tr-st}}$:

$$r_{0\text{-st}} \approx \frac{8}{\frac{1}{D} + \rho_{\text{FIS-tr-st}} \cdot \frac{10}{0.59 \cdot \lambda \cdot f}}; \qquad r_{0\text{-st}} \leq D. \qquad (8)$$

Of course, a short-term $r_{0\text{-st}}$ derived from a single FIS has no statistical meaning; however, any longer term r_0 values can be derived from vectors of $r_{0\text{-st}}$ by averaging, where a sliding-mean algorithm is recommendable for smoother data representation.

Obviously, the transition region from real short-term measurements "st" (single samples with exposure times below the atmospheric coherence time) to a real long-term estimation of r_0 needs to be accounted for. We suggest, therefore, to put the averaging time in seconds for the calculation of a modified (*intermediate-term*) Fried parameter into the subscript to clearly state the averaging effect, e.g., $r_{0\text{-}0.1}$ or $r_{0\text{-}10}$. The effect of two different averaging times is shown in Fig. 6.

It shall be clarified again that the introduction of the tilt-removed r_0 parameters in this paper is for derivation purposes only because they have no real physical meaning.

5. Summary

We have deduced a method to find an estimate for r_0 in a dynamic link scenario. A classical r_0 cannot be derived in unstable scenarios where the atmospheric volume traversed by the link changes very rapidly (in seconds or even shorter), such as in a satellite downlink. To account for the impracticality of measuring the real aAoA in such mobile (tracked) link scenarios, a way of determining r_0 from the size of the short-term *tilt-removed* FISs has been derived. The introduction of a short-term $r_{0\text{-st}}$ is somewhat contradictory to the basic meaning of the Fried parameter, which is, by definition, a long-term parameter. By stretching the definition of a long-term r_0 to average values of only n seconds (→ "$r_{0\text{-}n}$"), we are able to estimate this essential parameter also in dynamic or mobile scenarios.

Further work should involve a relation from these short-term parameters to other parameters for the atmospherically distorted optical wave, namely, the instantaneous heterodyning efficiency $\eta_{\text{het}}(t)$, and so to determine the performance of a coherent or fiber-coupled receiver.

Appendix A: Abbreviations and Symbols

Abbreviation/ Symbol	Term
AoA	Angle of arrival
aAoA	Atmospheric AoA
D	Receiver aperture diameter
f	focal length of the telescope system
FIS	Focal intensity speckle pattern (short-term exposure)
FSD	Focal seeing disk (long-term exposure)
FWHM	Full width at half-maximum

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FIS	Focal intensity speckle pattern (short-term exposure)
FSD	Focal seeing disk (long-term exposure)
FWHM	Full width at half-maximum
IRT	Index-of-refraction turbulence
KIODO	Kirari Optical Downlinks to Oberpfaffenhofen
r_0	Fried parameter, atmospheric coherence length
$r_{0\text{-st}}$	Short-term r_0 (for exemplification only)
$r_{0\text{-tr-st}}$	Tilt-removed short-term r_0 (for exemplification only)
ρ_{dl}	Sigma radius of diffraction limited (ideal) focal spot size
ρ_{aAoA}	Focal angular deviation value due to atmospheric angle-of-arrival tilt
$\rho_{\text{FIS-tr-st}}$	Sigma radius of one FIS sample
σ_{FSD}	Sigma radius of long exposure FSD
$\sqrt{\langle\beta^2_{\text{aAoA}}\rangle} = \sigma_{\text{aAoA}}$	Sigma of aAoA over the Rx aperture in rad (assuming normal statistics)
$\sqrt{\langle\rho^2_{\text{aAoA}}\rangle} = f \cdot \sigma_{\text{aAoA}}$	Sigma radius of focal spot broadening by aAoA

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