Scalable Distributed Schur Complement Solvers for Internal and External Flow Computations on Many-Core Architectures

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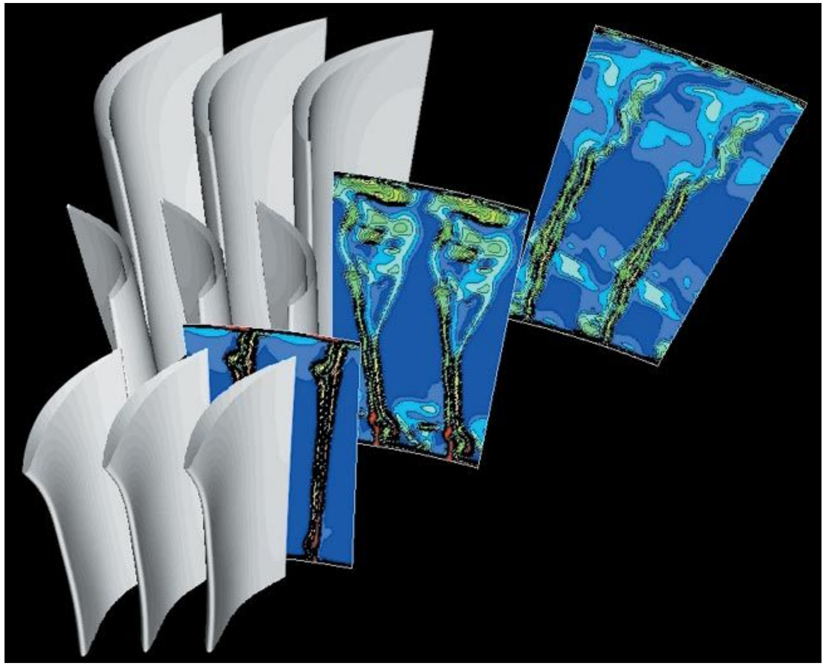
Survey

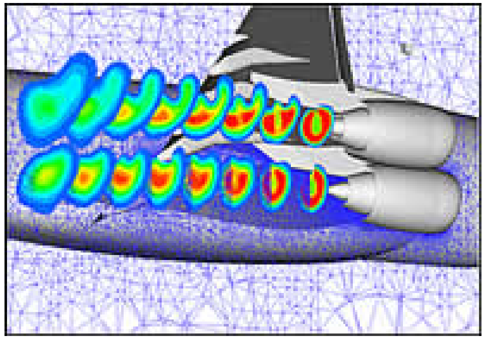
- Internal and external flow computations at DLR
- Storage Schemes for sparse matrices
- The *Distributed Schur Complement* method (DSC)
- Experiments with TRACE and TAU matrices
- Conclusions

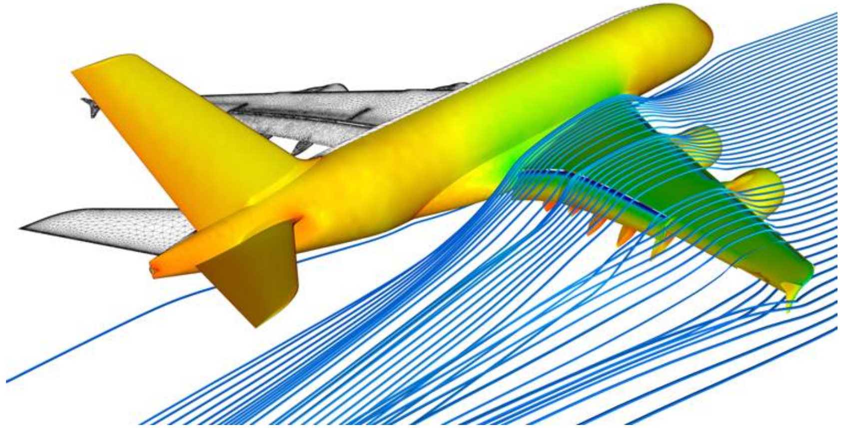
Parallel Simulation System TRACE



- TRACE: Turbo-machinery Research Aerodynamic Computational Environment
- Developed by the Institute for Propulsion Technology of the German Aerospace Center (DLR-AT)
- Calculates internal turbo-machinery flows
- Finite volume method with block-structured grids
- The linearized TRACE modules require the parallel, iterative solution with preconditioning of large, sparse, non-symmetric real or complex systems of linear equations

Preconditioners for TAU: Background





- TAU: developed for the aerodynamic design of aircrafts by the DLR Institute of Aerodynamics and Flow Technology
- Unstructured RANS solver (Reynolds-averaged Navier-Stokes), exploits finite volumes
- Requires the parallel, iterative solution with preconditioning of large, sparse, real, non-symmetric systems of linear equations
- Solvers used: geometric Multigrid, AMG preconditoned GMRes
- Here: experiments with DSC methods

Storage Schemes for Sparse Matrices

Compressed Row Storage (CSR) and Block Compressed Row Storage (BCSR)

Matrix:

1	0	0	2	0	0
0	3	4	5	0	0
0	0	0	0	6	7
0	0	0	0	8	9

Non-zero values, row-wise:

1	2	3	4	5	6	7	8	9

Column indices, row-wise:

1	4	2	3	4	5	6	5	6

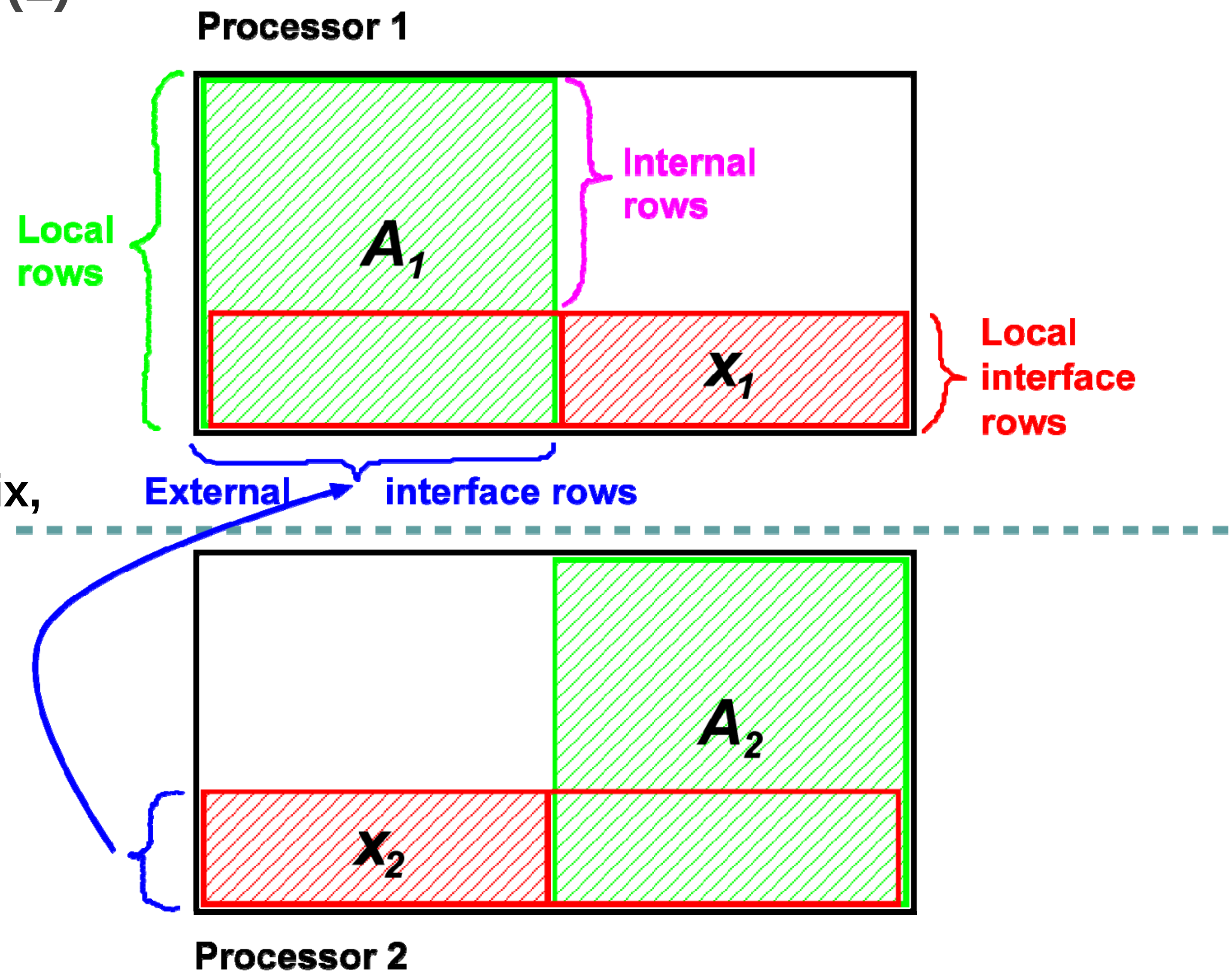
Row pointers:

1	3	6	8	10

1	0	0	2	0	0
0	3	4	5	0	0
0	0	0	0	6	7
0	0	0	0	8	9

- TRACE and TAU apply BCSR with 5x5 blocks.
- Avantage: less indirect addressing
- Disadvantage: A few zeros are stored.

DSC Method (1)

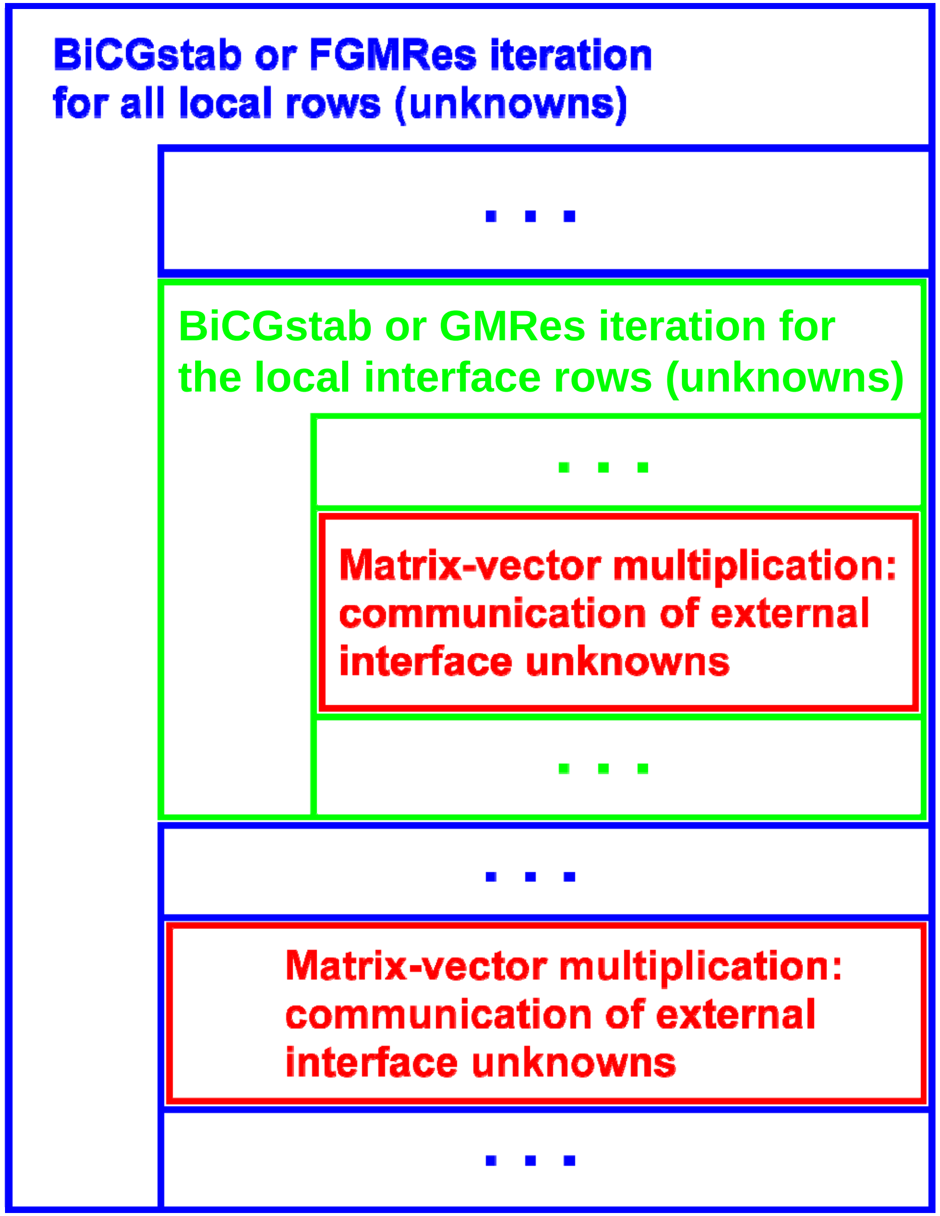


Distributed matrix,
2 processors

DSC Method (2)

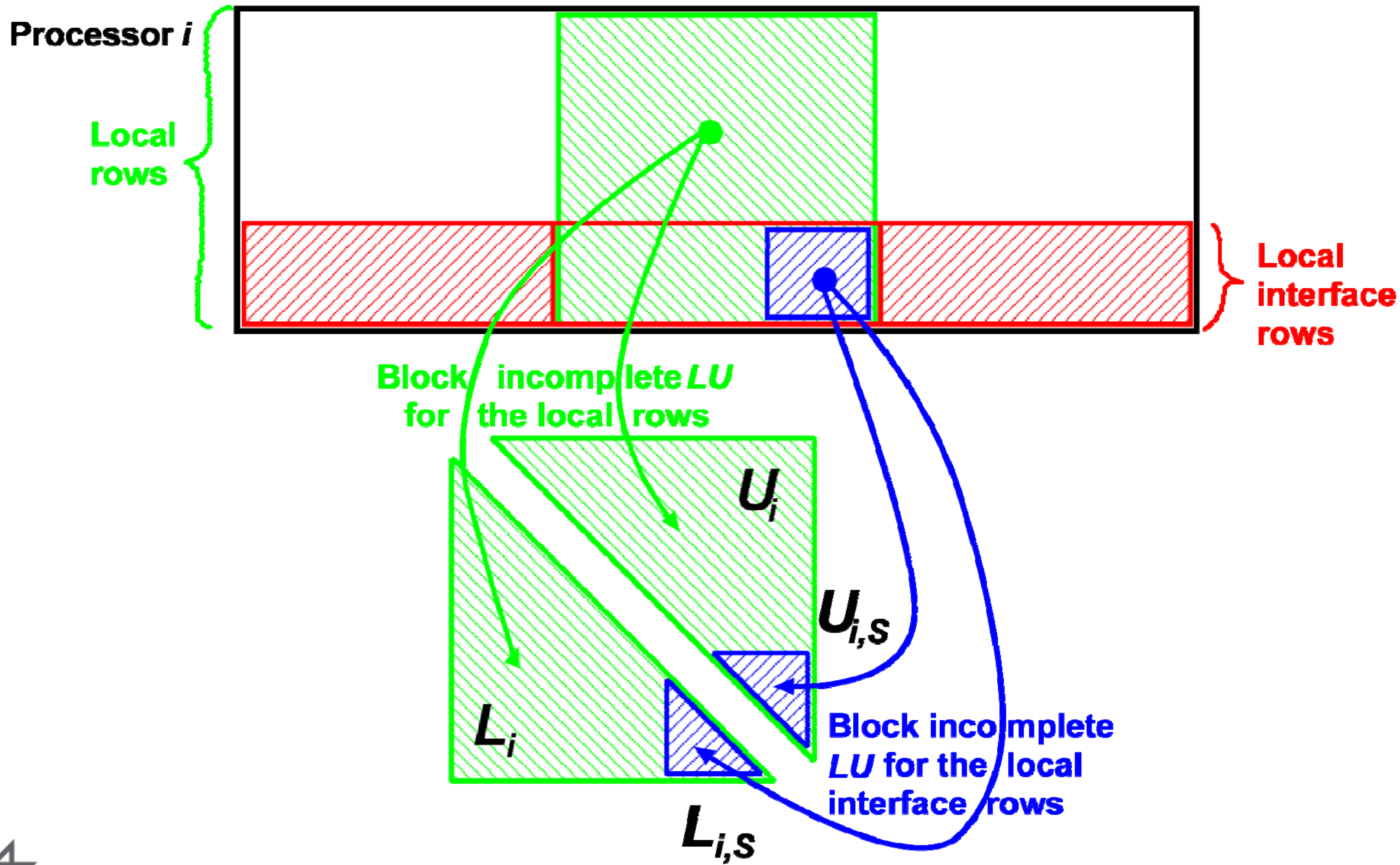
DSC Algorithm

Schematic view on each processor

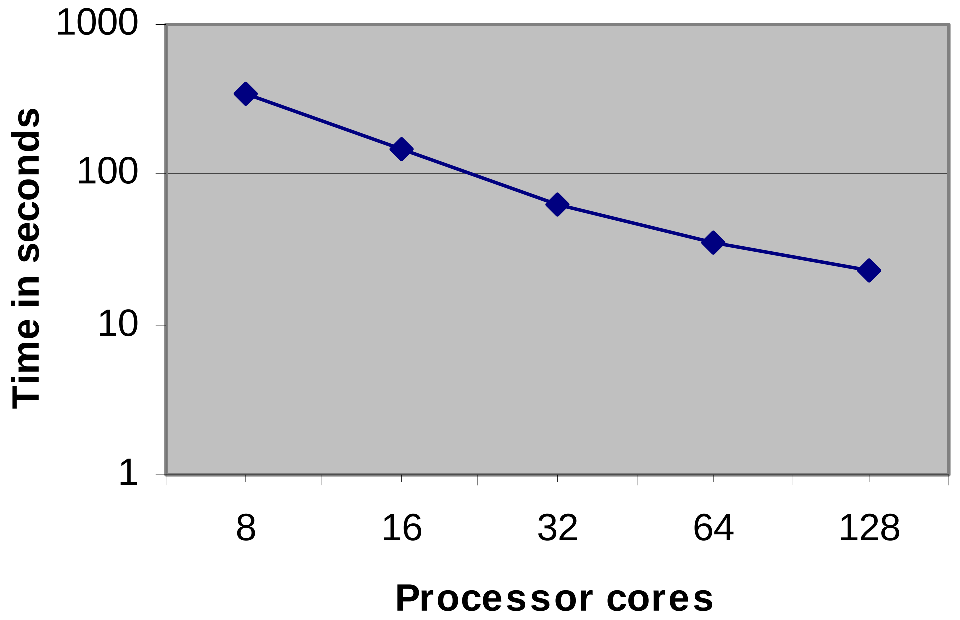


DSC Method (3)

Preconditioning within the DSC algorithm



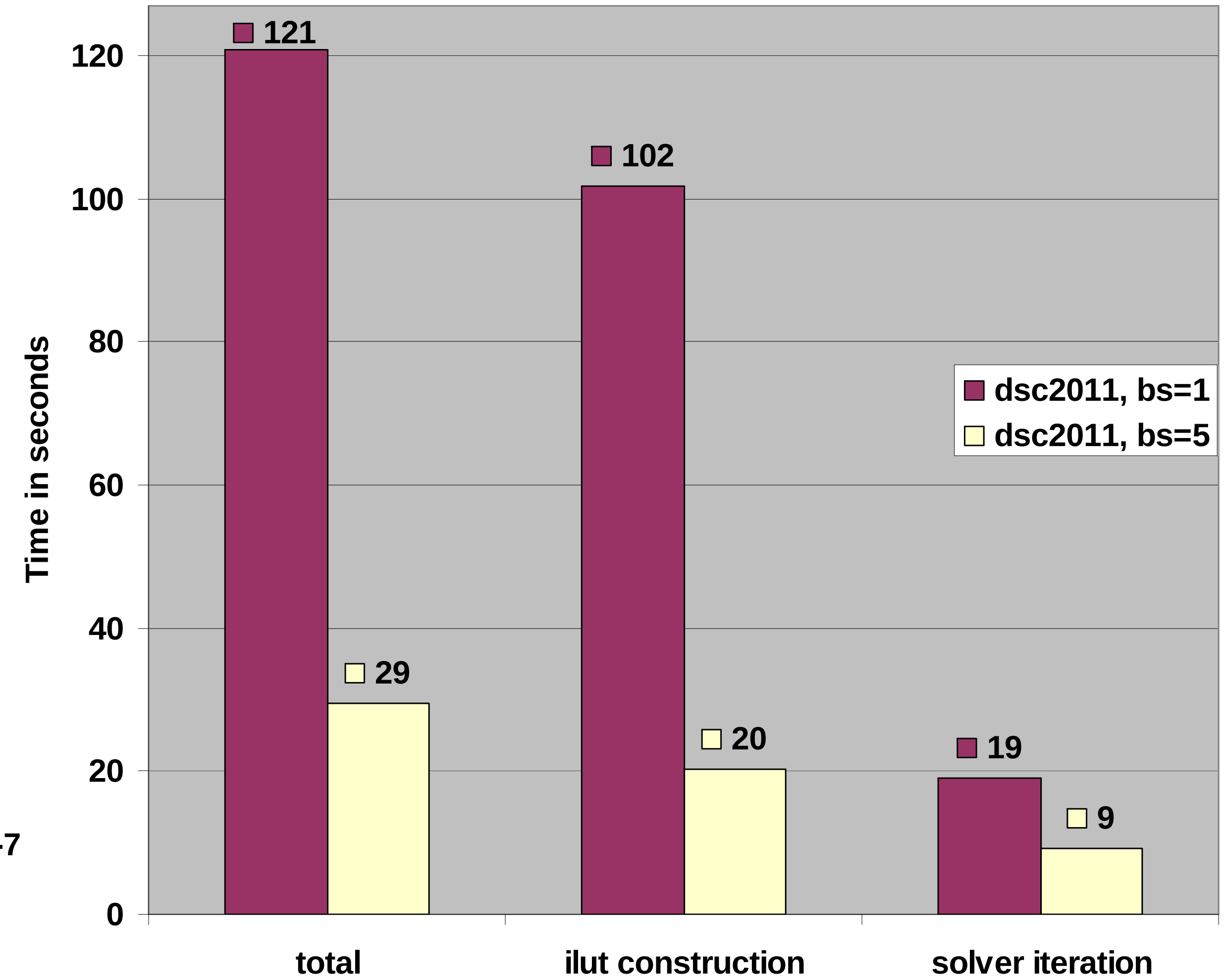
DSC Method: Strong Scaling (CSR, real)
(Dual-processor nodes; Quad-Core Intel Harpertown; 2.83 GHz)



TAU matrix: n=541,980; nz=170,610,950; threshold = 10^{-3}; |rel. residual| < 10^{-5}

DSC Method: CSR versus BCSR Format (real)
(2x Intel XEON E5520 with 4 cores each, 2.26 GHz)

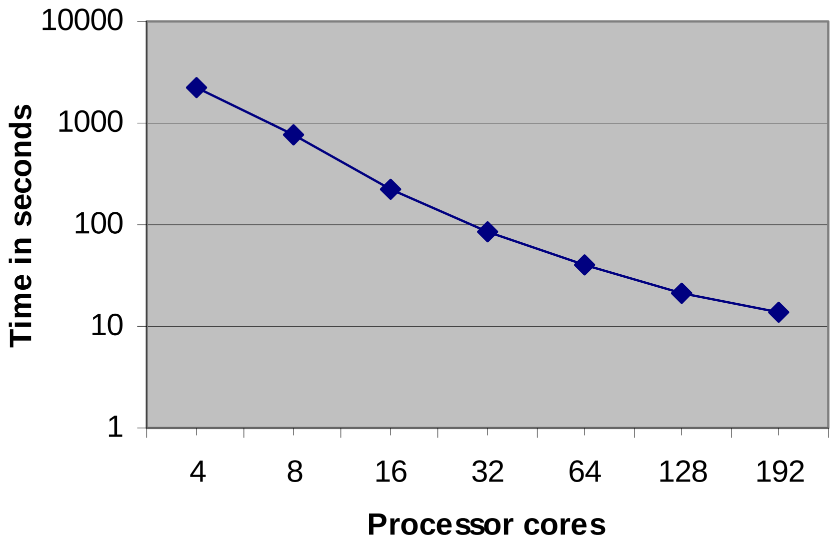
Results on 8 cores



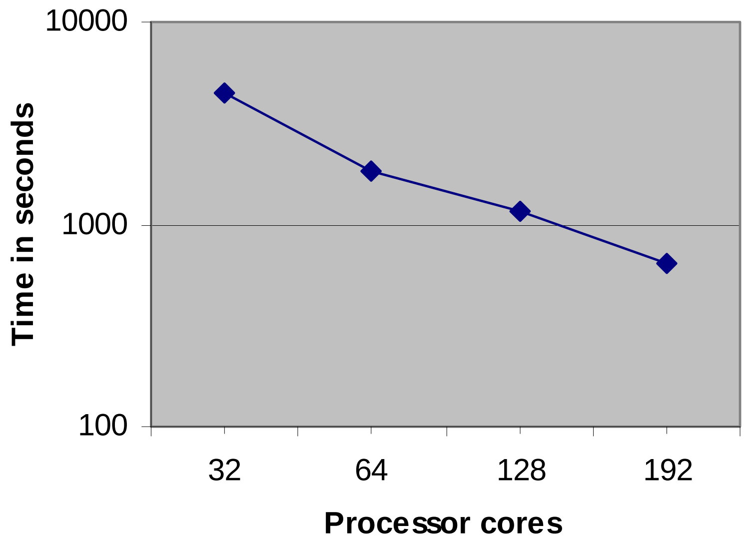
TAU matrix:
n=541,980;
nz=170,610,950;
threshold = 10^{-3};
|rel. residual| < 10^{-7}

DSC Method: Strong Scaling (CSR, complex)

(Dual-processor nodes; Quad-Core Intel Harpertown; 2.83 GHz)



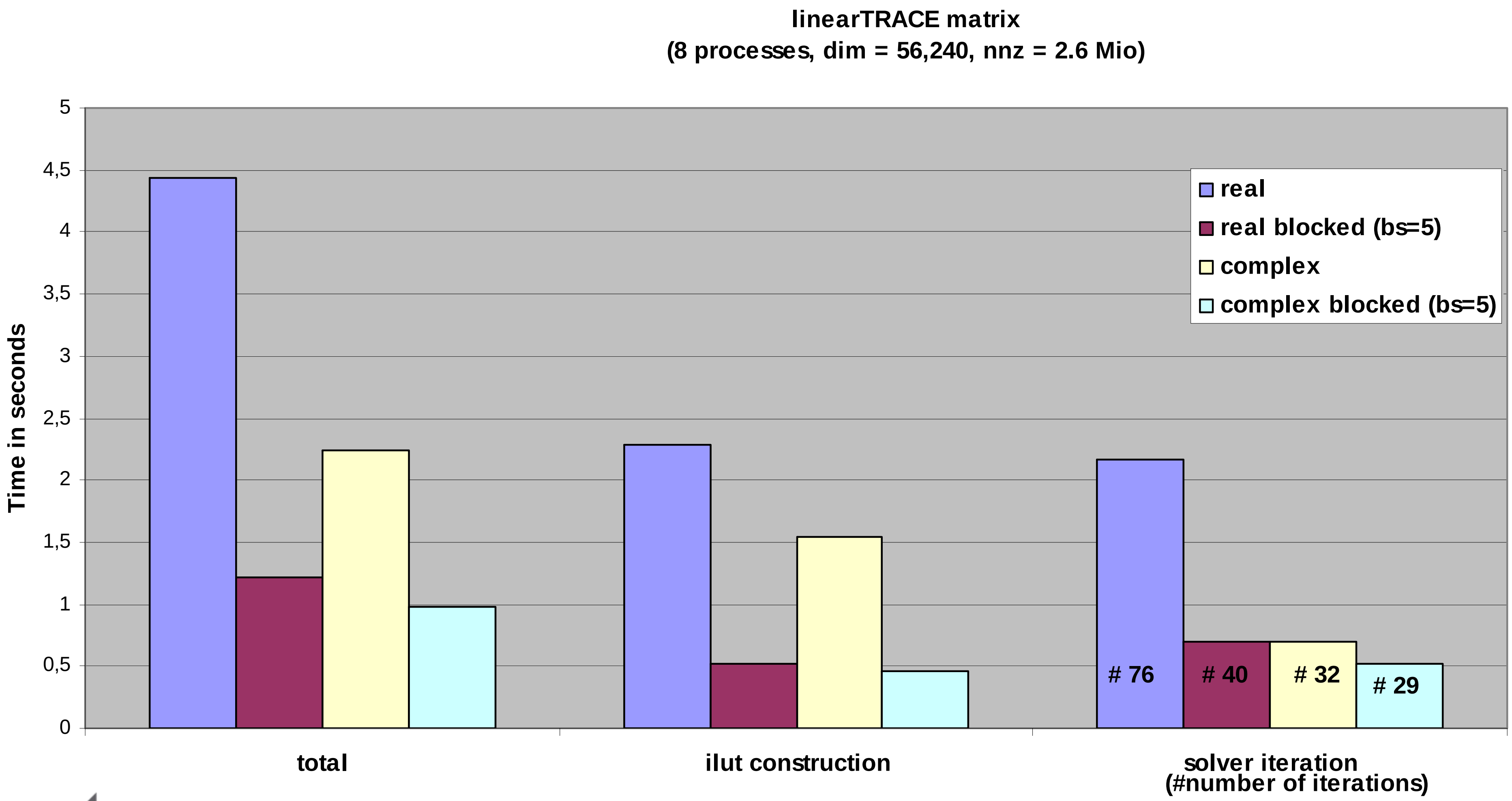
TRACE matrix THD
(n=378,400; nz=45,456,500;
threshold = 10^{-3}; |rel. residual| < 10^{-5})



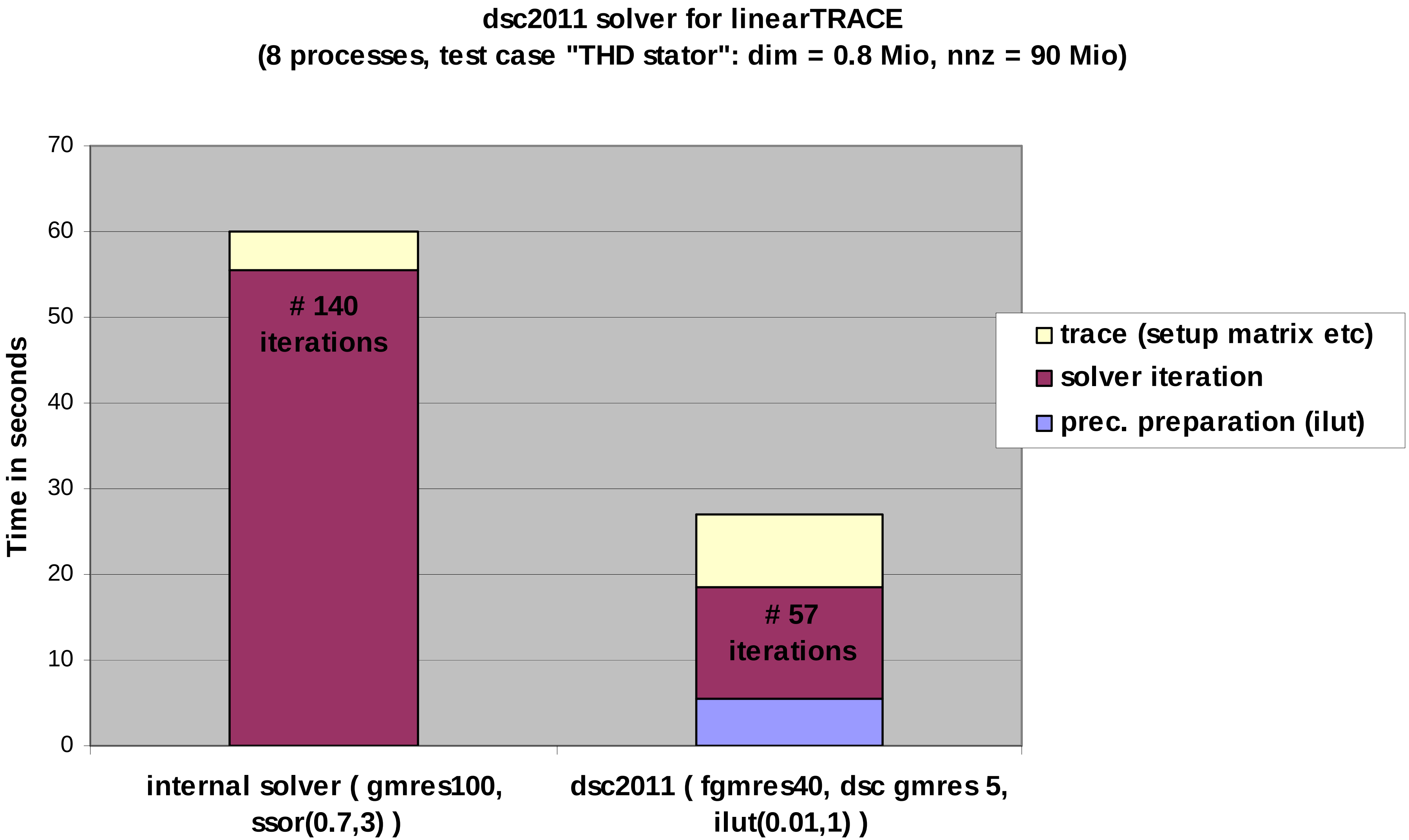
TRACE matrix UHBR
(n=4,497,520; nz=552,324,700;
threshold = 10^{-3}; |rel. residual| < 10^{-10})

DSC Solver: CSR versus BCSR Format

(2x Intel XEON E5520 with 4 cores each, 2.26 GHz)



linearTRACE Performance: Internal versus DSC Solver

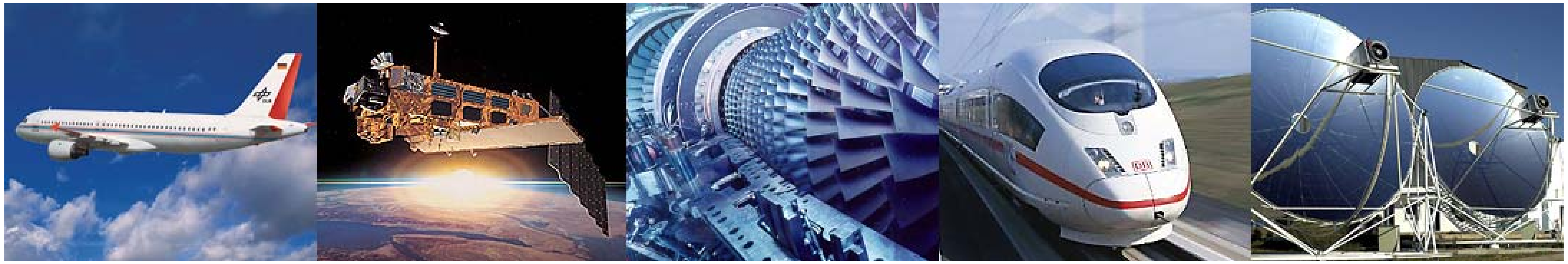


Conclusions

- Complex computations distinctly faster than real ones
- BCSR format application significantly outperforms CSR format application for TRACE and TAU problems.
- DSC method distinctly faster than block-local methods
- DSC method very suitable for TRACE and TAU problems

Questions?

DLR
German Aerospace Center



- Research Institution
- Space Agency
- Project Management Agency

Locations and employees

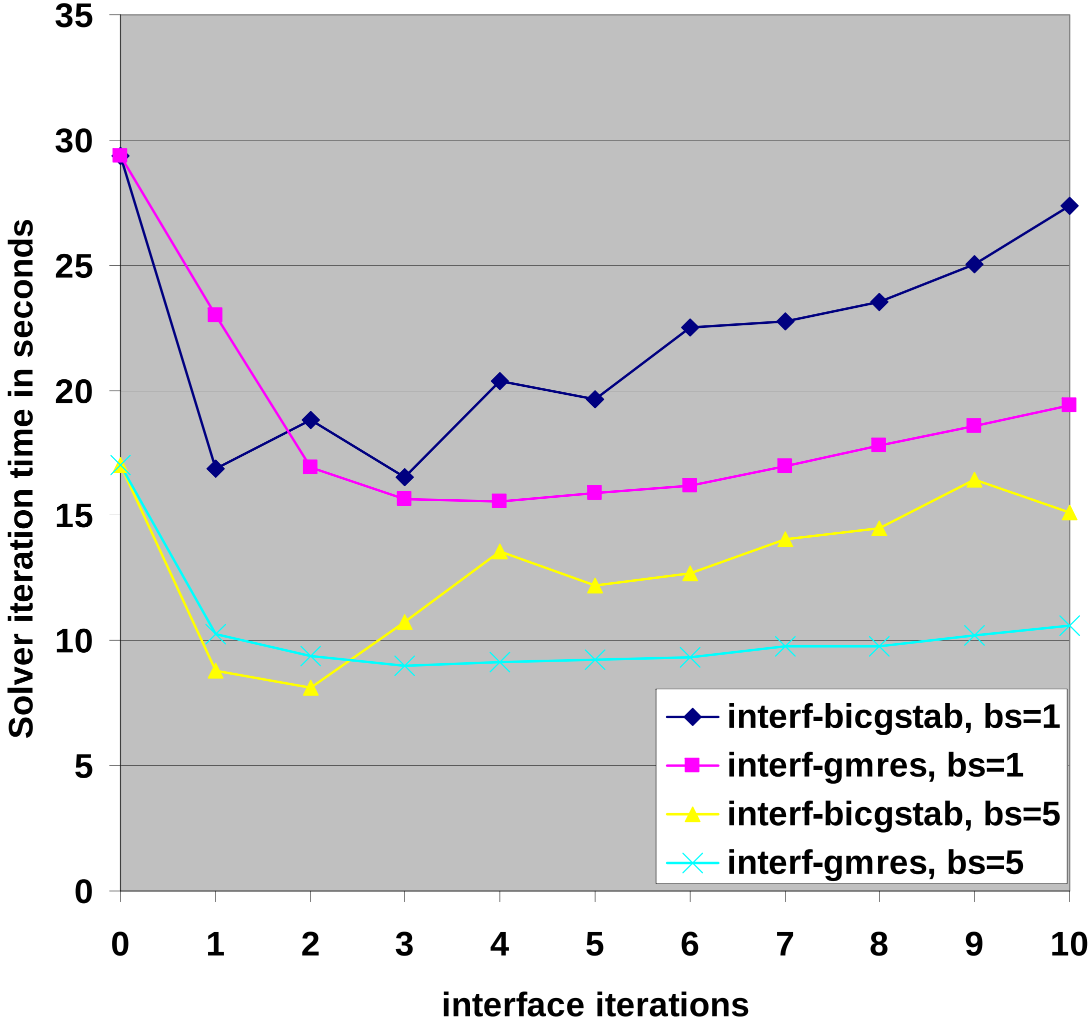
Germany: 6,900 employees across 33 research institutes and facilities at

- 15 sites.

Offices in **Brussels**, **Paris** and **Washington**.

DSC Method: Effect of the Interface Iteration (real)
(2x Intel XEON E5520 with 4 cores each, 2.26 GHz)

Results on 8 cores



TAU matrix:
n=541,980;
nz=170,610,950;
threshold = 10^{-3};
|rel. residual| < 10^{-7}