Assessment of resolution quality for LES

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Outline

Introduction. Motivation

Description of the simulation method DLR THETA code

Steps toward reliable assessment of quality of LES results using so-called single-grid estimators (or: sensor) for resolution quality of LES

Sensor 1 based on resolved turbulent kinetic energy

Sensor 2 based on resolved shear stress

Conclusion. Outlook
Introduction

- During last few years, strongly growing tendency and interest to apply LES-type methods (wall-modelled LES, DES) to flows at **high-Reynolds number** in **complex geometries**
- Even for simple test case, e.g. channel flow at $Re_T=395$, it is well known that
  - Quality of LES results strongly depends on proper time step size and mesh design
  - “Best practice ranking” of different SGS models can only be done on sufficiently fine mesh (Kravchenko and Moin, JCP 1997, Morinishi & Vasiljev 2001)
  - Overall observation: Sensitivity of LES results on numerical resolution (temporal and spatial) is much larger than for RANS modelling
- However: trends how to use LES in industry and research are opposite
  - increase amount of small-scale geometric details
  - increase large-scale complexity of the test-cases (e.g., DES/LES of full aircraft, full car)
  - But: Question of proper spatial and temporal resolution often not addressed
- For complex configurations of industrial relevance, a convergence study using **global mesh refinement** is **prohibitively expensive**
- Remedy: **Local mesh refinement** in most important regions (boundary layers, free shear layers,…)
- Ultimate goal: **Automatic grid refinement using single-grid estimators as refinement indicators** (sensors) to ensure a proper resolution quality of LES
Numerical method and LES modelling

- DLR THETA Code
  - Unstructured finite-volume solver for flows with small compressibility effects
  - Projection scheme using the interpolation scheme by Rhie and Chow
  - Discretization of convective fluxes using central differencing scheme (CDS)
  - Time discretization using 2nd order backward differencing formula (BDF-2)
  - Subgrid-scale models: Smagorinsky model (with van Driest damping), WALE model

Demonstration of LES capability of THETA code using the standard Smagorinsky model:

Experiment by Comte-Bellot for decaying isotropic turbulence
Resolution requirements for wall-resolved LES at low Re

Turbulent channel flow \( Re_T = 395 \)

- Motivation: Wall-resolved LES avoids possible additional problems (e.g., „log-layer mismatch“) due to near-wall modelling
- Required time step size: \( \delta t^+ = \delta t u_T^2/\nu = 0.4 \) (precursor study, Choi & Moin JCP 1994)
- Insufficient resolution even on 64x64x64
- Only on 96x96x96 mesh, results are close to DNS data
- No simulation on 128³ mesh yet

Standard Smagorinsky model with van Driest damping
Resolution requirements for wall-modelled LES at high Re
Turbulent channel flow $Re_T = 4800$

➤ First step: Investigation of time-discretisation error
➤ Standard Smagorinsky model and hybrid wall functions as near-wall model, matching node at $y^+=50$
➤ Too large time step causes log-layer mismatch
➤ Required time step size
\[ \delta t^+ = \delta t \frac{u_\tau^2}{\nu} = 1.75 \]

Standard Smagorinsky model with van Driest damping and hybrid wall functions as near-wall model
Resolution requirements for wall-modelled LES at high Re
Turbulent channel flow $Re_T = 4800$

- Second step: Investigation of spatial resolution on three computational meshes
- "Convergence" of mean and fluctuation profiles on the 128x24x128 mesh
- This corresponds to $\Delta x^+=235$, $\Delta z^+=117$
- This is in agreement with best-practice guidelines, e.g., Sagaut (2000)

$\Rightarrow$ Regarding spatial & temporal resolution requirements, we have best practice guidelines for attached boundary layer flows
$\Rightarrow$ However: Are these guidelines still valid for strongly accelerated/decelerated flow?
Question: What are proper resolution requirements for free shear layers and regions of separated flow?

- **Aim:** Assess the resolution quality of LES in regions of free shear layers and separated flow
- **Basic test case:** Flow over backward facing step by Driver & Seegmiller
Idea of single-grid estimator (refinement indicator) for LES

- Single-grid estimator as refinement indicator (sensor) to measure the resolution quality
- \( S \) is a map: \( \text{Grid} \rightarrow [0,1] \), for each grid point \( x \rightarrow S(x) \) in \([0,1]\)

- Aim: Define
  - \textbf{sensor} \( S \) being a function of the discrete numerical solution and
  - \textbf{threshold values} \( s_1, s_2 \) in \((0,1)\)
    such that
  - \( S(x) < s_1 \) indicates that resolution is not sufficient for node \( x \)
  - \( S(x) > s_2 \) indicates that resolution is fine enough for node \( x \)

- First conceptual proposal of indicator for resolved turbulent kinetic energy by Pope (2000)
- First operational proposals for indicator by Celic (2005), Klein (2005): Both the contribution of SGS model and the numerical error are considered.
- Present approach: Consider only contribution of resolved scales and subgrid scales
Single-grid estimators for LES

- Refinement indicator (sensor) to measure the resolution quality of the LES
- Indicator for resolved turbulent kinetic energy based on idea by Pope (2000)
  - Scale similarity assumption used for the subgrid scale turbulent kinetic energy

\[
S_k(x) = \frac{k}{k + k_{sgs}}, \quad k = \frac{1}{2} \langle (u - \langle u \rangle)^2 \rangle, \quad k_{sgs} = \frac{1}{2} \langle (u - u)^2 \rangle \\
= u_{resolved}^t \approx u_{sgs}^t
\]

\[
\bar{u}(x, t) = \int_{\mathbb{R}^d} g_\Delta(x - y) u(y, t) dy \quad g_\Delta : \text{top hat filter function}
\]

Note: So far no special treatment of the filtering operator near walls and no study of the role of the specific filter function

- Indicator for resolved shear stress

\[
S_s(x) = \frac{\tau}{\tau + \tau_{sgs}}, \quad \tau = \langle u'u' \rangle, \quad \tau_{sgs} = -\langle \nu_t \rangle \frac{du}{dy}.
\]
Evaluation of sensor for turbulent channel flow $Re_T = 395$

- Time step size: $\delta t^+ = \delta t\frac{u_1^2}{\nu} = 0.4$, meshes: 32x32x32, 48x48x48, 64x64x64, 96x96x96
- Insufficient resolution even on 64x64x64 mesh, $\Delta x^+=39, \Delta z^+=19.5$
- Only on 96x96x96 mesh, more than 90% of turbulent shear stress resolved corresponding to $\Delta x^+=26, \Delta z^+=13$

Standard Smagorinsky model

Indicator of resolved shear stress
Evaluation of sensor for turbulent channel flow $Re_T = 4800$

- Time step size: $\delta t^+ = \delta t \frac{u_T^2}{\nu} = 1.75$, meshes: 64x24x64, 96x24x96, 128x24x128
- Only on 128x24x128 mesh, more than 85% of turbulent shear stress resolved, corresponding to $\Delta x^+ = 235$, $\Delta z^+ = 117$

Study of space-discretisation error
Smag. model + wall functions

Indicator of resolved shear stress
Resolution requirements for flow over a backward-facing step at Reₜₐ₉=37500 (experiment by Driver and Seegmiller)

- First step: Investigation of required time steps size
- Second step: Convergence study on successively refines grids
- Synthetic turbulence at inlet by Klein, Sadiki, Janicka (2003)

![Diagram showing flow behavior over a backward-facing step](image)

On finest mesh (219x89x32), resolution almost sufficient, but medium mesh (110x47x32) not fine enough

⇒ **Question**: Can this be seen also from the behaviour of the refinement indicators?
Computational meshes for flow over backward facing step

Coarse mesh 78x31x16 nodes

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<td>420</td>
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<tr>
<td>Δz+</td>
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<td>260</td>
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Medium mesh 110x47x32 nodes

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<td>220</td>
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<tr>
<td>Δz+</td>
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Computational meshes for flow over backward facing step

**Fine mesh 169x71x32 nodes**

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<tr>
<td>∆z+</td>
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**Very fine mesh 219x89x32 nodes**

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<td>∆z+</td>
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</table>
Single-grid estimator for resolved turbulent kinetic energy

$$S_k(x) = \frac{k}{k + k_{sgs}} , \quad k = \frac{1}{2} \langle (u - \langle u \rangle)^2 \rangle , \quad k_{sgs} = \frac{1}{2} \langle (u - \bar{u})^2 \rangle$$

Coarse mesh

Medium mesh
Single-grid estimator for resolved turbulent kinetic energy

\[ S_k(x) = \frac{k}{k + k_{sgs}}, \quad k = \frac{1}{2} \langle (\mathbf{u} - \langle \mathbf{u} \rangle)^2 \rangle, \quad k_{sgs} = \frac{1}{2} \langle (\mathbf{u} - \bar{\mathbf{u}})^2 \rangle \]

Fine mesh

Very fine mesh
Steps toward a reliable operational single-grid estimator

- Indicator values for $S_k$ increase monotoneously if refining the mesh (this is obviously a necessary requirement) √

- How to choose threshold values $s_1$, $s_2$?
  - $S(x) < s_1$ indicates that resolution is insufficient for control volume at $x$
  - $S(x) > s_2$ indicates that resolution is fine enough for control volume at $x$

- In order to **calibrate the single-grid estimator**, we use findings of convergence study using global mesh refinement indicating that sufficient „convergence“ has been achieved only on the finest mesh

  ⇒ This suggests to choose threshold value $s_2 = 0.9$
Conclusion. Outlook

- Reliable LES results require to avoid insufficient resolution due to too coarse mesh and too coarse time step
  - Implication: Calibration of model parameters and best-practice guidelines on subgrid-scale models only after reliable results in terms of numerical error have been obtained
- Assessment of refinement indicators (sensors) to measure the resolution quality of LES by performing a study of mean- and fluctuation flow quantities on successively refined meshes
  - Sensor based on resolved turbulent kinetic energy appears to be suited for free-shear layers and regions of separated flows
  - Sensor based on resolved turbulent shear stress seems to be suited for regions of attached boundary layers
- Next steps:
  - Blending between sensor based on sensor based on resolved shear stress (in regions of attached BLs) and resolved turbulent kinetic energy (else)
  - Application of refinement indicators to testcases of larger complexity
  - Use refinement indicators for automatic grid refinement for LES
Steps toward reliable assessment of quality of LES

- **Step 1**: Preliminary convergence study:
  - Step 1a: Investigation of time-discretization error
  - Step 1b: Investigate spatial discret. error: Simulations on globally refined meshes

- **Step 2**: Presentation of single-grid estimators (called sensor).
  Aim: Given a grid and a LES solution, assess its resolution quality

- **Step 3**: Evaluation and calibration of the sensors
  - Step 3a: Evaluation of single-grid estimators (sensors) for test-cases
  - Step 3b: Calibration by comparison between predictions of single-grid estimators and grid convergence study using globally refined meshes
Indicator of resolved turbulent kinetic energy

**Shortcoming:** Indicator of resolved turbulent kinetic energy not suited for attached boundary layers

⇒ Underresolution causes large overprediction of streamwise component $<u'^2>$ which dominates the corresponding underresolution of $<v'^2>$ and $<w'^2>$

- Not very clear indication of improved resolution in region of attached boundary layer
  
  + Sensor indicates of increased resolution of turbulent kinetic energy in the region of free shear layer and recirculating flow

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Combination with indicator of resolved turbulent shear stress

Sensor of resolved turbulent shear stress suitable for attached boundary layers but not for regions of free-shear layers and recirculating flow

⇒ Combination of both sensors for operational single-grid estimator

Sensor indicates an increased resolution of turbulent shear stress in the attached boundary layer

Not very clear indication of improved resolution in region of free-shear layer and recirculating flow

Mesh refinement