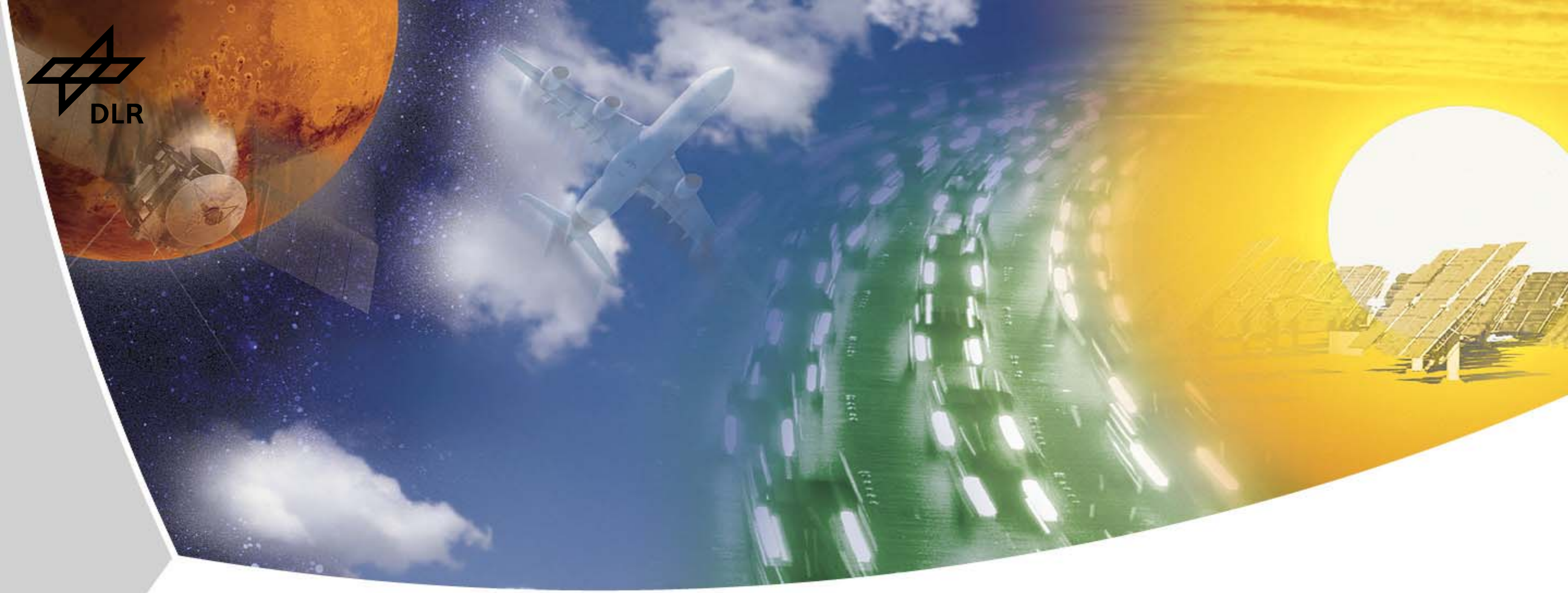

Assessment of resolution quality for LES

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Outline

- Introduction. Motivation
- Description of the simulation method DLR THETA code
- Steps toward reliable assessment of quality of LES results using so-called single-grid estimators (or: sensor) for resolution quality of LES
    - Sensor 1 based on resolved turbulent kinetic energy
    - Sensor 2 based on resolved shear stress
- Conclusion. Outlook

Introduction

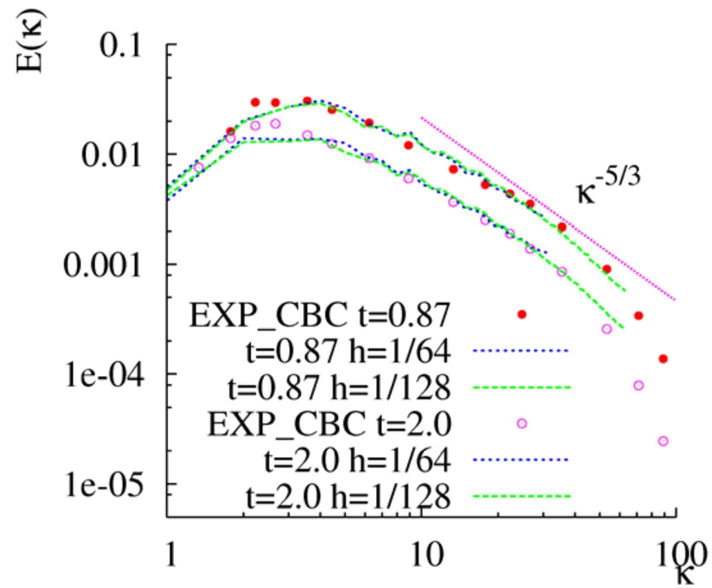
- During last few years, strongly growing tendency and interest to apply LES-type methods (wall-modelled LES, DES) to flows at **high-Reynolds number** in **complex geometries**
- Even for simple test case, e.g. channel flow at Re_τ=395, it is well known that
  - Quality of LES results strongly depends on proper time step size and mesh design
  - “Best practice ranking” of different SGS models can only be done on sufficiently fine mesh (Kravchenko and Moin, JCP 1997, Morinishi & Vasiljev 2001)
  - Overall observation: Sensitivity of LES results on numerical resolution (temporal and spatial) is much larger than for RANS modelling
- However: trends how to use LES in industry and research are opposite
  - increase amount of small-scale geometric details
  - increase large-scale complexity of the test-cases (e.g., DES/LES of full aircraft, full car)
  - But: Question of proper spatial and temporal resolution often not addressed
- For complex configurations of industrial relevance, a convergence study using **global mesh refinement** is **prohibitively expensive**
- Remedy: **Local mesh refinement** in most important regions (boundary layers, free shear layers,…)
- Ultimate goal: **Automatic grid refinement using single-grid estimators as refinement indicators** (sensors) to ensure a proper resolution qualitiy of LES

Numerical method and LES modelling

- DLR THETA Code
  - Unstructured finite-volume solver for flows with small compressibility effects
  - Projection scheme using the interpolation scheme by Rhie and Chow
  - Discretization of convective fluxes using central differencing scheme (CDS)
  - Time discretization using 2nd order backward differencing formula (BDF-2)
  - Subgrid-scale models: Smagorinsky model (with van Driest damping), WALE model

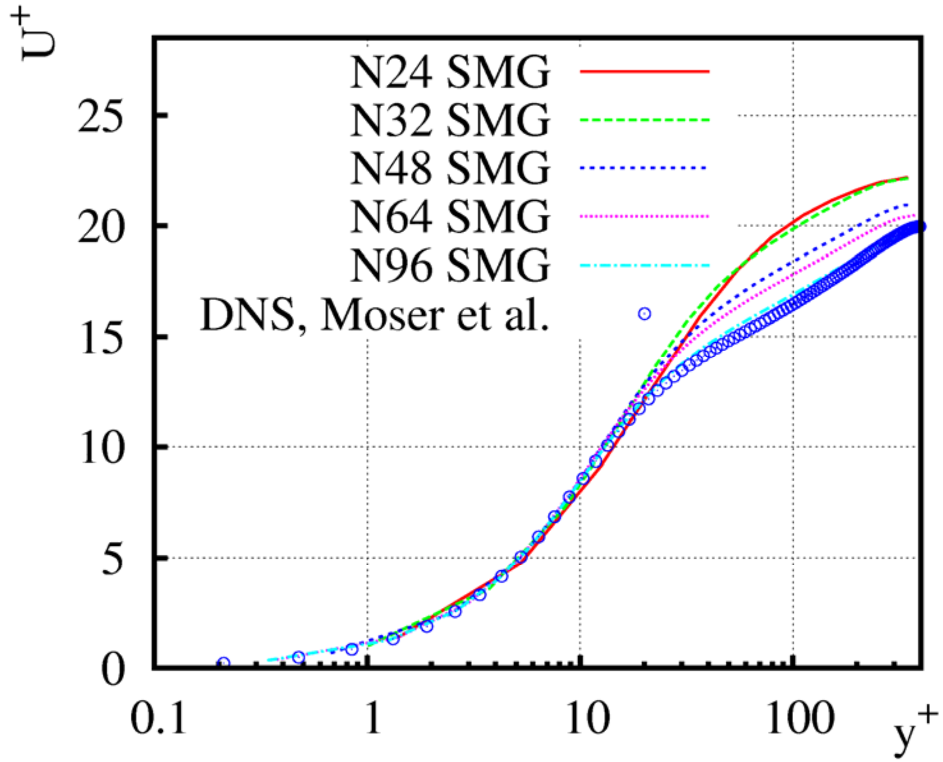
Demonstration of LES capability of THETA code using the standard Smagorinsky model:

Experiment by Comte-Bellot for decaying isotropic turbulence



Resolution requirements for wall-resolved LES at low Re
Turbulent channel flow $Re_\tau = 395$

- Motivation: Wall-resolved LES avoids possible additional problems (e.g., „log-layer mismatch“) due to near-wall modelling
- Required time step size : $\delta t^+ = \delta t\, u_\tau^2/\nu = 0.4$ (precursor study, Choi & Moin JCP 1994)
- Insufficient resolution even on 64x64x64
- Only on 96x96x96 mesh, results are close to DNS data
- No simulation on 128^3 mesh yet

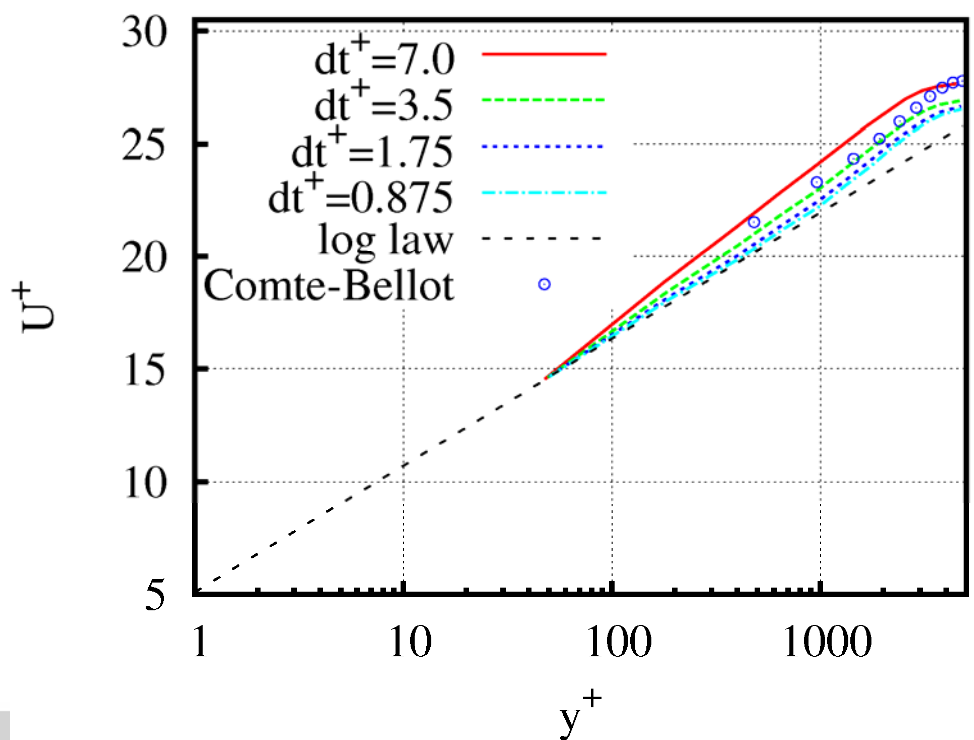


Standard Smagorinsky model with van Driest damping

Resolution requirements for wall-modelled LES at high Re
Turbulent channel flow Re_τ = 4800

- First step: Investigation of time-discretisation error
- Standard Smagorinsky model and hybrid wall functions as near-wall model, matching node at y^+=50
- Too large time step causes log-layer mismatch
- Required time step size

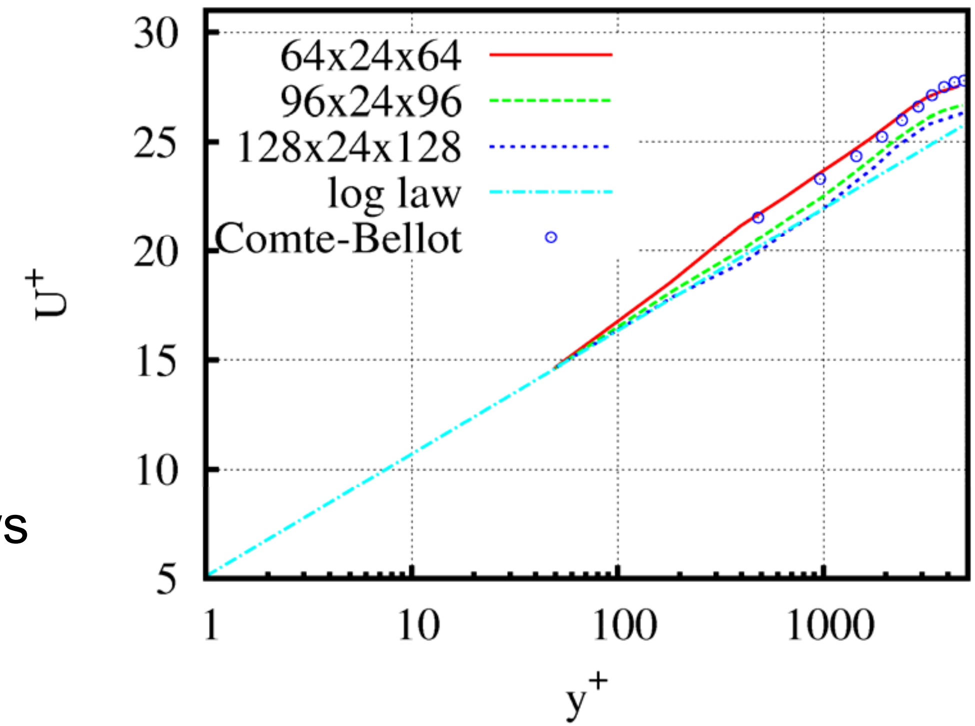
  $\delta t^+ = \delta t\, u_\tau^2/\nu = 1.75$



Standard Smagorinsky model with van Driest damping and hybrid wall functions as near-wall model

Resolution requirements for wall-modelled LES at high Re
Turbulent channel flow $Re_\tau = 4800$

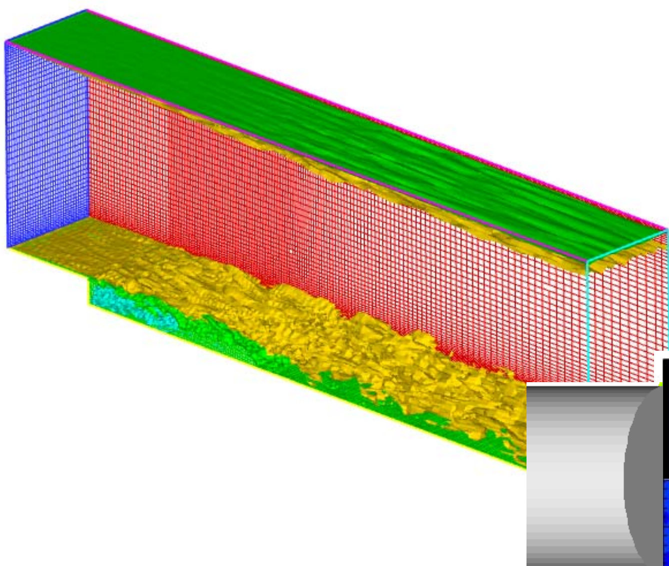
- Second step: Investigation of spatial resolution on three computational meshes
- „Convergence“ of mean and fluctuation profiles on the 128x24x128 mesh
- This corresponds to $\Delta x^+=235$, $\Delta z^+=117$
- This is in agreement with best-practice guidelines, e.g., Sagaut (2000)



⇒ Regarding spatial & temporal resolution requirements, we have best practice guidelines for attached boundary layer flows
⇒ However: Are these guidelines still valid for strongly accelerated/decelerated flow?

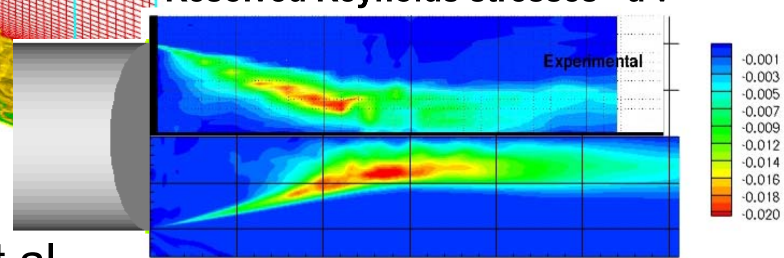
Question: What are proper resolution requirements for free shear layers and regions of separated flow?

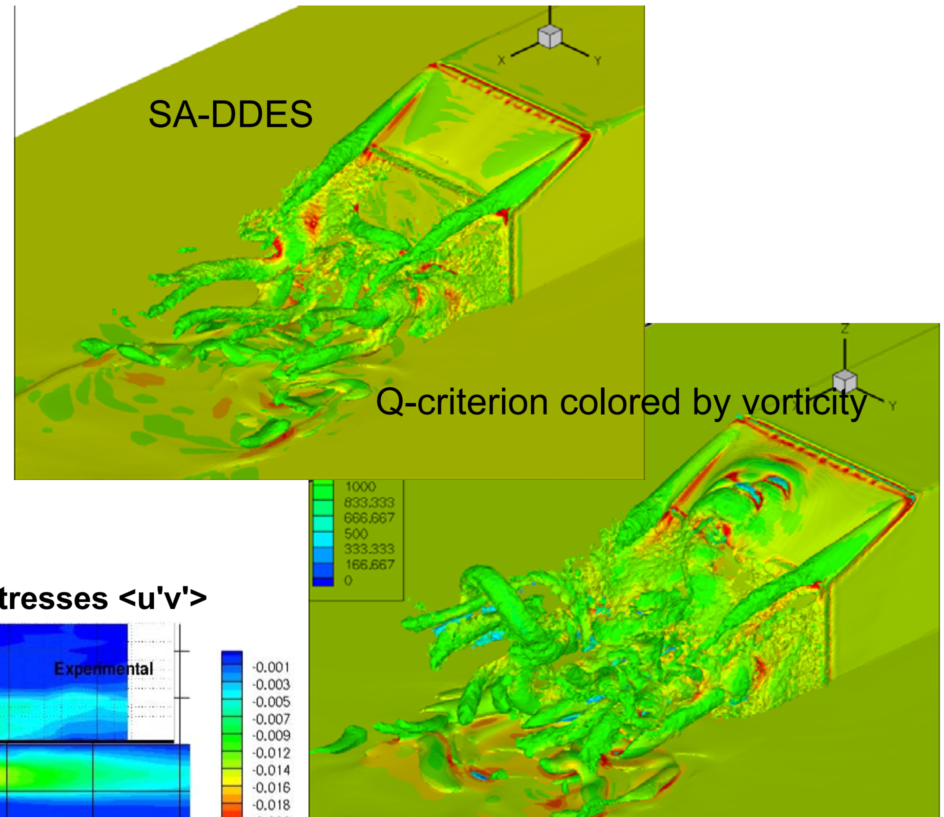
- **Aim: Assess the resolution quality of LES in regions of free shear layers and separated flow**
- Basic test case: Flow over backward facing step by Driver & Seegmiller



Courtesy by Togiti et al.

Resolved Reynolds stresses <u'v'>





Courtesy by Schwamborn

Idea of single-grid estimator (refinement indicator) for LES

- Single-grid estimator as refinement indicator (sensor) to measure the resolution qualitiy
- S is a map: Grid → [0,1], for each grid point x → S(x) in [0,1]

- Aim: Define
  - **sensor S** being a function of the discrete numerical solution and
  - **threshold values s_1, s_2** in (0,1)

  such that
- $S(x) < s_1$ indicates that resolution is not sufficient for node x
- $S(x) > s_2$ indicates that resolution is fine enough for node x

- First conceptual proposal of indicator for resolved turbulent kinetic energy by Pope (2000)
- First operational proposals for indicator by Celic (2005), Klein (2005): Both the contribution of SGS model and the numerical error are considered.
- Present approach: Consider only contribution of resolved scales and subgrid scales

Single-grid estimators for LES

- Refinement indicator (sensor) to measure the resolution qualitiy of the LES
- **Indicator for resolved turbulent kinetic energy** based on idea by Pope (2000)
  - Scale similarity assumption used for the subgrid scale turbulent kinetic energy

$$S_k(\boldsymbol{x}) = \frac{k}{k + k_{\text{sgs}}}, \quad k = \frac{1}{2}\langle(\underbrace{\boldsymbol{u} - \langle\boldsymbol{u}\rangle}_{= u'_{\text{resolved}}})^2\rangle, \quad k_{\text{sgs}} = \frac{1}{2}\langle(\underbrace{\boldsymbol{u} - \overline{\boldsymbol{u}}}_{\approx u'_{\text{sgs}}})^2\rangle$$

$$\overline{\boldsymbol{u}}(x,t) = \int_{\mathbb{R}^d} g_\Delta(\boldsymbol{x} - \boldsymbol{y})\boldsymbol{u}(\boldsymbol{y},t)\mathrm{d}\boldsymbol{y}$$

g_Δ : top hat filter function

Note: So far no special treatment of the filtering operator near walls and no study of the role of the specific filter function

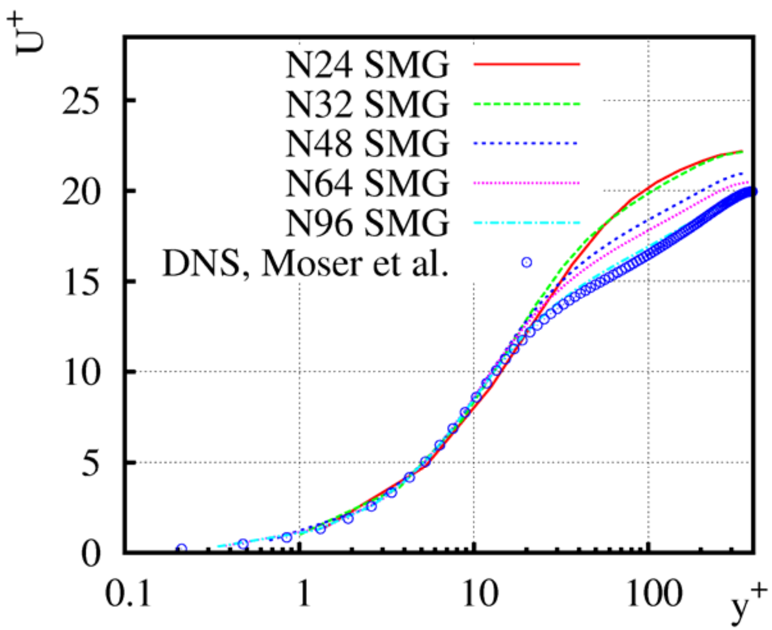
- **Indicator for resolved shear stress**

$$S_s(\boldsymbol{x}) = \frac{\tau}{\tau + \tau_{\text{sgs}}}, \quad \tau = \langle u'v'\rangle, \quad \tau_{\text{sgs}} = -\langle\nu_t\rangle\langle\frac{du}{dy}\rangle.$$

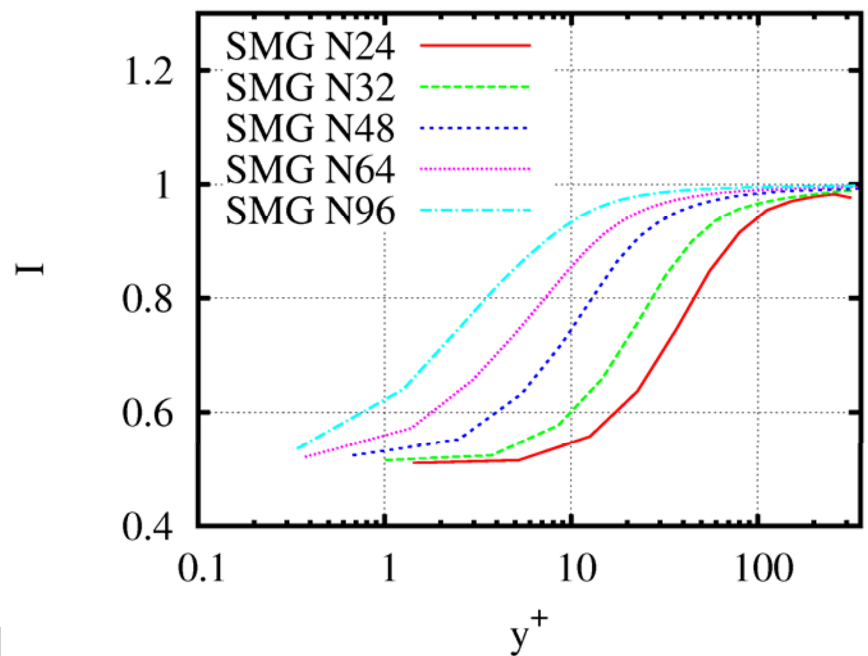
Evaluation of sensor for turbulent channel flow Re_τ = 395

- Time step size : $\delta t^+ = \delta t\, u_\tau^2/\nu = 0.4$, meshes: 32x32x32, 48x48x48, 64x64x64, 96x96x96
- Insufficient resolution even on 64x64x64 mesh, $\Delta x^+=39$, $\Delta z^+=19.5$
- Only on 96x96x96 mesh, more than 90% of turbulent shear stress resolved corresponding to $\Delta x^+=26$, $\Delta z^+=13$

Standard Smagorinsky model



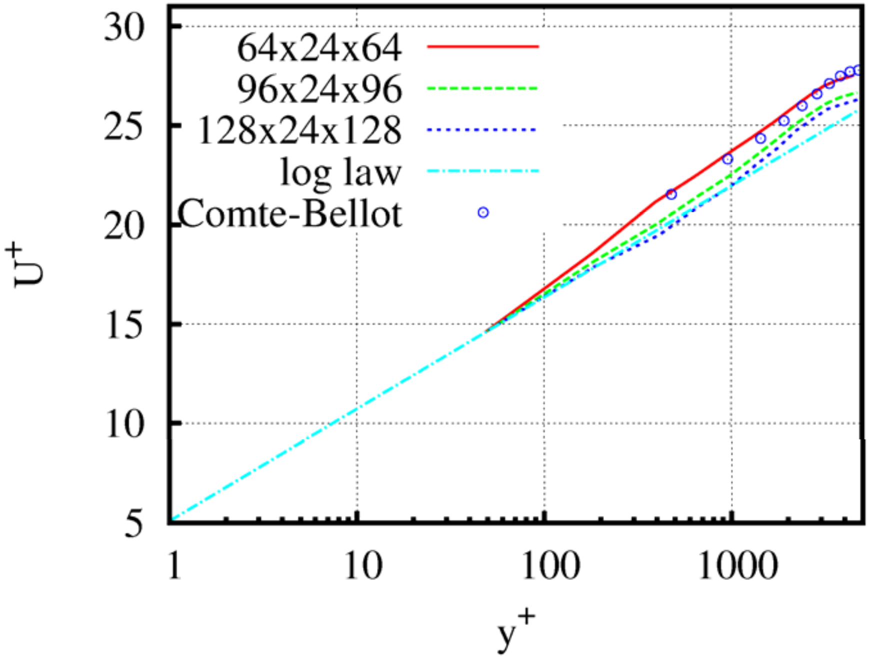
Indicator of resolved shear stress



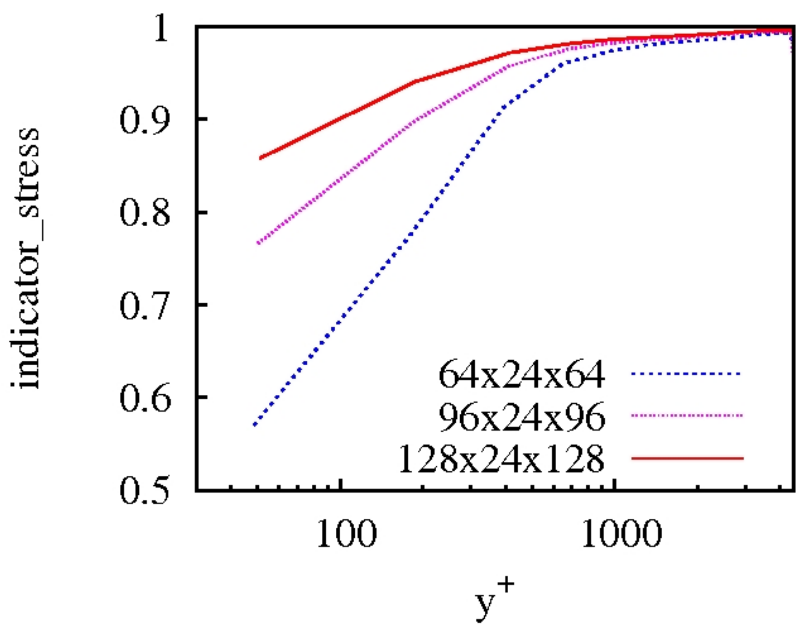
Evaluation of sensor for turbulent channel flow Re_τ = 4800

- Time step size : $\delta t^+ = \delta t\, u_\tau^2/\nu = 1.75$, meshes: 64x24x64, 96x24x96, 128x24x128
- Only on 128x24x128 mesh, more than 85% of turbulent shear stress resolved, corresponding to Δx^+=235, Δz^+=117

Study of space-discretisation error
Smag. model + wall functions

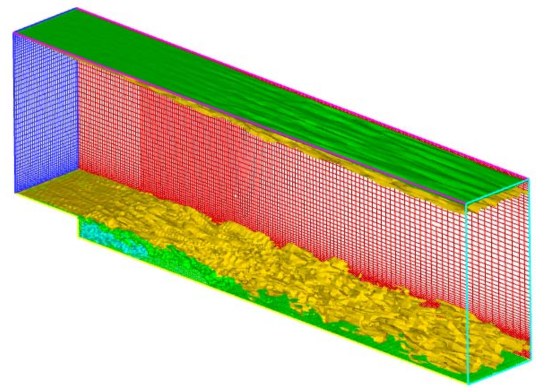


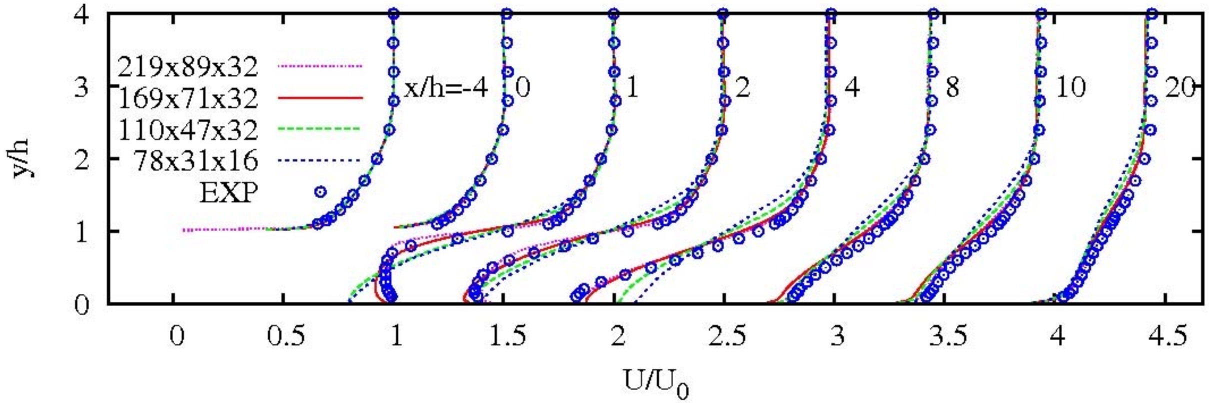
Indicator of resolved shear stress

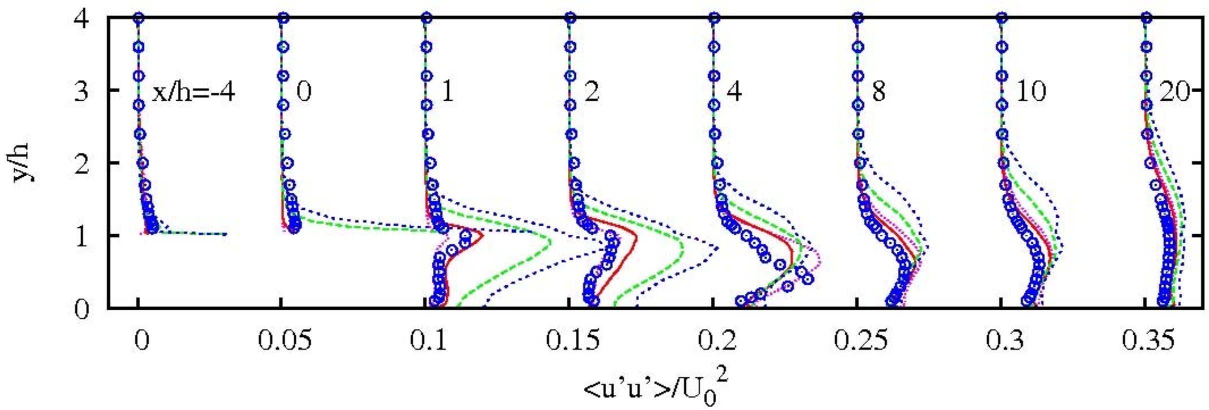


Resolution requirements for flow over a backward-facing step at Re_h=37500 (experiment by Driver and Seegmiller)

- First step: Investigation of required time steps size
- Second step: Convergence study on successively refines grids
- Synthetic turbulence at inlet by Klein, Sadiki, Janicka (2003)







On finest mesh (219x89x32), resolution almost sufficient, but medium mesh (110x47x32) not fine enough

⇒ **<u>Question</u>**: Can this be seen also from the behaviour of the refinement indicators?

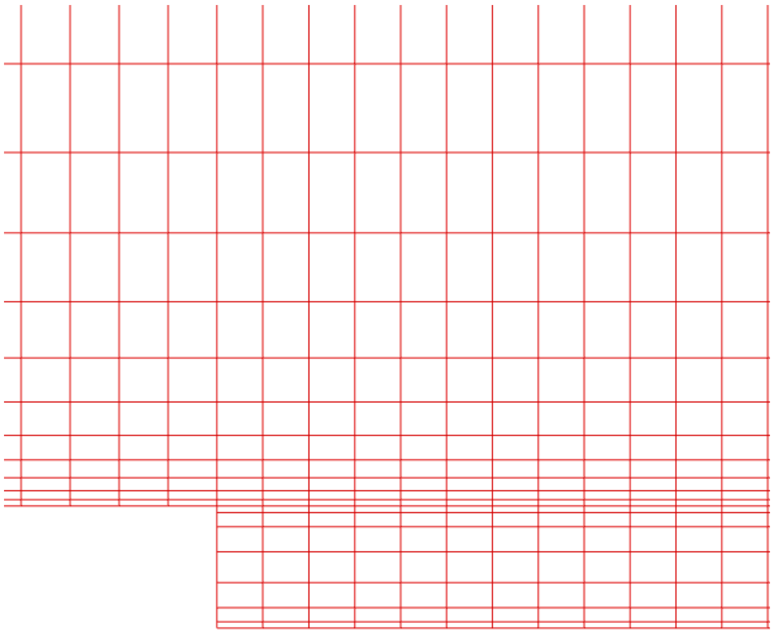
Computational meshes for flow over backward facing step

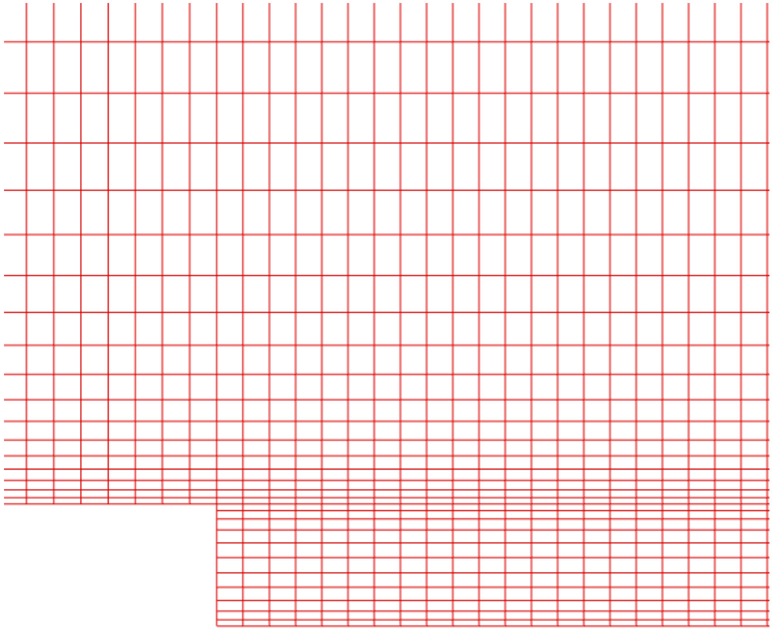
Coarse mesh 78x31x16 nodes

x/h	-0.5	10
Δx+	660	420
Δz+	300	260

Medium mesh 110x47x32 nodes

x/h	-0.5	10
Δx+	300	220
Δz+	150	130

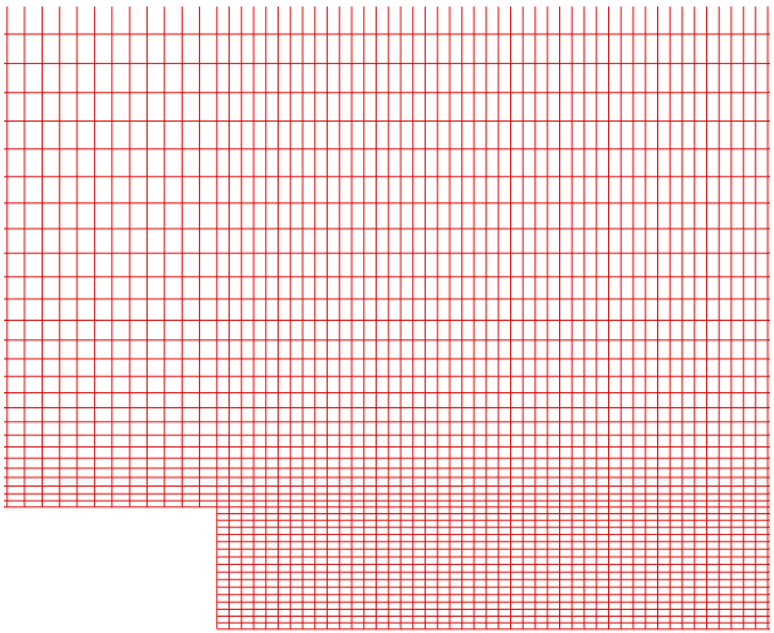




Computational meshes for flow over backward facing step

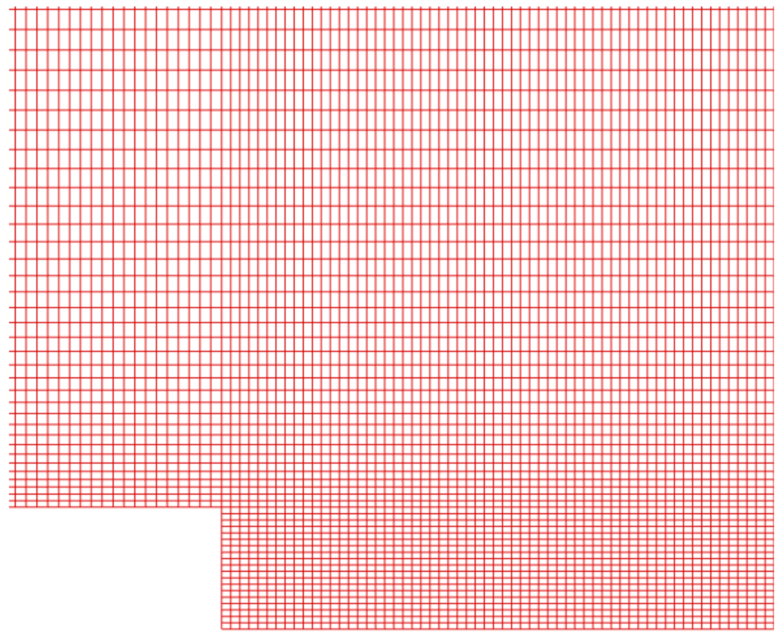
Fine mesh 169x71x32 nodes

x/h	-0.5	10
Δx+	180	180
Δz+	150	130



Very fine mesh 219x89x32 nodes

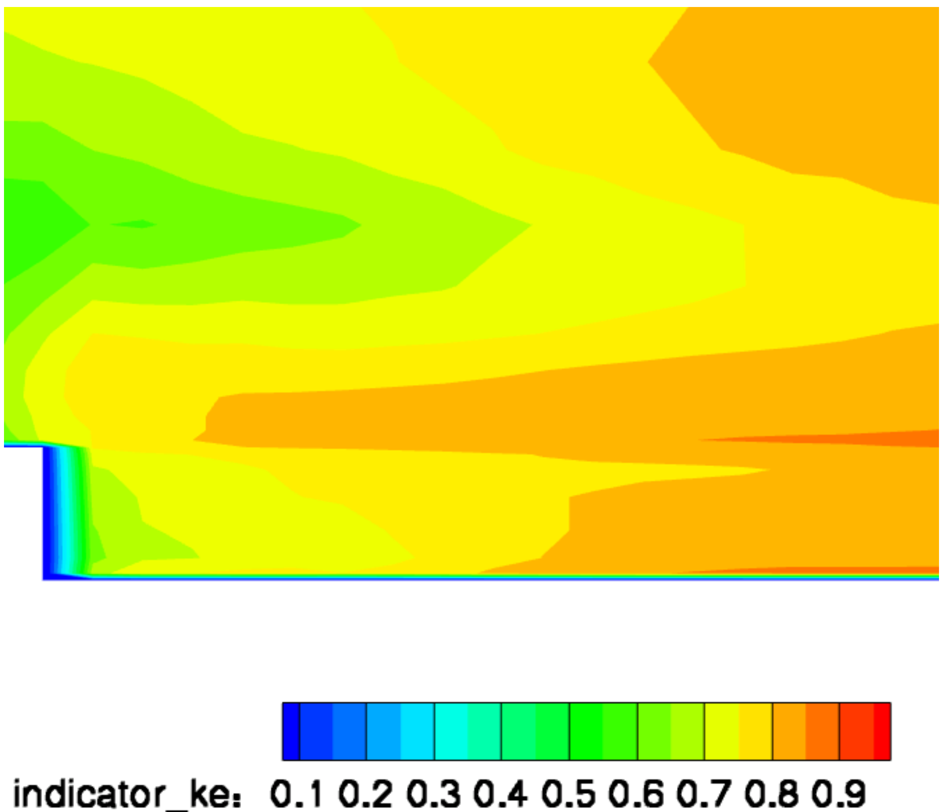
x/h	-0.5	10
Δx+	110	160
Δz+	150	130



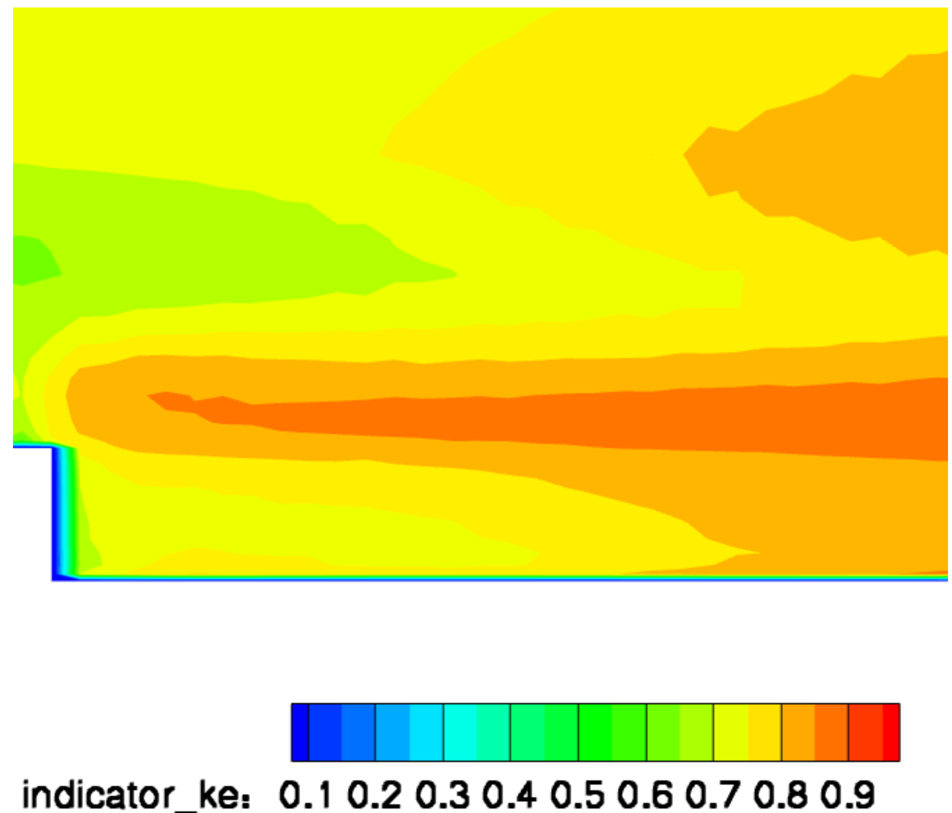
Single-grid estimator for resolved turbulent kinetic energy

$$S_k(\boldsymbol{x}) \;=\; \frac{k}{k + k_{\mathrm{sgs}}}\,, \qquad k = \frac{1}{2}\langle(\boldsymbol{u} - \langle\boldsymbol{u}\rangle)^2\rangle\,, \qquad k_{\mathrm{sgs}} = \frac{1}{2}\langle(\boldsymbol{u} - \overline{\boldsymbol{u}})^2\rangle$$

Coarse mesh



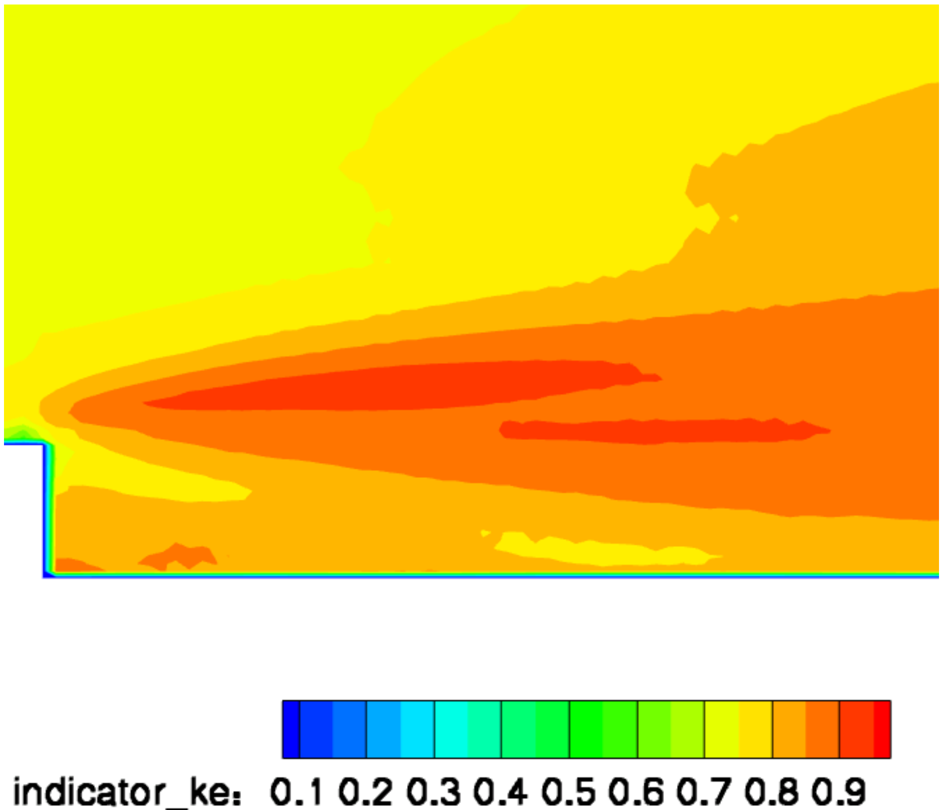
Medium mesh



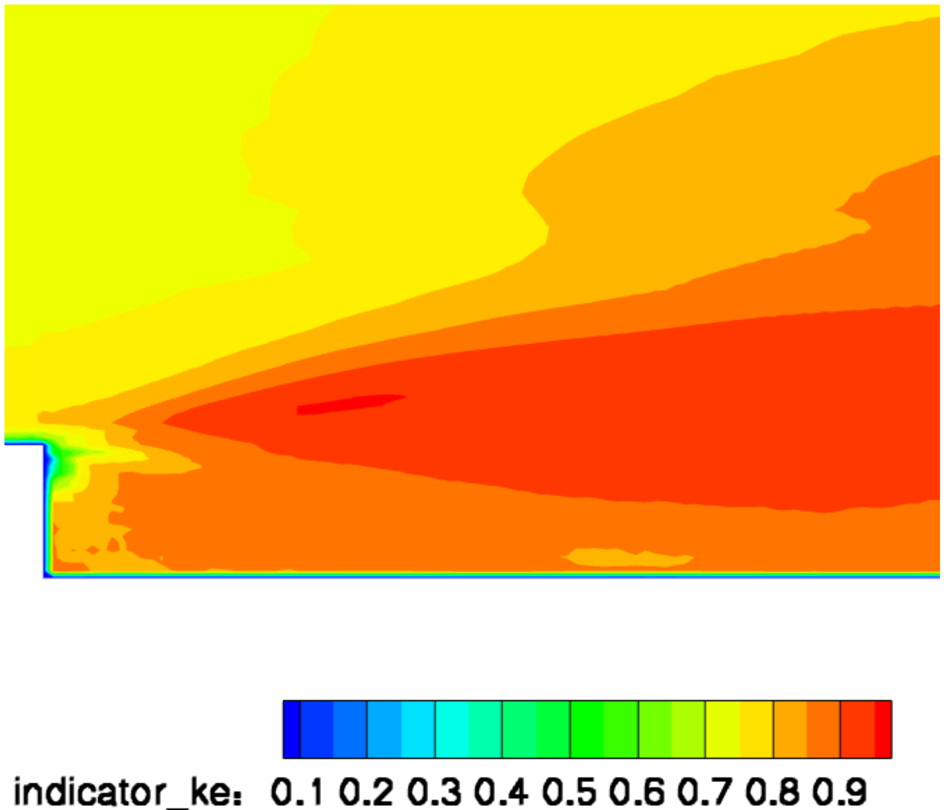
Single-grid estimator for resolved turbulent kinetic energy

$$S_k(\boldsymbol{x}) = \frac{k}{k + k_{\mathrm{sgs}}}\,, \qquad k = \frac{1}{2}\langle(\boldsymbol{u} - \langle\boldsymbol{u}\rangle)^2\rangle\,, \qquad k_{\mathrm{sgs}} = \frac{1}{2}\langle(\boldsymbol{u} - \overline{\boldsymbol{u}})^2\rangle$$

Fine mesh



Very fine mesh



Steps toward a reliable operational single-grid estimator

- Indicator values for S_k increase monotoneously if refining the mesh (this is obviously a necessary requirement) √
- How to choose threshold values s_1, s_2?
  - $S(x) < s_1$ indicates that resolution is insufficient for control volume at x
  - $S(x) > s_2$ indicates that resolution is fine enough for control volume at x
- In order to **calibrate the single-grid estimator**, we use findings of convergence study using global mesh refinement indicating that sufficient „convergence“ has been achieved only on the finest mesh

⇒ This suggests to choose threshold value s_2=0.9

Conclusion. Outlook

- Reliable LES results require to avoid insufficient resolution due to too coarse mesh and too coarse time step
  - Implication: Calibration of model parameters and best-practice guidelines on subgrid-scale models only after reliable results in terms of numerical error have been obtained
- Assessment of refinement indicators (sensors) to measure the resolution quality of LES by performing a study of mean- and fluctuation flow quantities on successively refined meshes
  - Sensor based on **resolved turbulent kinetic energy** appears to be suited for **free-shear layers and regions of separated flows**
  - Sensor based on **resolved turbulent shear stress** seems to be suited for **regions of attached boundary layers**

- Next steps:
  - Blending between sensor based on sensor based on resolved shear stress (in regions of attached BLs) and resolved turbulent kinetic energy (else)
  - Application of refinement indicators to testcases of larger complexity
  - Use refinement indicators for automatic grid refinement for LES

Steps toward reliable assessment of quality of LES

- **Step 1**: Preliminary convergence study:
  - Step 1a: Investigation of time-discretization error
  - Step 1b: Investigate spatial discret. error: Simulations on globally refined meshes
- **Step 2**: Presentation of single-grid estimators (called sensor).

  Aim: Given a grid and a LES solution, assess its resolution quality
- **Step 3**: Evaluation and calibration of the sensors
  - Step 3a: Evaluation of single-grid estimators (sensors) for test-cases
  - Step 3b: Calibration by comparison between predictions of single-grid estimators and grid convergence study using globally refined meshes

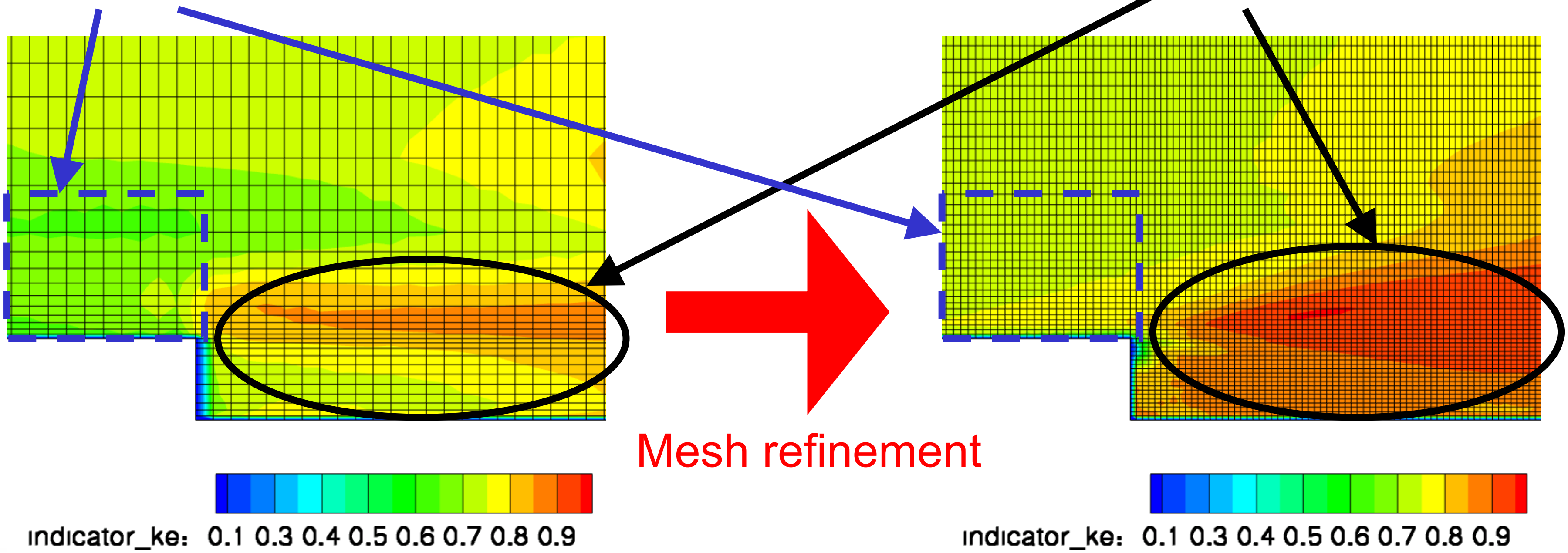
Indicator of resolved turbulent kinetic energy

Shortcoming: Indicator of resolved turbulent kinetic energy not suited for **attached boundary layers**

$\Rightarrow$ Underresolution causes large overprediction of streamwise component $\langle u'^2 \rangle$ which dominates the corresponding underresolution of $\langle v'^2 \rangle$ and $\langle w'^2 \rangle$

– Not very clear indication of improved resolution in region of attached boundary layer

+ Sensor indicates of increased resolution of turbulent kinetic energy in the region of free shear layer and recirculating flow



Combination with indicator of resolved turbulent shear stress

Sensor of resolved turbulent shear stress suitable for attached boundary layers but not for regions of free-shear layers and recirculating flow

$\Rightarrow$ Combination of both sensors for operational single-grid estimator

+ Sensor indicates an increased resolution of turbulent shear stress in the attached boundary layer

– Not very clear indication of improved resolution in region of free-shear layer and recirculating flow