Optimal traffic states in a cellular automaton model for city traffic

Robert Barlovic[1], Elmar Brockfeld[2], Andreas Schadschneider[3], and Michael Schreckenberg[1]

[1] Theoretische Physik Fakultät 4, Gerhard-Mercator-Universität Duisburg, D-47048 Duisburg, Germany
[2] Deutsches Zentrum für Luft- und Raumfahrt e. V. (DLR), D-12489 Berlin, Germany
[3] Institut für Theoretische Physik, Universität zu Köln, D-50937 Köln, Germany

Abstract. The impact of global traffic light control strategies for city networks is analyzed in a recently proposed cellular automaton model. The model combines basic ideas of the Biham-Middleton-Levine model for city traffic and the Nagel-Schreckenberg model for highway traffic. The city network has a simple square lattice geometry. All streets and intersections are treated equally, i.e., there are no dominant streets.

1 Introduction

Nowadays mobility is regarded as one of the most significant ingredients of a modern society. Unfortunately, the capacity of the existing street networks is often exceeded. In urban networks the flow is controlled by traffic lights and traffic engineers are often forced to question if the capacity of the network is exploited by the chosen control strategy. One possible method to answer such questions could be the use of vehicular traffic models in control systems as well as in the planning and design of transportation networks. For almost half a century there were strong attempts to develop a theoretical framework of traffic science (for an overview see [1–3]).

In this paper we analyze the impact of global traffic light control strategies, in particular synchronized traffic lights, traffic lights with random offset, and with a defined offset in a recently proposed cellular automaton (CA) model for city traffic. Chowdhury and Schadschneider [5,6] combine basic ideas from the Biham-Middleton-Levine (BML) [7] model of city traffic and the Nagel-Schreckenberg (NaSch) [8] model of highway traffic. In order to take into account the more detailed dynamics of the NaSch model, they extended the BML model by inserting finite streets between the cells. On the streets vehicles drive in accordance to the NaSch rules.

2 Model

As one can see from Fig. 1, the network of streets build a $N \times N$ square lattice. All intersections are assumed to be equitable, i.e., there are no main roads in

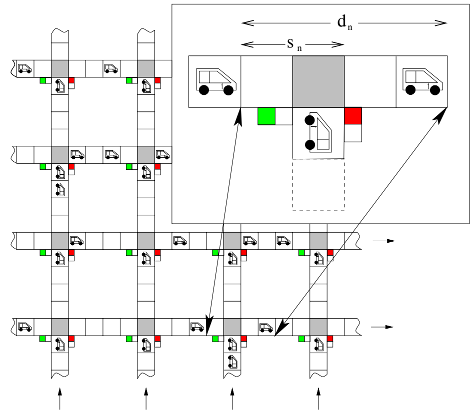

Fig. 1. Snapshot of the underlying lattice of the model. In this case the number of intersections in the quadratic network is set to $N \times N = 16$. The length of the streets between two intersections is chosen to $D - 1 = 4$.

the network where the traffic lights have a higher priority. In accordance with the BML model streets parallel to the $x-$axis allow only single-lane east-bound traffic while the ones parallel to the $y-$axis manage the north-bound traffic. The separation between any two successive intersections on every street consists of $D-1$ cells. The traffic lights are chosen to switch after a fixed time period T. The length of the time periods for the green lights does not depend on the direction and thus the “green light” periods are equal to the “red light” periods. In addition to the ChSch model [5], we improved the traffic lights by assigning an offset parameter to every one (see [9] for an detailed explanation). This modification can be used for example to shift the switch of two successive traffic lights in a way that a “green wave” can be established in the complete network.

In analogy to the NaSch model the speed v of the vehicles can take one of the $v_{max} + 1$ integer values in the range $v = 0, 1, ..., v_{max}$. The dynamics of vehicles on the streets is given by the maximum velocity v_{max} and the randomization parameter p of the NaSch model which is responsible for the movement. The state of the network at time $t+1$ can be obtained from that at time t by applying the following rules to all cars at the same time (parallel dynamics):

- Step 1: *Acceleration:* $v_n \to \min(v_n + 1, v_{\max})$
- Step 2: *Braking due to other vehicles or traffic light state:*
  - Case 1: The traffic light is red in front of the n-th vehicle:
    $v_n \to \min(v_n, d_n - 1, s_n - 1)$
  - Case 2: The traffic light is green in front of the n-th vehicle:
    If the next two cells directly behind
    the intersection are occupied
    $v_n \to \min(v_n, d_n - 1, s_n - 1)$
    else $v_n \to \min(v_n, d_n - 1)$
- Step 3: *Randomization with probability p:* $v_n \to \max(v_n - 1, 0)$
- Step 4: *Movement:* $x_n \to x_n + v_n$

Here x_n denotes the position of the n-th car and $d_n = x_{n+1} - x_n$ the distance to the next car ahead (see Fig. 1). The distance to the next traffic light ahead is given by s_n. The length of a single cell is set to 7.5 m in accordance to the NaSch model. The maximal velocity of the cars is set to $v_{max} = 5$ throughout this paper. Since this should correspond to a typical speed limit of 50 km/h in cities, one time-step approximately corresponds to 2 sec in real time. Note, that Case 2 of Step 2 is modified slightly in comparison to [6]. Due to this modification, a driver will only occupy an intersection if it is assured that he can leave it again. This is done to avoid the transition to a completely blocked state (gridlock) that is undesireable when exploring high densities.

3 Strategies

In order to improve the overall traffic conditions in the considered model different global traffic strategies are investigated. At this point it has to be taken into account that there are no dominant streets. This makes the optimization much more difficult and implies that the green and red phases for each direction should have the same length. For a main road intersection with several minor roads the total flow usually can be improved easily by optimizing the flow on the main road.

3.1 Synchronized Traffic Lights

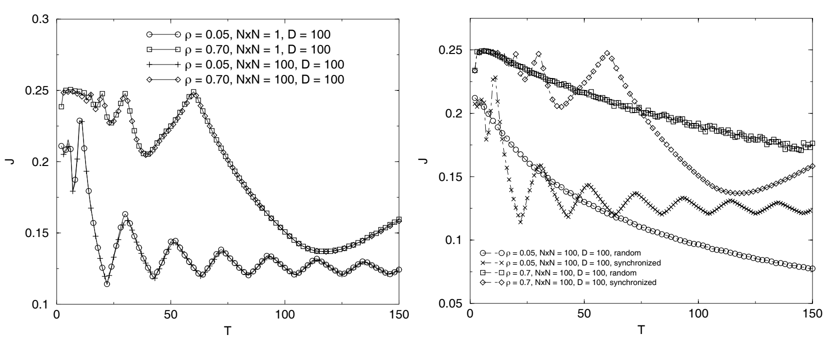

Fig. 2. Left: The mean flow strongly oscillates in the case of synchronized traffic lights. A small mini network segment shows the same dynamics like a large network. **Right:** A random shift in the switching leads to a more flexible strategy, e.g., without oscillations. The mean flow is remarkable higher in comparison to synchronized traffic lights.

The typical dependence between the time periods of the traffic lights and the mean flow in the system is shown in Fig. 2 (left) for synchronized traffic lights. For low densities one finds a strongly oscillating curve with maxima and minima at regular distances whereby the optimal traffic states are determined by the travel times between the intersections. The traffic light cycle time corresponding to the maximum system flow is equal to $T_{max} = D/2v_{free}$.[1] Similar oscillations can be even found at very high densities. It is further interesting that if considering a "mini network" with only one single intersection the same results are obtained as for large networks. In [9] it is shown that a simple phenomenological approach based on the description of the dynamics in such a "mini network" is capable to explain the impact of the cycle times in very good agreement with the numerical results even for large networks. Moreover it is very interesting that although the vehicle movement is stochastic (NaSch model) and the mean density on the streets in the network fluctuates, there is no local conglomeration of vehicles in the network leading to remarkable deviations in the flow in comparison to the idealized "mini network" where the density on the streets is fixed (see Fig. 2 (left)). It seems that the signalized intersections of the model interact with the density fluctuations in a way that the vehicles are equally distributed in the network. Also for high densities one can find a strong dependence of the mean flow in the system for the chosen cycle times (see Fig. 2). Obviously for high densities this dependence is not caused by free flowing vehicles, but determined by the movement of jams. The fraction of time the green light is not blocked by a jam controls the overall flow.

3.2 Random Offset Strategy

In this section we want to point out that switching successive traffic lights with a random shift can lead to a more flexible strategy, e.g., without oscillations. Moreover, in contrast to a system with synchronized traffic lights a random shift between the intersections can lead to a remarkable higher global system flow. To give an insight into the effects induced by the random offset we depicted the throughput in the network in dependence of the cycle times in Fig. 2 (right). Obviously the strong oscillations found in the curves corresponding to the synchronized strategy are suppressed by the randomness in the switching. Thus the random offset strategy leads to a smoothed curve which is very useful when adjusting the optimal cycle times in a network. For free-flow densities the random offset strategy outperforms the synchronized strategy for relatively low cycle times because unfavorable states (states with minimal global flow) are avoided by the randomness. However, at high densities the oscillations are suppressed in a similar manner as for the low density case. Hence, as for low densities, this strategy gives an improved flexibility when adjusting optimal cycle times in the network and an improved overall flow in a wide area.

3.3 Green Wave Strategy

For the "green wave" strategy the individual offset parameter that is assigned to every intersection is used to implement a certain time delay between the traffic light phases of two successive intersections. This delay can be constituted in the

[1] $v_{free} = v_{max} - p$ is the velocity of free-flowing vehicles.

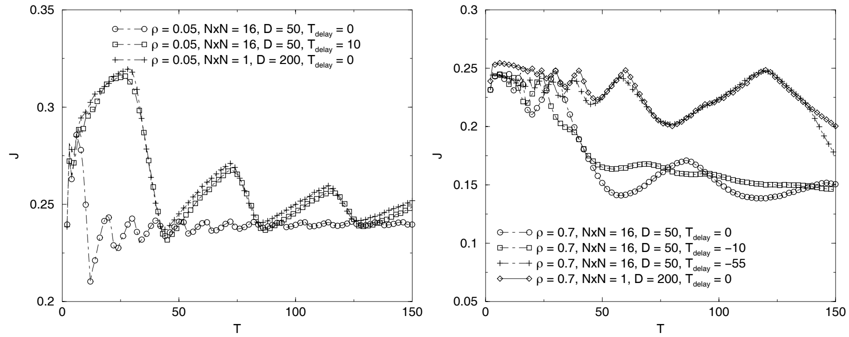

Fig. 3. The global flow is plotted for the "green wave" strategy and compared to a system with synchronized strategy. The left diagram shows the free-flow case of the system while in the right diagram the influence of the green wave strategy in the high density state is plotted.

whole network [9]. To quantify the improvement obtained by the "green wave" strategy the overall network flow is plotted against the cycle time (see Fig. 3) and compared with the synchronized strategy. The left diagram corresponds to the free-flow case of the system. Obviously, the green wave strategy with a properly chosen offset parameter shows reasonable improvements over the strategy with synchronized traffic lights. The optimal "green wave strategy" is to adjust the time delay such that the first vehicle tresspassing an intersection will arrive at the next one exactly when it switches to green. The corresponding optimal delay time is given by $T_{delay} = D/v_{free}$. In this way, the "green wave" strategy is capable to pipe all the vehicles through the streets as if there is only one intersection left in the system. This agreement to a system with only one intersection but an equal total street length is demonstrated in Fig. 3 for low as well as high densities. By definition no "green wave" can be established at high densities, but a suitable offset in the switching between successive traffic lights can lead anyhow to an improved flow. Obviously the dynamics for high densities is governed by the motion of large jams that move oppositely to the driving direction. The optimal system state would be reached if a jam moves backward from one intersection to the one before and blocks it while the traffic light is red anyway so that afterwards moving vehicles (outflow of the jam) can take advantage of the green phase as much as possible. For high densities the optimal delay time is equal to $T_{delay} = D/v_{jam}$.[2]

4 Summary and Discussion

We have analyzed the ChSch model which combines basic ideas from the Biham-Middleton-Levine (BML) model of city traffic and the Nagel-Schreckenberg (NaSch) model of highway traffic. In our investigation we focused on global traffic

[2] v_{jam} is the velocity of backwards moving jams.

light control strategies and tried to find optimal model parameters in order to maximize the network flow. For this purpose we started with the original formulation of the ChSch model where the traffic lights are switched synchronously. It is shown that the global throughput of the network strongly depends on the cycle times, i.e, one finds strong oscillations in the global flow in dependence of the cycle times for low as well as for high densities. In order to allow a more flexible traffic light control the ChSch model was enhanced by an additional model parameter. This new parameter is assigned to every intersection representing a time offset, so that the traffic lights are not enforced to switch simultaneously anymore. Consequently to avoid the strong oscillations we analyzed a network where traffic lights are switched at random. It is shown that the strong oscillations are completely suppressed by randomness. Thus the random offset strategy can be very useful if a control strategy is required which is not very sensitive to the adjustment of the cycle times. This is in particular the case if the density on the streets is strongly fluctuating. The random offset strategy outperforms the standard ChSch model with synchronized traffic lights at low densities for small cycle times and at high densities for all cycle times. An explanation for the profit at high densities is the fact that some parts of the network are completely jammed while in other parts of the network the cars can move nearly undisturbed. This additional gain due to the inhomogeneous allocation of vehicles indicates that an autonomous traffic light control based on local decisions could be more effective than the analyzed global schemes. Moreover a two dimensional "green wave" is implemented with the help of the offset parameter. The "green wave" gives much improvement to the flow in comparison to the synchronized strategy and the random offset at low densities and has even an incisive impact on the throughput at high densities. Although the "green wave" strategy is capable to give a strong improvement, the dependence between flow and the cycle time found in the original ChSch model remains.

References

1. D. Helbing, H.J. Herrmann, M. Schreckenberg, and D.E. Wolf (eds.), *Traffic and Granular Flow '99* (Springer, 2000).
2. D. Helbing, *Verkehrsdynamik: Neue Physikalische Modellierungskonzepte*, (in German) (Springer, 1997).
3. D. Chowdhury, L. Santen, and A. Schadschneider, Phys. Rep. **329**, 199 (2000); Curr. Sci. **77**, 411 (1999) and Comp. Science & Techn. **2**(5), 80 (2000).
4. S. Wolfram, *Theory and applications of cellular automata* (World Scientific, 1986).
5. D. Chowdhury and A. Schadschneider, Phys. Rev. E **59**, R1311-1314 (1999).
6. A. Schadschneider, D. Chowdhury, E. Brockfeld, K. Klauck, L. Santen, and J. Zittartz, in: *Traffic and Granular Flow '99* (Springer, 2000).
7. O. Biham, A.A. Middleton, and D. Levine, Phys. Rev. A **46**, 6124 (1992).
8. K. Nagel and M. Schreckenberg, J. Physique I **2**, 2221 (1992).
9. E. Brockfeld, R. Barlovic, A. Schadschneider, and M. Schreckenberg, Phys. Rev. E **64**, 056132 (2001)