On the decay and amplification of turbulence in Sexl–Womersley type flows

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Introduction

Decay and amplification of shear flow turbulence in oscillatory fluid motions is of theoretical interest and practical relevance, since the onset of turbulence can drastically change the transport properties and mixing efficiency. To supplement former theoretical and experimental investigations on the transition to turbulence in Sexl–Womersley (SW) type flows we perform three-dimensional direct numerical simulations (DNS) of oscillatory pipe flows at various Womersley numbers $Wo \in \{5, 13, 26, 52\}$ and Reynolds numbers $Re_\tau \in \{1440, 2880, 5760, 11520\}$ based on the friction velocity u_τ.

Sexl–Womersley flow

- control parameters

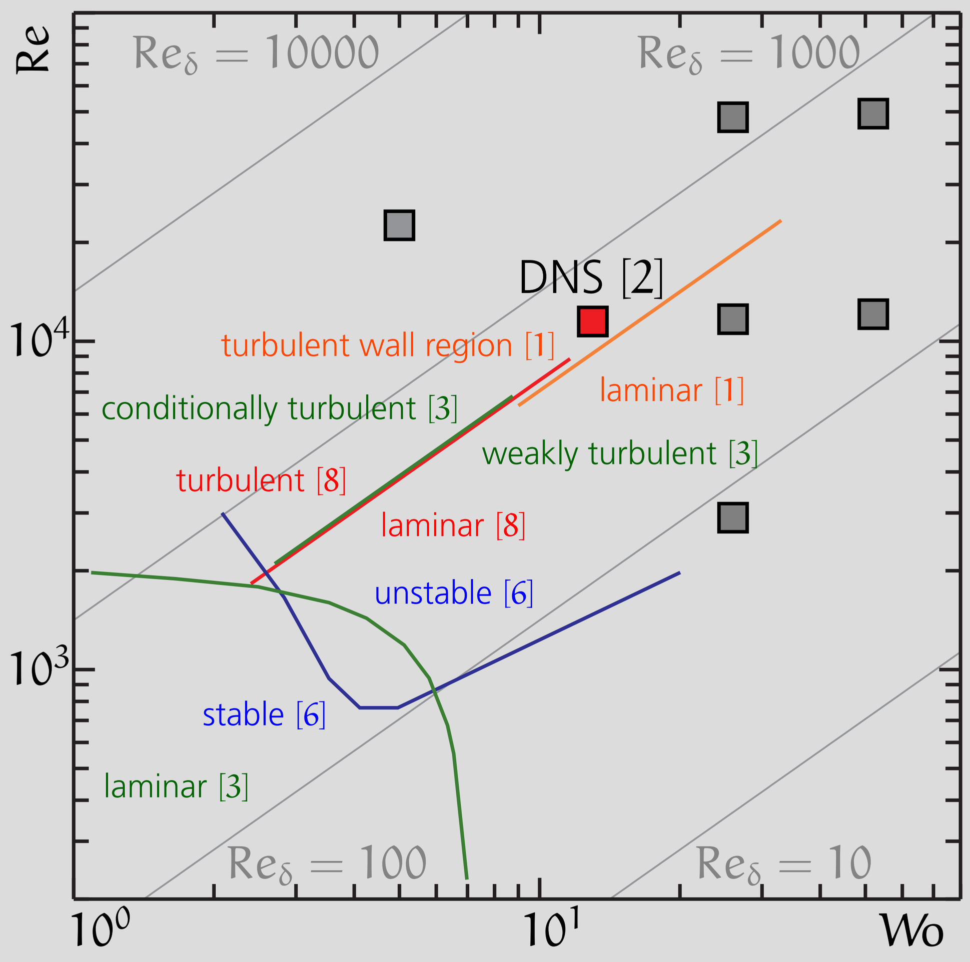
$$Wo = \frac{D}{2}\sqrt{\frac{\omega}{\nu}} \quad \text{and} \quad Re_\tau = \frac{u_\tau \cdot D}{\nu}$$

$$\rightsquigarrow Re = \frac{\hat{u} \cdot D}{\nu} \quad \text{and} \quad Re_\delta = \frac{\hat{u} \cdot \delta}{\nu} = \frac{1}{\sqrt{2}} \frac{Re}{Wo}$$

- analytical (laminar) SW solution [4, 7]

$$u_z(r,t) = -\frac{C}{\omega} e^{i\omega t} \left[1 - \frac{J_0\left(\frac{2r}{D} Wo \sqrt{i}\right)}{J_0\left(Wo\sqrt{i}\right)} \right]$$

- DNS results [2] for $Wo = 13$ and $Re_\tau = 1440$
- resulting in $Re = 11460$ and $Re_\delta = 625$
- investigate the decay and amplification of turbulence close to the transitional regime



Direct numerical simulation (DNS)

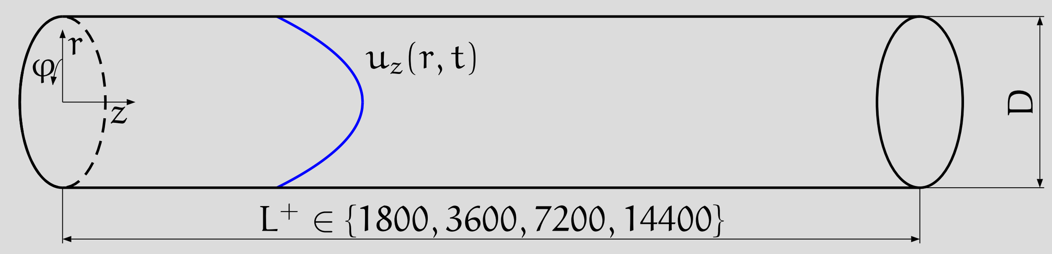
- incompressible Navier–Stokes equations

$$\nabla \cdot \mathbf{u} = 0$$

$$\text{and} \quad \partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p' - \frac{\Delta \mathbf{u}}{Re_\tau} = -\left[\nabla_r, \nabla_\varphi, \cos\left(\frac{4Wo^2}{Re_\tau} t\right) \nabla_z\right]^T \langle p \rangle$$

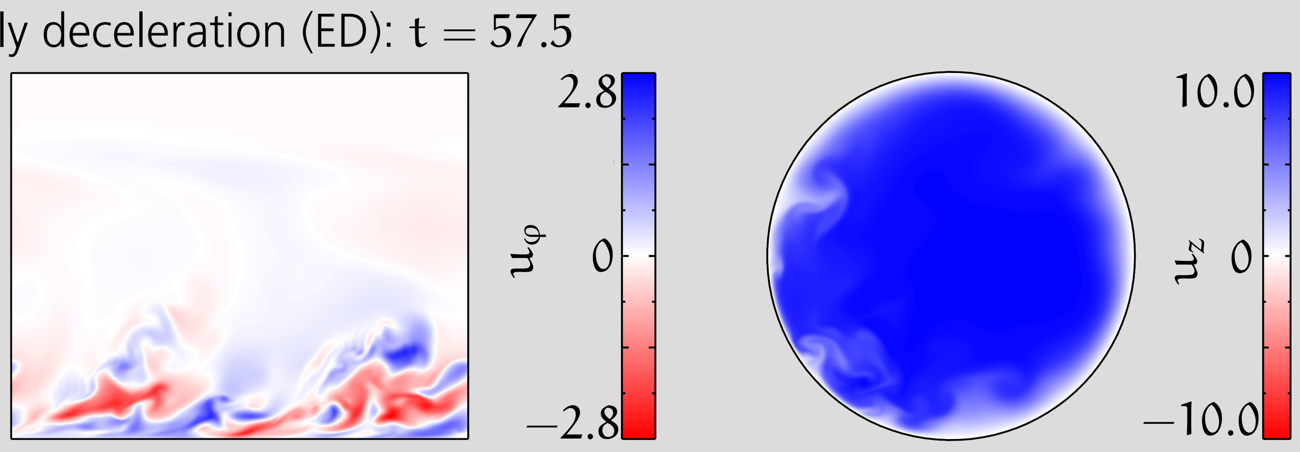
$$\text{with} \quad p = p' + \langle p \rangle$$

- directly solved in cylindrical coordinates
- fourth order accurate finite volume method [5]
- staggered grid with $1024 \times 256 \times 128$ points to resolve Kolmogorov scales [2]
- implicit/explicit leapfrog–Euler time integration [5]
- well-correlated initial flow field at $Wo = 0$ and $Re_\tau = 1440$ [2]
- periodic boundary condition (BC) in φ and z
- no-slip and impermeability BC at $r = \frac{D}{2}$

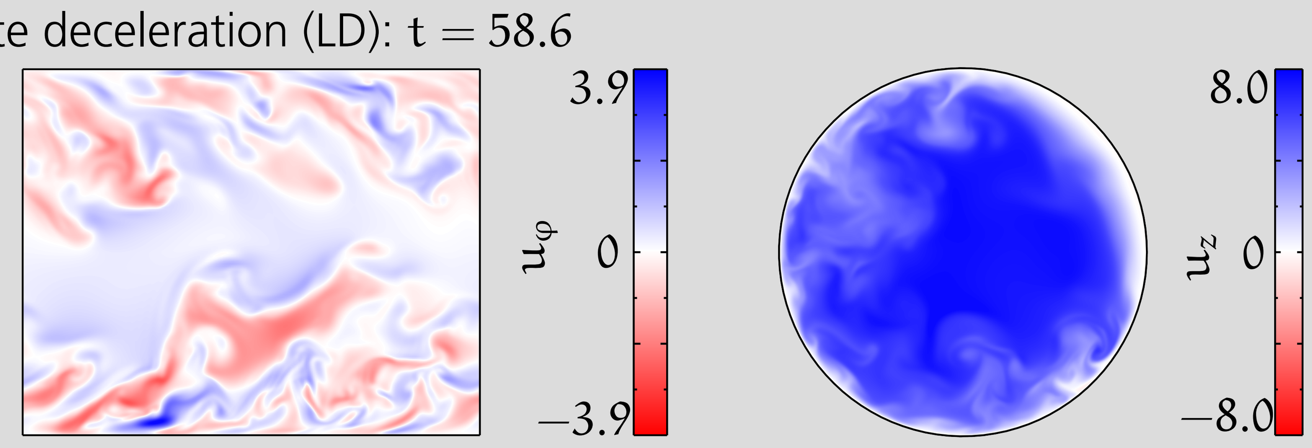


Instantaneous velocities — $Wo = 13$ and $Re = 11460$

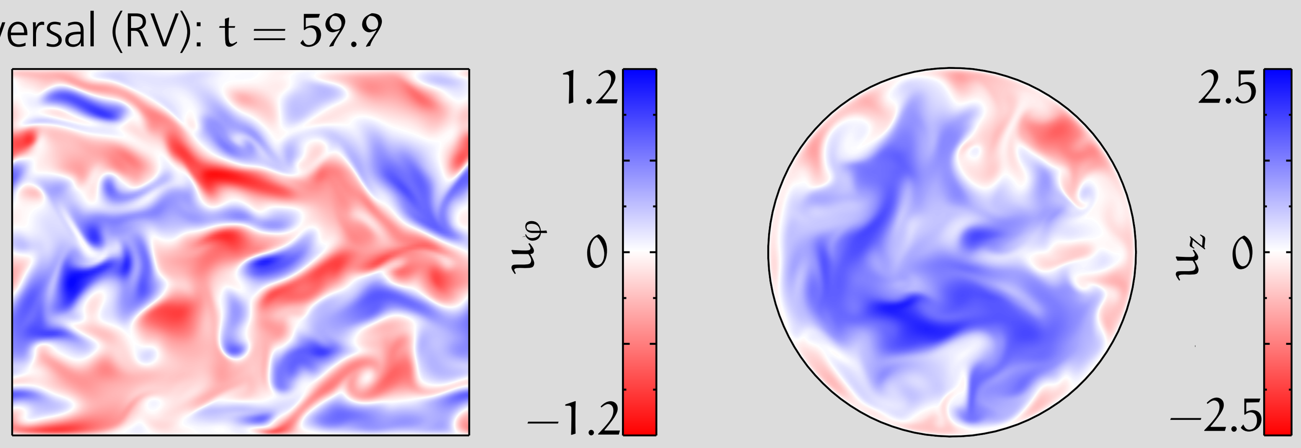
Early deceleration (ED): $t = 57.5$



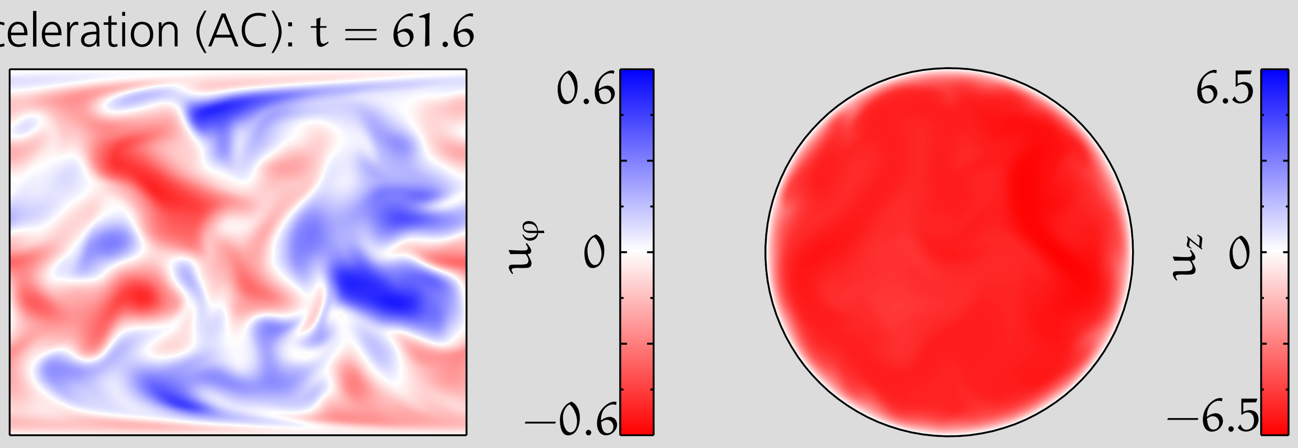
Late deceleration (LD): $t = 58.6$



Reversal (RV): $t = 59.9$



Acceleration (AC): $t = 61.6$

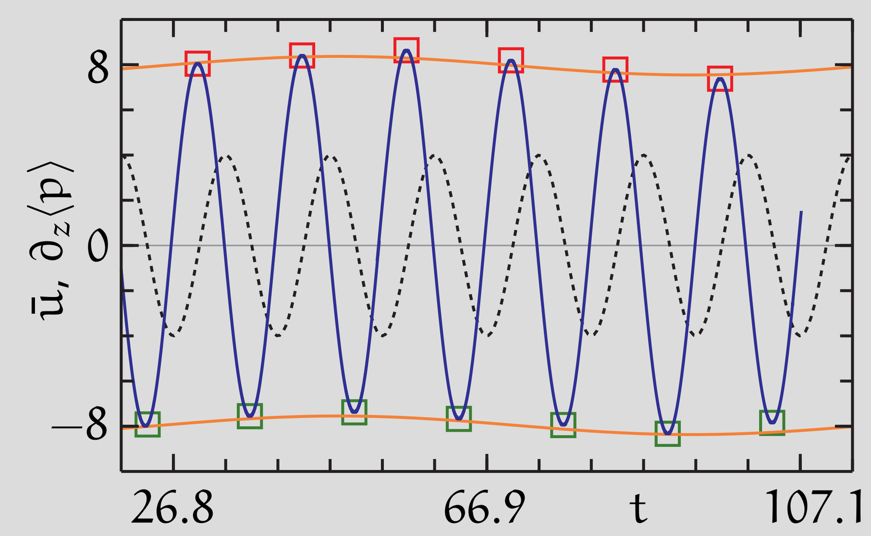


- localised turbulent bursts close to the wall (ED)
- amplification of turbulent velocity fluctuations (ED, LD)
- elongated structures inclined to the wall (LD)
- relaminarisation due to low bulk flow values (RV, AC)
- no turbulence despite of increasing bulk flow (AC)

Time lines — $Wo = 13$ and $Re = 11460$

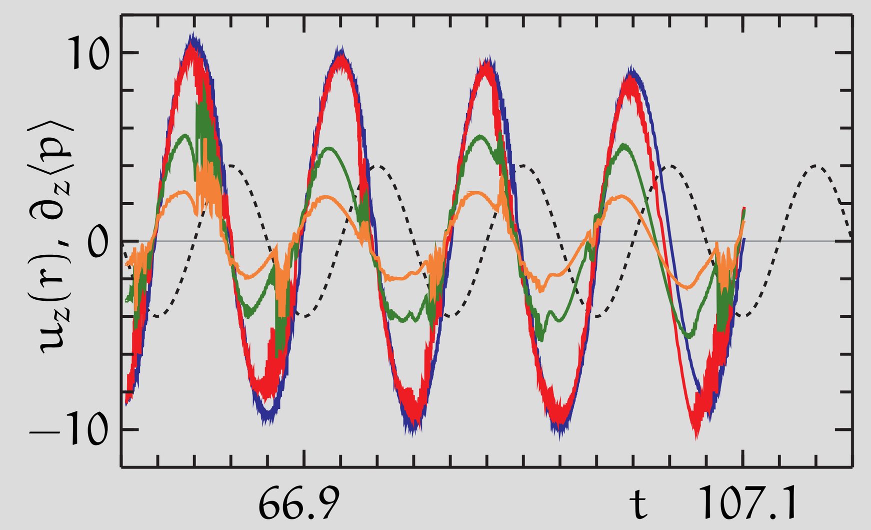
Temporal evolution of the mean flow $\bar{u}$ and the driving pressure gradient $\partial_z \langle p \rangle$

- phase lag of about $\pi/2$ between bulk flow and driving pressure as predicted for SW flow
- periodic variation in peak flow values $\hat{u}$, red (positive) and green (negative) symbols
- orange line: sinusoidal fit with $T \approx 7 \cdot \frac{4Wo^2}{Re}$



Temporal evolution of the axial velocity $u_z(r)$

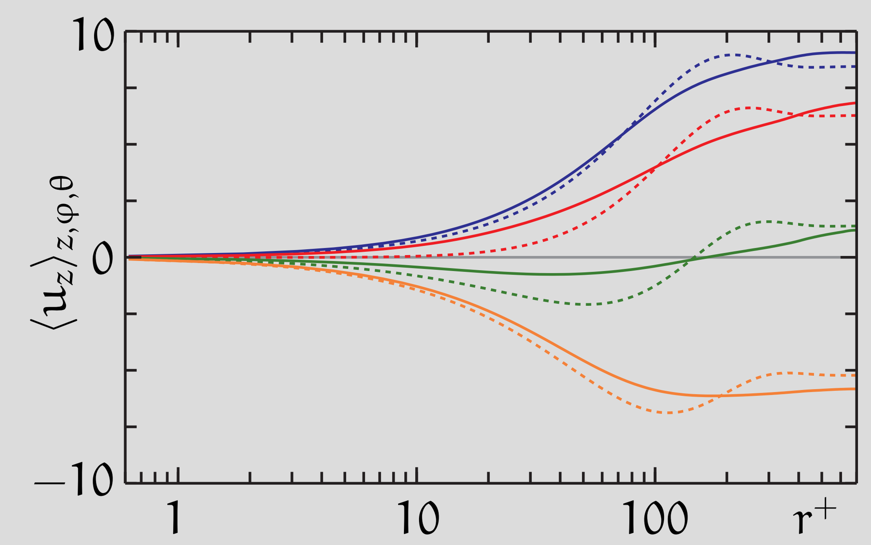
- at four radial probe locations from $r^+ = 7$ (orange) to $r^+ = 353$ (blue)
- turbulent near-wall bursts during ED and LD slightly differ in strength and phase
- laminarisation during AC



Phase averages — $Wo = 13$ and $Re = 11460$

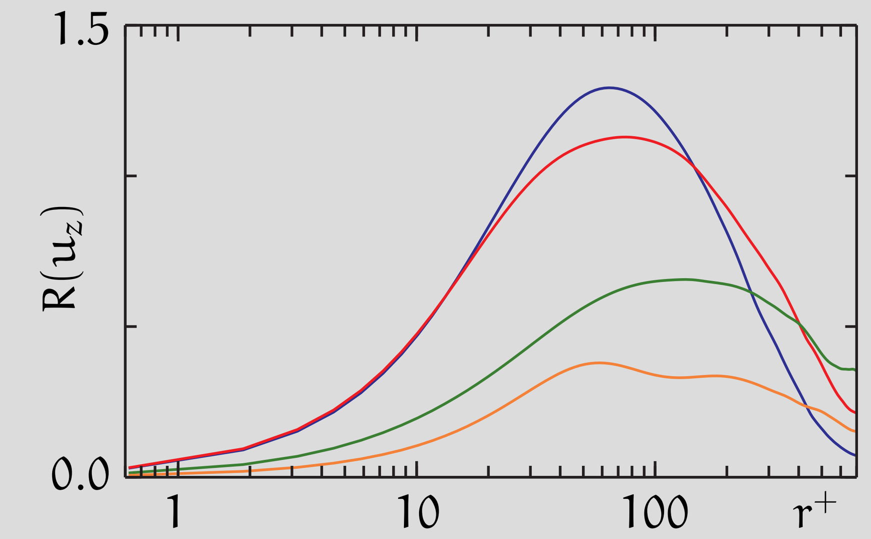
Phase averaged mean velocity profiles $\langle u_z \rangle_{z,\varphi,\theta}$

- at four instants during oscillation from ED (blue) to AC (orange)
- laminar SW profiles (dashed)
- no inflection points typical for SW at high Wo
- coaxial counter flow during RV less pronounced compared to SW



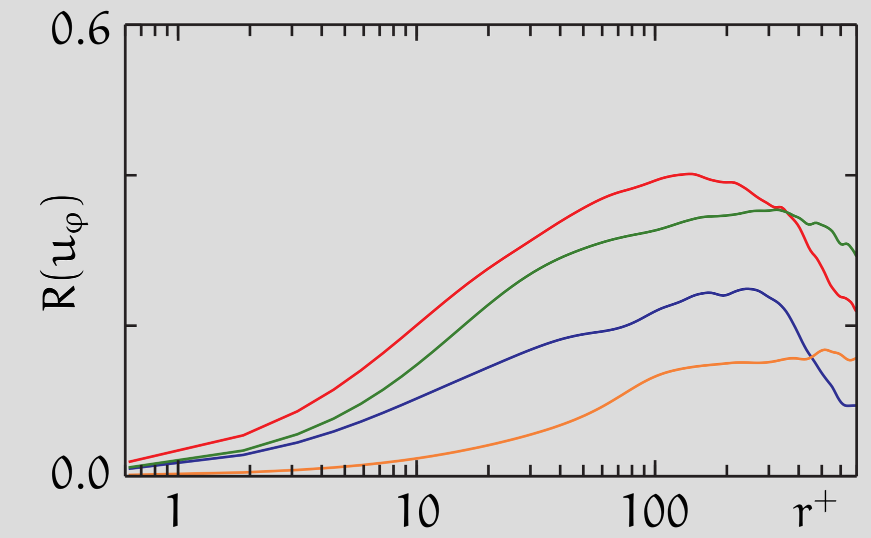
Phase averaged axial RMS fluctuations $R(u_z)$

- highest $R(u_z)$ during ED and LD when turbulent bursts occur in an annular region close to the wall
- afterwards decreasing $R(u_z)$ until AC



Phase averaged azimuthal RMS fluctuations $R(u_\varphi)$

- turbulence is distributed towards the centre line during ED and LD
- highest $R(u_\varphi)$ persists in the core flow, whereas highest $R(u_z)$ persists closer to the wall during AC



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Acknowledgements

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