Air Transport Research Society World Conference 2010, Porto, Portugal

A MODEL OF COMPETITION IN PASSENGER AIR TRANSPORT MARKETS

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Abstract

This paper presents a model of passenger air transport markets that generally focuses on three main groups of stakeholders: air travellers, airlines and airports. Air travellers' choice of air transport services is modelled by a demand function considering product differentiation. Thus, demand for passenger air transport is not a fixed model-exogenous input parameter but is determined model-endogenously and depends on the supply of flights and their various characteristics. Competitive relationships are modelled as Cournot quantity competition with heterogeneous products. Airlines and airports only have incomplete information about the nature of each other, which is then updated by a dynamic learning process in each period. The model is of particular interest in evaluating business strategies on behalf of airlines and airports and for public institutions that wish to analyse various market scenarios and evaluate politico-economic actions.

KEYWORDS: airline competition, game theory, nonlinear programming

1. Introduction

In recent years, there has been an increasing interest in the modelling of competitive relationships within the passenger air transport markets. Aside from improvements in modelling techniques and the steadily increasing computational power of personal computers (an essential prerequisite to employ such models effectively in praxis), the key drivers of this trend were the growing importance of deregulated and thus more competitive passenger air transport markets and the rise of new business models in aviation. It is now more than 30 years since the US Airline Deregulation Act was approved by the US Congress (e.g., Goetz and Vowles, 2009) and more countries have followed in subsequent years. For example, deregulation began in Australia in the early 1980’s (e.g., Hooper, 1998) and in Europe in the late 1980’s (e.g., Ehmer et al., 2000). Prior to deregulation, national air transport markets were mainly monopolies, with only one national carrier or a very few regulated carriers serving their national air transport markets. International air travel was governed exclusively by strict bilateral national agreements.

According to the competitive environment of passenger air transport markets, we differentiate between three classes of models: models of monopolistic competition, models of oligopolistic competition (but without extensive network optimisation capabilities and thus only applicable to some problems) and models of oligopolistic competition with extensive network optimisation capabilities, which are therefore applicable to a wide range of problems.

The first class of models serves to optimise flight structures between airports and is largely applied to complex hub-&-spoke systems, where coordination costs tend to be high. Such models include extensive network optimisation capabilities, in order to produce feasible flight schedules as dictated by passenger demand, flight restrictions and capacity constraints. These models are usually applied under monopolistic conditions and examples of such models include Gordon (1974), Jacquemin (2006), Jeng (1987) and Miller (1963). Passenger demand is assumed to be fixed in most models and their objective is to optimise flight structures, in order to meet a certain demand. There are some models of monopolistic competition, without extensive network optimisation, that operate on simpler network structures. These models are mainly used for specialised studies; for example, Berechman and Shy (1996) and Brueckner and Zhang (2001) compare flight frequency, fares and social welfare in point-to-point and hub-&-spoke networks.

The second class of models comprises of models predominantly tailored to analyse a particular question in an oligopolistic market environment: they typically focus on point-to-point traffic and simple hub-&-spoke networks of low complexity, without considering special flight restrictions and capacity constraints. A popular is the analysis of market equilibrium and social welfare in deregulated and regulated markets. Examples include Douglas and Miller (1974), Panzar (1979, 1980), Schipper et al. (2003) and Zhang (1996). Further analysis comprises network competition, network invasion and entry deterrence. Pels (2009) considers point-to-point and hub-&-spoke traffic, in order to analyse the effects of network competition between two airlines (with regards to the invading of each other's network). Aguirregabiria and Ho (2009) model a dynamic game of airline competition, with a focus on entry deterrence in hub-&-spoke networks: they applied the theory of Markov's perfect equilibrium, in order to find a solution. Zhang (1996) analyses fortress hubs in airline networks and the impact of local competition on social welfare in hub-&-spoke systems. Oum et al. (1995) compare the growing number of hub-&-spoke systems in the USA and Canada after deregulation with point-to-point systems, which were popular before.

The third class of model is associated with competitive relationships and includes extensive network optimisation capabilities; however, the members of this class still differ to some degree, in their ability to model market structures and complex network structures. It is thus sometimes difficult to decide whether a particular model belongs to class two or class three. Nevertheless, the ability to include competitive relationships between market actors and more flexible network structures enhance model practicality in a multitude of real-life problems. Models which focus more on (multi-) hub-&-spoke systems, with an exogenously-given passenger demand, are Dobson and Lederer (1993), Kanafani and Ghobrial (1985), Hansen (1990) and Hansen and Kanafani (1990). Takebayashi (2009) employ a bi-level approach, with demand and prices fixed and airlines controlling frequency and aircraft choice. Takebayashi and Kanafani (2005) employ a bi-level approach to model competition between hub-&-spoke and point-to-point carriers: passenger demand is fixed, with airlines controlling fares and flight frequency. Hong and Harker (1992) apply a variational inequality approach to the proper pricing of capacity. Evans et al. (2008), Evans and Schäfer (2009) and Adler (2001, 2005) develop models of airline competition which are applicable to a wide range of different network structures and a number of competitors.

Just how relevant is the feature of a model in allowing for arbitrary airline network structures? Hendricks et al. (1995) analyse network choice (hub-&-spoke vs. point-to-point, not just air transport markets) of a monopoly carrier and highlight that this choice depends on the degree of economics of density in an origin-destination (O-D) market. In their enhanced model, they analyse the deterrence effect of a hub-&-spoke network on small-scale entry (Hendricks et al., 1997) and duopolistic competition between two large carriers, who are not restricted in their choice of networks (Hendricks et al., 1999). Hendricks et al. reason that hub-&-spoke carriers enjoy an advantage over point-to-point carriers as a result of their higher productivity: they can attain economies of density more easily than point-to-point carriers. Thus, they conclude, the number of point-to-point carriers tend to decrease over time.

However, in airline markets, there is growing empirical and theoretical evidence that a hub-&-spoke network structure may not always be optimal and that the impact of low-cost carriers, which mainly rely on the point-to-point concept, is significant: they invade the networks of traditional carriers that focus on the hub-&-spoke system (e.g., Dennis, 2007; Gillen and Morrison, 2005; Mason, 2001; Mason and Alamdari, 2007; Pels et al., 2000; Vowles, 2001).

In this paper we present a model of passenger air transport markets that can handle any number of airlines and airports and complex network structures. Demand for passenger air transport is not a fixed model-exogenous input parameter but is determined model-endogenously and therefore depends on, amongst other factors, the supply of flights and their various characteristics. Competitive relationships between airlines and airports are modelled on a game-theoretic framework and there are three major innovations, when compared to existing approaches: the method of modelling air passenger demand, the handling of incomplete information and learning and how market equilibrium is computed.

The decision problems of each airline and airport are modelled as a nonlinear programme and game theory is employed to model competitive relationships. Incomplete information and dynamic learning play a central role in this model; however, this makes the model computationally difficult to handle. Thus, the concepts of a so-called empirical reaction function and market entry/exit probability function are presented: each competitor's marginal behaviour is learned on the basis of a smoother version of their past moves and serves as input for the empirical reaction function to forecast future actions. The market entry/exit probability function describes the relationship between the profitability of a particular market and the probability of market entry and exit. Furthermore, the modelling approach employed in this paper means a partial departure from assuming perfect rational individuals with unlimited computing abilities towards behaviour which is more likely to be based on observed past actions of opponents.

One of the central objectives of the model is to explain the dynamic developments of air transport markets and their competitive forces. In this context, incomplete information and learning play a critical role, with regards to the profitability of deterrence and entry strategies.

The outline of this paper is as follows: chapter two serves as a brief introduction to the foundations of game theory and discrete choice, as employed in the model. Chapter three subsequently describes the model in detail. Finally, the paper closes with a model discussion and a summary of the results.

2. Methodical background

2.1 Game theory

Game theory refers to the modelling of interactive decision-making. A game-theoretic model comprises a finite set of N players, for each player *i* a nonempty action set A_i

with elements a_i and a preference relation $\succsim_i$ on the set of action profiles $a=\left(a_j\right)_{j\in N}$. An action profile a represents an outcome and the set of outcomes and action profiles, respectively, is denoted $A=\times_{j\in N} A_j$. Here, we can see how a strategic game differs from a decision problem: each player does not only care about his own actions but also those taken by the other players. Thus, the preference relation $\succsim_i$ of each player *i* is defined, with regards to the set of action profiles A, rather than his action set A_i (Osborne and Rubinstein, 1994, p.11).

The most popular solution concept employed in game theory is that of Nash equilibrium (Nash, 1950). A Nash equilibrium can briefly be described as an action profile in which each player's action is a best response, given the actions of the other players, and thus no player can profitably deviate. More formally, a Nash equilibrium of a strategic game $\left\langle N,\left(A_i\right),\left(\succsim_i\right)\right\rangle$ is a profile $a^* \in A$ of actions with the property that for every player $i \in N$ we have $\left(a_{-i}^*, a_i^*\right) \succsim_i \left(a_{-i}^*, a_i\right)$ for all $a_i \in A_i$. Here, a_{-i} describes an action profile exclusive of the action of player *i*: each player is assumed to have complete information about the relevant characteristics of the strategic game and thus acts rationally. However, the concept of a Nash equilibrium only describes a steady state of the play of a strategic game and does not say anything about how this steady state has been reached (Osborne and Rubinstein, 1994, pp.14).

In a repeated game, the so-called stage game is played in each of the periods $t \in \{0,1,\ldots\}$. The number of periods can take any finite number or be infinite. A player's choice in the stage game is denoted as an 'action', whilst their behaviour in the repeated game is termed a 'strategy'. In this paper, we look at repeated games of perfect monitoring; i.e. that all players observe the chosen action profile at the end of each period (Mailath and Samuelson, 2006, pp.15). In every period, the repetition of the stage game Nash equilibrium also represents a Nash equilibrium of the repeated game (Mailath and Samuelson, 2006, p.191).

Strategy games are dominated by equilibrium analysis; however, in many cases, the assumptions that players immediately and unerringly identify and play an equilibrium strategy, thus the equilibrium being common knowledge (Aumann, 1976, pp.1236;

Milgrom, 1981, pp.219) to the players, may be questionable (Milgrom and Roberts, 1991, p.82). Learning dynamics become even more important if players acquire new decision-relevant information in the course of play, which is typical if the same or a similar game is repeated several times. These learning dynamics may even have an impact on the equilibrium finally reached, if one is reached at all. The learning mechanism developed later in this paper is founded on ideas of fictitious play and smooth fictitious play.

Fictitious play (Brown, 1951; Robinson, 1951) is one of the earliest learning rules, yet it was initially not proposed as a pure learning model but, rather, as an iterative method for computing Nash equilibria in zero-sum games. However, due to its intuitive update rule, it is commonly viewed as a simple learning model: every player is assumed to choose a best response to the assessed strategies of his opponents in every period of the game while he believes that his opponents are playing a mixed strategy, which is given by the empirical distribution of their past actions. Note that players know only their individual payoffs and are oblivious to the payoffs obtained or obtainable by their opponents (Shoham and Leyton-Brown, 2009, pp.195). The essential idea behind this approach is that, at least asymptotically, past choices of opponents serve, to some extent, as a sound guide to their future behaviour (Fudenberg and Kreps, 1993, p.334). Smooth fictitious play was first analysed by Fudenberg and Kreps (1993): in this, players choose a perturbed version of their best response, but perturbation diminishes as the game progresses. The random utility model is one of the reasons for employing smooth fictitious play: players choose to randomise, even when they are not indifferent between their actions, as a means of protection from mistakes in their model of opponent’s play (Fudenberg and Levine, 1998, p.117). For further models of learning (such as Bayesian learning, reinforcement learning and evolutionary learning), the interested reader is referred to Fudenberg and Levine (1998) and Shoham and Leyton-Brown (2009, pp.189).

2.2 Discrete choice theory

The fundamental hypothesis of discrete choice models is the assumption of individual utility maximisation. The decision maker is assumed to rate alternatives of his choice set by means of a particular utility function and will choose the one with the highest utility. However, from an outside perspective, the utility of an alternative for a specific individual represents a random variable. Thus, utility U_i for alternative *i* is decomposed into a deterministic component V_i and a random component ε_i (McFadden, 1974, p.108):

(1) $$U_i = V_i + \varepsilon_i$$

The random component of the utility function is introduced for various reasons; for example, incomplete measurability of the decision-relevant alternative attributes and limited rationality (Maier and Weiss, 1990, pp.98; Manski, 1977, p.229). Hence, from an external point of view, only evidence in terms of the probability of an alternative being the one with the highest utility is possible.

Different concepts of discrete choice models differ, in terms of their specification of the random component. The most prominent member is the logit-model, with independently and identically distributed random components. The choice probability of an alternative *i* is computed as (McFadden, 1974, p.110):

(2) $$P_i = \frac{e^{\mu U_i}}{\sum_j e^{\mu U_j}}$$

The scale parameter μ of the Gumbel distribution is usually fixed to a value of one, in order to enable identification of the model parameters of the utility function (Ben-Akiva and Lerman, 1985, p.107). For our purpose, the logit-model is completely adequate and we will therefore refer to it later in the paper. However, for an introduction to the more sophisticated discrete choice models, the interested reader is referred to, for example, Ben-Akiva and Lerman (1985) and Train (2003).

3. The model

The modelling approach chosen in this paper represents a mixture of both game-theoretic and decision-theoretic elements and this becomes especially apparent if we look at how competitors' future actions are assessed and market entry and exit is modelled. Therefore, this section is subdivided in two parts: the first part describes, for each period, the static decision problems of each class of players; i.e., airlines, airports and air passengers. The second part of this chapter deals with the modelling of market dynamics of the game and includes learning and assessing opponents' strategies, market entry and exit and the equilibrium concept applied to the game.

The model is modular in build, so that it can be customised to the particular problem at hand. For example, airports may be disregarded as players in the game if they are government owned or regulated and do not follow a truly competitive strategy. In this case, capacity is, in many cases, not a decision taken by the airports themselves and is thus fixed, from their point-of-view. Instead, capacity decisions may be based on political or environmental considerations of the government, rather than on a competitive airport strategy (Adler, 2005, p.64). Thus, airport capacities and airport charges represent fixed inputs for the decision process of each airline; however, the model is flexible enough to account for airports pursuing an individual competitive strategy.

3.1 Player's one-period decision problem

Notation

C^{ac}_{atijm}	Other variable aircraft costs of airline *a* in period *t* on flight route *i*→j for aircraft of type *m*
C^{P}_{atiklj}	Other variable passenger costs of airline *a* in period *t* on flight route *i*→*k*→*l*→*j*
C^{ac}_{itam}	Other variable costs per aircraft of type *m* and airline *a* for airport *i* in period *t*
C^{rf}_{it}	Fixed costs of supplying runway capacity at airport *i* in period *t*
C^{rv}_{it}	Variable costs of supplying runway capacity at airport *i* in period *t*
C^{tf}_{it}	Fixed costs of supplying terminal capacity at airport *i* in period *t*
C^{tv}_{it}	Variable costs of supplying terminal capacity at airport *i* in period *t*
f_{atijkl}	Flight frequency offered by airline *a* in period *t* on flight route *i*→*k*→*l*→*j*
F_{mi}	Runway capacity consumption of an aircraft of type *m* at airport *i*

G_{ai}	Set of airlines which take precedence over airline *a* at airport *i*
i	Interest rate
P_{atiklj}	Airline's *a* ticket price in period *t* for flight route *i*→*k*→*l*→*j*
P_{it}^{f}	Full passenger charges for departing passengers at airport *i* in period *t*
P_{it}^{t}	Transfer passenger charges for stopover passengers at airport *i* in period *t*
P_{itm}^{l}	Landing charges at airport *i* in period *t* for aircraft of type *m*
$P_{atikljn}$	Probability of *n* competitors being active on flight route *i*→*k*→*l*→*j* in period *t,* from the viewpoint of airline *a*
$q_{atikljn}$	Element *n* of the vector of service quality variables for flight route *i*→*k*→*l*→*j* of airline *a* in period *t*
S_{aijm}	Seat capacity of aircraft of type *m* of airline *a,* operating on flight leg *i*→*j*
S_{tiklj}	Number of competitors on flight route *i*→*k*→*l*→*j* in period *t*
V	Set of feasible combinations of *i*, *k*, *l* and *j*
$V_t\left(S_t\right)$	Value of landing in state S_t
x_{atiklj}	Number of seats offered by airline *a* in period *t* on flight route *i*→*k*→*l*→*j*
x_t	Decision in period *t*
y_{atijm}	Number of aircraft of type *m* of airline *a* operating on flight leg *i*→*j* in period *t*
z_{it}^{r}	Runway capacity supplied at airport *i* in period *t*
z_{it}^{t}	Terminal capacity supplied at airport *i* in period *t*
$\alpha_{atikljn}$	Coefficient *n* of inverse demand function P_{atiklj}
α_{it}^{f}	Coefficient of inverse demand function P_{it}^{f}
α_{it}^{t}	Coefficient of inverse demand function P_{it}^{t}
α_{itm}^{l}	Coefficient of inverse demand function P_{itm}^{l}
$\phi_{atikljn}$	Coefficient *n* of market entry/exit probability function (MEEP) $P_{atikljn}$
β_{btmopn}^{atiklj}	Homogeneity coefficient of the inverse demand function P_{atiklj}, measuring similarities between the particular flights on flight route

	i→k→l→j of airline *a* in period *t* and flights on flight routes *m→o→p→n* of airline *b* in period *t*
$\Delta^{atmopn} f_{btiklj}$	Airline *b*'s increase of flight frequency on flight route *i→k→l→j* in period *t,* if airline *a* increases her number of seats offered on flight route *m→o→p→n* in period *t* by one unit
$\Delta^{atmopn} x_{btiklj}$	Airline *b*'s increase of number of seats supplied on flight route *i→k→l→j* in period *t,* if airline *a* increases her number of seats offered on flight route *m→o→p→n* in period *t* by one unit
$\Delta^{atmopn} x^{t}_{b(t+1)iklj}$	Prediction of $\Delta^{atmopn} x_{b(t+1)iklj}$ in period *t* for period *(t+1)*
π_{atiklj}	Profit of airline *a* in period *t* on flight route *i→k→l→j*
$\pi_{atikljn}$	Profit of airline *a* in period *t* on flight route *i→k→l→j* with *n* airlines being active
π_a^{ltp}	Long-term profit (LTP) of airline *a*
π_a^{stp}	Short-term profit (STP) of airline *a*
π_t	Profit in period *t*

Airlines

In this section, we present the one-period decision problem for each airline; i.e. one-period profit-maximisation. In this, each airline views the values of the strategic decision variables of competing airlines and airports as input data for their decision process.

(3) $$Max\ \pi_{at}^{al}\left(x_{at}, y_{at}\right) = \sum_{\substack{i,k,l,j \\ i\neq j, j\neq k, k\neq l, \\ i\neq k, j\neq l, i\neq l}} \left(\left(P_{atiklj}\left(\ldots\right) - C^{p}_{atiklj} - P^{f}_{it}\left(z^{t}\right) - P^{t}_{kt}\left(z^{t}\right) - P^{t}_{lt}\left(z^{t}\right)\right) * x_{atiklj}\right) +$$

(4) $$\sum_{\substack{i,k,j \\ i\neq k, \\ k\neq j, i\neq j}} \left(\left(P_{atikkj}\left(\ldots\right) - C^{p}_{atikkj} - P^{f}_{it}\left(z^{t}\right) - P^{t}_{kt}\left(z^{t}\right)\right) * x_{atikkj}\right) +$$

(5) $$\sum_{\substack{i,j \\ i\neq j}} \left(\left(P_{atiijj}\left(\ldots\right) - C^{p}_{atiijj} - P^{f}_{it}\left(z^{t}\right)\right) * x_{atiijj}\right) -$$

(6)
$$\sum_{\substack{i,j,m \\ i\neq j}} \left(C^{ac}_{atijm} + P^{l}_{jtm}\left(z^{r}\right)\right) * y_{atijm}$$

Subject to:

(7)
$$\sum_{\substack{k,l \\ i\neq j,k\neq l, \\ i\neq l}} \left(x_{atijkl} + x_{atkijl} + x_{atklij}\right) \leq \sum_{m} S_{aijm} * y_{atijm} \quad \forall i,j$$

(8)
$$y_{atijm} = y_{atjim} \quad \forall i,j,m; i\neq j$$

(9)
$$2 * \sum_{\substack{a\in G\cup\{a\},j,m \\ i\neq j}} F_{mi} * y_{atjim} \leq z^{r}_{it} \quad \forall i$$

(10)
$$\sum_{\substack{a\in G_{ai}\cup\{a\},k,l,j \\ i\neq j, j\neq k, k\neq l, \\ i\neq k, j\neq l, i\neq l}} \left(2 * \left(x_{atkijl} + x_{atkjil}\right) + x_{atikjl} + x_{atklji}\right) +$$
$$\sum_{\substack{a\in G_{ai}\cup\{a\},k,l \\ i\neq k, k\neq l, \\ i\neq l}} \left(2 * x_{atkiil} + x_{atikkl} + x_{atklli}\right) + \sum_{\substack{a\in G_{ai}\cup\{a\},j,d \\ i\neq j}} \left(x_{atiijj} + x_{atjjii}\right) \leq z^{t}_{it} \quad \forall i$$

(11)
$$\sum_{m} y_{atijm} \geq f_{atijkl} \quad \forall i,k,l,j \quad i\neq j, i\neq k, i\neq l$$

(12)
$$\sum_{m} y_{atijm} \geq f_{atkijl} \quad \forall i,k,l,j \quad i\neq j, k\neq l$$

(13)
$$\sum_{m} y_{atijm} \geq f_{aklij} \quad \forall i,k,l,j \quad i\neq j, i\neq k, j\neq k, j\neq l$$

(14)
$$x_{atiklj} \geq 0 \quad \forall i,k,l,j,$$
$$y_{atijm} \in N \cup 0 \quad \forall i,j,m$$

Rows (3) to (6) describe the one-period objective function of each airline. If integer restrictions are computationally too expensive, they may be neglected by coarsening the time-scale of the model; for example, moving from a monthly to a yearly view, so that fractional numbers (in particular the number of flights) do not pose a serious problem. Strategic decision variables are the number of seats offered on each flight route and the number of flights operating on each flight leg. With regards to uniform model presentation, the number of seats offered is indexed redundant and independent of the number of stopovers by four lower subscript letters describing flight route structure. Passenger costs are subdivided into passenger charges paid by the airline to the airport and other variable passenger costs. Like Adler and Berechman (2001, p.380), we have subdivided airport charges into passenger charges paid to the departure airport for each passenger carried and landing charges paid to the arrival airport, based on aircraft type and size. Passenger charges are further subdivided into full price, paid to the first departure airport, and transfer price, which is paid at each subsequent hub, if the flight route to the chosen destination of a passenger includes at least one stopover. This airport charges schedule is clearly arranged but also offers enough flexibility to include other relevant charges, such as handling, night and noise charges. The demand function and the inverse demand function $P^a(\ldots)$, respectively, are defined for each particular flight route. (…) represents the independent variables of the inverse demand function, such as the number of seats offered, flight frequency, time of flight and number of stopovers, in addition to the number of seats and flights offered by competing airlines. Thus, row (3) applies to flights with two stopovers, row (4) corresponds to flights with one stopover and row (5) relates to nonstop flights. Expanding the model to flights with more than two stopovers is straightforward; however, the majority of flights have, at most, two stopovers and thus explicitly including such flights only complicates presentation without adding anything substantially new. Row (6) describes the fixed costs of each flight, composed of aircraft operating costs and landing charges at the arriving airport.

Rows (7) to (14) represent the constraints each airline has to comply with in their planning process. Constraint (7) ensures that aircraft capacity restrictions are fulfilled on each flight leg, whilst constraint (8) balances the number of aircrafts in both directions between two airports, in order to support subsequent tactical and operational network planning (Jacquemin, 2006, p.175). Constraint (9) limits the available runway capacity at each airport. The sigma sign includes airline *a* and all competing airlines that take precedence over airline *a*; for example, because of grandfather rules. Each aircraft uses the runway of an airport for arrival and departure, whilst F_{mi} allows for different levels of runway capacity consumption, depending on aircraft type and airport. Constraint (10) limits each airport's terminal capacity available to each airline for flight routes with two, one and no stopovers. Like constraint (9), the sigma sign includes airline *a* and all competing airlines that take precedence over airline *a*. Transfer passengers use the terminal for arrival and departure, whereas passengers emplaned and deplaned use the terminal only once. Rows (11) to (13) require the number of flights between two airports to be above the corresponding flight frequency of the corresponding inverse demand function. (14) describes the domain of the strategic decision variables.

As we are looking at networks from a strategic and long-term point of view, no complicated airline pricing strategies are included (Adler, 2005, p.61). Different fare segments (for example, economy, business and first class) are not included in the model formulation presented above, but introducing different seat categories is straightforward and essentially the same as duplicating existing O-D relations and introducing a particular inverse demand function for each copy. However, for ease of presentation, we neglect different seat categories here.

Airports

In this section, we present the one-period decision problem of each airport pursuing a competitive strategy; i.e., one-period profit maximisation in this chapter.

(15) $$Max\ \pi_{it}^{ap}\left(z_{it}^{r}, z_{it}^{t}\right) = \sum_{a,j,m}\left(P_{itm}^{l}(\ldots) - C_{itam}^{ac}\right) * y_{atjim} +$$

(16) $$\sum_{\substack{a,k,l,j \\ i\neq j, j\neq k, k\neq l, \\ i\neq k, j\neq l, i\neq l}}\left(\left(P_{it}^{t}(\ldots) - C_{it}^{t}\right) * \left(x_{atkijl} + x_{atkjil}\right) + \left(P_{it}^{f}(\ldots) - C_{it}^{f}\right) * x_{atikjl}\right) +$$
$$\sum_{\substack{a,k,l \\ i\neq k, \\ k\neq l, i\neq l}}\left(\left(P_{it}^{t}(\ldots) - C_{it}^{t}\right) * x_{atkiil} + \left(P_{it}^{f}(\ldots) - C_{it}^{f}\right) * x_{atikkl}\right) +$$
$$\sum_{\substack{a,j \\ i\neq j}}\left(\left(P_{it}^{f}(\ldots) - C_{it}^{f}\right) * x_{atiijj}\right) -$$

(17) $$C_{it}^{tf} - C_{it}^{tv} * z_{it}^{t} -$$

(18) $$C_{it}^{rf} - C_{it}^{rv} * z_{it}^{r}$$

Subject to:

(19) $$2 * \sum_{\substack{a,j,m \\ i\neq j}} F_{mi} * z_{atjim} \leq z_{it}^{r}$$

(20)
$$\sum_{\substack{a,k,l,j \\ i\neq j, j\neq k, k\neq l, \\ i\neq k, j\neq l, i\neq l}} \left(2*\left(x_{atkijl}+x_{atkjil}\right)+x_{atikjl}+x_{atklji}\right)+ \sum_{\substack{a,k,l \\ i\neq k, k\neq l, \\ i\neq l}} \left(2*x_{atkiil}+x_{atikkl}+x_{atklli}\right)+\sum_{\substack{a,j \\ i\neq j}}\left(x_{atiijj}+x_{atjjii}\right)\leq z_i^t$$

(21)
$$z_{it}^r, z_{it}^t \geq 0$$

Rows (15) to (18) describe the one-period objective function of each airport. An airport's decision variables are terminal and runway capacity that is supplied. However, airports communicate their passenger and landing charges to airlines, which are dependent on the level of terminal and runway capacity at each airport. These charges are described by inverse demand functions with independent variables (…). For ease of presentation, passenger charges have been only subdivided into full and transfer without referring to, for example, destination type: however, including more details poses no difficulties. The same is true for landing charges, which are simply categorised by aircraft type. Row (15) specifies aircraft-related revenues and costs, whereas row (16) describes passenger-related revenues and costs. Rows (17) and (18) specify the cost of supplying terminal and runway capacity: they are composed of a fixed-cost pool and a variable part, according to the level of capacity supplied.

Rows (19) to (21) represent the constraints each airport has to fulfil. Constraint (19) ensures that the number of aircraft movements does not violate runway capacity restrictions of the airport, whilst constraint (20) requires the number of passengers handled at the airport to be below its maximum terminal capacity. Constraint (21) describes the domain of the strategic decision variables.

Air passengers

Passengers' air travel demand is modelled for each combination of airline and flight route. The demand function approach in this paper is based on the full price demand model (De Vany, 1974, pp.77; Oum et al., 1995, p.841; Panzar, 1979, p.92) and the product characteristics approach by Lancaster (1966). The demand a carrier attracts on a specific flight route depends on the number of seats offered and quality of service provided and this also applies to flights that serve as a substitute; however, the degree of substitution may vary, depending on the quality of service supplied. Passenger demand is influenced by many attributes that form the quality of service provided (e.g., Alamdari and Black, 1992; Hess et al., 2007; Bieger et al., 2007). These attributes can roughly be subdivided into the following categories: travel cost, travel time, number of (daily) connections, number of stops and comfort. The model approach chosen allows the capturing of competition with homogeneous products, in addition to the more general case of product differentiation. Likewise, Oum et al. (1995, pp.838) state that airlines are modelled as multiproduct firms, choosing the O-D destinations and flights routes they serve; however, including the network configuration of each airline affects the nature of interaction between the markets served.

As airlines are modelled as Counot competitors and thus the number of flights and seats provided represent their strategic decision variables, we work directly with the inverse demand function, which we define as follows:

$$(22) \qquad P_{atiklj} = \alpha_{atiklj1} - \alpha_{atiklj2} * \sum_{\substack{b,\\ mopn \in V}} \beta_{btmopn}^{atiklj} * x_{btmopn} + \sum_{n>2} \alpha_{atikljn} * q_{atikljn}$$

The last sum in (22) is the weighted sum of the service quality attributes of a particular route, which describes the preferences of air passengers. The coefficient β_{btiklj}^{atiklj} serves as a measure of homogeneity and is therefore defined for values between zero and one. The closer the value approaches one, the more passengers view two flight routes as almost perfect substitutes. In turn, similarity between two flight routes depends on the Euclidean distance between their characteristics; thus, β_{btiklj}^{atiklj} is defined as:

$$(23) \qquad \beta_{btmopn}^{atiklj} = \frac{1}{1 + \sqrt{\sum_{n>2} \alpha_{atikljn} * \left(q_{btmopnn} - q_{atikljn}\right)^2}}$$

To reduce computational costs, especially in large-scale applications with thousands of flight routes, only those flight routes with a β_{btiklj}^{atiklj} beyond some predefined value may be considered in (22).

(22) and (23) show that the price-setting behaviour of airlines on their operated flight routes is influenced by their own flights supplied on different routes, in addition to flights operated by competing carriers. The more air passengers perceive different flight routes as substitutable, the higher the pressure on prices. Thus, ticket prices on a particular flight route of an airline depend both on the quality of service provided and the quality of potential substitutes.

3.2 Modelling market dynamics

Empirical reaction function and learning dynamics

Competitors' responses to own actions are not assessed through introspection; i.e., by taking the role of each competitor, solving their optimisation problems (including all interdependencies between competitors), analytically deriving their best response function and inserting them into their own decision problem, which is then solved. Such an approach would, in a number of cases, be computationally intractable for problems of a modest size and, moreover, it assumes that players are perfect rational individuals with almost unlimited computing abilities. Thus, the approach adopted in this paper means a partial departure from assuming perfect rational individuals with unlimited computing abilities towards a behaviour based on observed actions of opponents: each airline directly assesses competitors' reactions by means of a so-called empirical reaction function (ERF) to approximate individual behaviour locally:

$$(24) \quad x_{btiklj} = Max\left\{x_{b(t-1)iklj} + \Delta^{atmopn} x_{btiklj} * \left(x_{atmopn} - x_{a(t-1)mopn}\right); 0\right\} \quad \forall b \neq a; mopn, iklj \in V$$

Rosenthal (1981, pp.93) suggests, in his paper, resorting to the paradigm of decision analysis and assessing each competitor's response directly, instead of a complex game-theoretic analysis. However, Kreps and Wilson (1982, pp.276) observe the ad-hoc assessment of competitors' behaviour in the approach of Rosenthal, but we build on observed past behaviour: every competitor is assumed to have some initial conjecture about the reactions of his fellow competitors, which is then updated each period by exponential smoothing:

$$(25) \quad \Delta^{atmopn} x^{t}_{b(t+1)iklj} = \left(1-\delta\right) * \Delta^{atmopn} x^{t-1}_{btiklj} + \delta * \Delta^{atmopn} x_{btiklj} \quad \forall b; mopn, iklj \in V$$

The parameter δ is bound between zero and one: the more δ approaches a value of one, the more recent observations have an influence on future conjectures.

Exponential smoothing allows accounting for noise in the data, as $\Delta^{atmopn}x_{btiklj}$ itself is a random observation (in a different context see Powell, 2007, pp.98) and the ERF is only a local approximation. An ERF based on exponential smoothing lends more weight to recent observations than past ones, as opposed to 'pure' fictitious play, which Milgrom and Roberts (1991, p.84) criticise because it puts equal weights on each observation, no matter how distant they are. More recent observations are often assumed be a better guide to future behaviour than distant observations. Furthermore, the exponential smoothing mechanism has a similar effect as smooth fictitious play (Shoham and Leyton-Brown, 2009, p.210; Fudenberg and Levine, 1998, pp.110) and accounts for the stochastic nature of opponents' actual behaviour observed each period: here, randomisation serves as a means of protection from mistakes in one's own model of opponent's play. Moreover, in deriving an iterative updating procedure, Powell (2007, pp.181) demonstrates how stochastic gradients and exponential smoothing are related to each other: we wish to find a prediction $\Delta^{atmopn}x_{btiklj}^{t-1}$ for period *t* that produces the smallest squared error between estimated marginal reactions of a competitor and his actual marginal reactions in period *t,* as shown in (26).

(26) $$Min\ F\left(\Delta^{atmopn}x_{btiklj}^{t-1}\right)=\frac{1}{2}\left(\Delta^{atmopn}x_{btiklj}^{t-1}-\Delta^{atmopn}x_{btiklj}\right)^2$$

Therefore:

(27) $$\nabla F\left(\Delta^{atmopn}x_{btiklj}^{t-1}\right)=\Delta^{atmopn}x_{btiklj}^{t-1}-\Delta^{atmopn}x_{btiklj}$$

is called a stochastic gradient because $\Delta^{atmopn}x_{btiklj}$ is a random observation. If we use a standard optimisation sequence to obtain an improved estimate $\Delta^{atmopn}x_{b(t+1)iklj}^{t}$, we obtain:

(28) $$\Delta^{atmopn}x_{b(t+1)iklj}^{t}=\Delta^{atmopn}x_{btiklj}^{t-1}-\delta*\left(\Delta^{atmopn}x_{btiklj}^{t-1}-\Delta^{atmopn}x_{btiklj}\right)$$

However, this is the same as (25). Powell (2007, pp.183) describes several heuristics to find appropriate values for δ.

Flight frequency determines service quality and thus affects homogeneity between different flight routes in (23). Thus, competitors' level of flight frequency on several flight routes influences the ticket price (22) an airline can realise on a particular flight route. Hence, the ERF concept is used to model competitors' reactions with regard to flight frequency on a given flight route, subject to the number of seats airline *a* offers on a particular flight route (compare with (24)):

(29) $$f_{btiklj} = Max\left\{f_{b(t-1)iklj} + \Delta^{atmopn} f_{btiklj} * \left(x_{atmopn} - x_{a(t-1)mopn}\right); 0\right\} \quad \forall b \neq a; mopn, iklj \in V$$

Individual conjectures, with regards to competitors' flight frequency, are updated along the lines of (25) every period. Further service quality variables include time of flight, number of stopovers, comfort and ticket price. However, the first three attributes are implicitly included in the definition of seat capacity per flight route: if an airline decides on a particular seating capacity on a given flight route, they automatically decide on a particular level of time of flight, stopovers and comfort. Here, we neglect different types of aircraft that differ, in terms of time of flight and comfort on a particular flight route, as differences are usually small. Accounting for different models of aircraft causes no problems but complicates model presentation without adding anything substantially new. However, ticket price represents a dependent variable, as we model Cournot quantity competition.

To conclude, from the point of a particular airline, their competitors are fully described by their ERFs. Competing airlines are subdivided into real airlines and virtual airlines; each of these may be further subdivided according to airline types: for example, full service network carrier and low-cost carrier. Real airlines are those that actually compete in a market, whereas virtual airlines do not currently compete in a market but may do so in the future. Real airlines are updated solely on the basis of their own observed behaviour, whereas virtual airlines are updated on the basis of the average observed behaviour of airlines of their type.

Demand for terminal and runway capacity represents derived demand from aircraft and passenger movements. These relationships are modelled on the basis of inverse demand functions $P_{it}^{f}\left(z_{it}^{t}\right)$, $P_{it}^{t}\left(z_{it}^{t}\right)$ and $P_{itm}^{l}\left(z_{it}^{r}\right)$, which depend on the terminal and runway capacity supplied by an airport. However, air passengers' inverse demand functions are based on a large data set, whereas airlines' inverse demand functions depend on the profit-maximising behaviour of a rather small sample of airlines, which may differ substantially in their individual attributes. Moreover, the relevant set of airlines can vary over time, as new airlines enter the market and present airlines leave the market or undergo a change. Therefore, we employ the general structure of the aforementioned ERFs to model individual conjectures and the updating of each airport:

(30)
$$\begin{aligned} P_{it}^{f} &= P_{i(t-1)}^{f} + \alpha_{it}^{f} * \left(z_{i(t-1)}^{t} - z_{it}^{t}\right), \\ P_{it}^{t} &= P_{i(t-1)}^{t} + \alpha_{it}^{t} * \left(z_{i(t-1)}^{t} - z_{it}^{t}\right), \\ P_{itm}^{l} &= P_{i(t-1)m}^{l} + \alpha_{itm}^{l} * \left(z_{i(t-1)}^{r} - z_{it}^{r}\right) \quad \forall m \end{aligned}$$

The parameters α_{it}^{f}, α_{it}^{t} and α_{itm}^{l} are updated along the lines of (25) every period. Competition between airports is assumed to be weak, compared to competition between airlines; thus, airports are modelled as local monopolists. The principal reason for this assumption is the fixed location of airports and thus they have only a limited scope of actions. An increased distance between airports and binding capacity constraints further reduce the level of competition between airports. Nevertheless, limited competitive relations between airports may be included in the parameters α_{it}^{f}, α_{it}^{t} and α_{itm}^{l} of (30), to some degree, but airports are not modelled to carry out specific reaction functions like (25) and (29).

Market entry/exit probability function

Many models of learning, with regards to games, force players to be rather 'unsophisticated'; i.e., the players can only use information about past play. However, experienced players make use of information about the past, in addition to considering competitors' information, payoffs and rationality (Milgrom and Roberts, 1991, p.84). In this paper, past play is essentially reproduced in the updating mechanism of the ERFs. In order to incorporate other factors, such as competitors' information, payoffs and rationality, we introduce in this section the so-called market entry/exit probability function (MEEP), which describes the likelihood of a specific number of competitors being active within a market in future periods. Market entry and exit is modelled as a choice of 'nature' and thus a chance event from the point of view of the individual market actor; however, this choice depends, to some degree, on the behaviour of market actors. Typically, market profitability is raised in the search of relevant variables that have a major impact on the future number of participants in a market: the higher the market profits, the higher the probability that market actors stay in the market or that new entrants are attracted and vice-versa (the probability of staying in the market respectively decreases with diminishing profits and market actors leaving the market). However, average market profitability is not very operational, from the point of view of a single market actor, as opponents' market profits and thus average market profits are usually not really observable to them; thus, as competitors' market profits tend to be positively correlated, we choose the proxy variable 'individual market profit' as a guide to future market structure from the point of view of each individual competitor. With regards to MEEPs, we employ the ideas suggested by Rosenthal (1981, pp.93) to model the entry and exit of competitors as a chance event, where competitors are assumed to arrive at some subjective probability distribution, with regards to those events. Hence, every market actor estimates market-specific relationships between the probability of meeting a particular number of next period market participants and his own current market profits:

(31) $$P_{atikljn} = \frac{e^{U_{atikljn}}}{\sum_m e^{U_{atikljm}}} \quad \forall iklj \in V$$

(32) $$U_{atikljn} = \phi_{atiklj1} + \phi_{atiklj2} * \pi_{a(t-1)ikljn} \quad \forall iklj \in V; \phi_{atiklj2} \geq 0$$

The profits in (32) are assumed to result from rational and profit-maximising behaviour, in order to serve as an indicator for future market entry and exit. If they resulted from discretionary inefficiency and gross wasteful behaviour, they would not serve as a reasonable indicator. Typically, $\phi_{atiklj1}$ decreases and $\phi_{atiklj2}$ increases with rising index *n,* if the number of future market actors increases when market profitability increases. Thus, the cost of deterring entry or promoting exit differs by airline characteristics, depending on whether a competitor is weak or strong, which is reflected in their parameters of (32). The optimal degree of entry deterrence and exit promotion is integrated into an airline's profit maximising behaviour and thus depends on their individual characteristics. The general mechanism generating market entry and exit is common knowledge but the individual MEEPs that describe the actual estimated relationship between one's own profits and the probability of market entry and exit is private to each competitor.

Heuristic equilibrium concept

Airports only base their strategies on inverse demand functions, such as (30). As they are assumed to be local monopolists, their actions do not vary in advance from period to period and thus lack inter-temporal interdependencies. Therefore, for airports, net present value maximisation is equivalent to maximising one-period profit.

In the case of airlines, however, it is more complicated: net present value maximisation is not equivalent to maximising one-period profit, due to inter-temporal strategic interdependencies modelled by ERFs (24), (29) and MEEPs (31). Entry deterrence is such an example of inter-temporal strategic interdependencies. Thus, we mainly focus on the airline case, as the airport case is straightforward.

Figure 1 illustrates a stochastic infinite horizon decision problem of a particular airline, where markets are defined by flight routes *i→k→l→j*. Different markets are connected by the passengers' inverse demand functions (22) and ERFs (24), (29). Circles represent states $S_{tiklj} = n$ and describe a single period decision problem of an airline for period *t* in market *i→k→l→j,* given a fixed number *n* of competitors (including (3) – (14), (24), (29) and (31)). Typically, only few airlines compete on a particular flight route and thus the maximum value of *n* is rather small. At the beginning of the planning cycle, airlines adjust their strategic variables for each period, in order to maximise their expected net present value: they consider any new information they may have learned up to the current period. Different states are distinguished by a time index *t* and a number *n* of competing airlines with certain decision-relevant characteristics. Airports are assumed to move prior to airlines in the game and their one-period decision problem is represented by (15) – (21) and (30).

Therefore, airlines are informed about airport capacities available, in addition to landing and passenger fees at each airport, prior to their decisions. Arrows between circles describe transition probabilities $P_{atikljn}$ (MEEPs) between states. Dotted arrows represent state probabilities for period 1 and arrows of the same colour add up to one. The decision-relevant characteristics of competing airlines are represented in each airline's ERFs, whereas capacity supply, landing and passenger fees represent the decision-relevant characteristics of an airport, from the point of view of airlines.

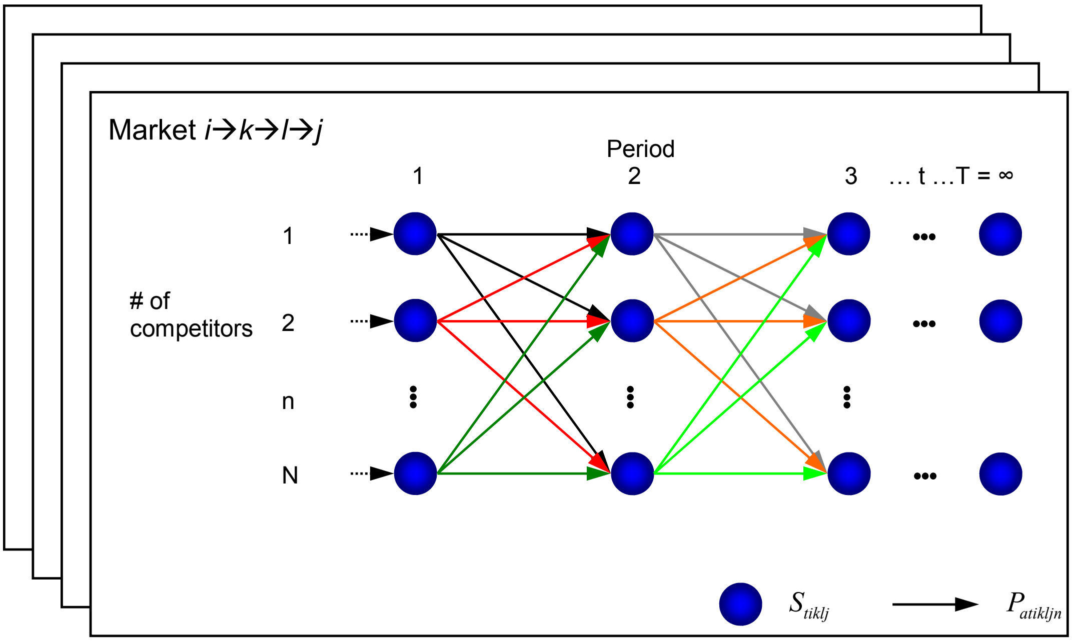

Figure 1: Strategic decision problem of an airline

A dynamic stochastic programme of a size to model real-world problems is typically manageable if it comprises only two to three periods (Schrage, 2006, p.355). The complexity of the decision problem depicted in Figure 1 grows quickly with each period added and is already barely manageable if the number of periods exceeds three for realistic sized problems. However, simply cutting off the problem after a certain number of periods is not an adequate solution, due to reputational effects: each airline tends to have an incentive to behave more aggressively in earlier periods, in order to encounter less competition in later periods and reap the rewards of such a strategy. If there is a last period, airlines have no more incentive to maintain their reputation beyond that period, which they then, possibly, 'milk'. However, the model's last period is, of course, not the last period of the real-world problem and thus the value of landing in a state with fewer competitors in the last period tends to be underrated (see (33)). This would cause no serious problems if we employed a continuous planning model, where the planning horizon lies well beyond the next planning cycle. However, the problem structure typically does not allow the adding of too many periods and the adding of only a few periods does not really resolve this issue. Therefore, the purpose of this section is to find a reasonable approximation for the decision problem depicted in Figure 1.

The background for our approach is Bellman's equation (e.g., Powell, 2007, pp.48):

$$x_t^*(S_t) = \max_{x_t}\left(\pi_t(S_t, x_t) + \frac{1}{1+i} * V_{t+1}(S_{t+1})\right) \tag{33}$$

x_t represents the decision in period *t* and S_t describes the current state in period *t*. x_t is chosen to maximise the sum of the one-period contribution $\pi_t(x_t, S_t)$ and the discounted value of landing in state S_{t+1}, which is represented by $\frac{1}{1+i} * V_{t+1}(S_{t+1})$. Figure 2 and Figure 3 illustrate the modelling approach. We subsequently denote the one-period contribution of (33) as short-term profit (STP) and the discounted value of landing in state S_{t+1} of (33) as long-term profit (LTP).

One-period contribution of (33) is modelled as a two-period game, in order to allow for reputational effects between two periods. The discounted value of landing in a state in (33) is modelled as the Nash equilibrium of an infinitely repeated game with discounted payoffs. However, the stage game again comprises of two periods, in order to model reputation effects between two adjoining periods. The repetition of stage game Nash equilibrium in every period is also equal to a Nash equilibrium of the repeated game (Mailath and Samuelson, 2006, p.191). As a side constraint, we require the strategic decisions to be equal in both periods. This has no effect, in terms of the strategy being a Nash equilibrium, if the interest rate takes a value of zero. However, if the interest rate is greater than zero, this strategy is no longer a Nash equilibrium but only an approximation, which becomes worse with an increasing interest rate. In spite of this, we employ this approximation, as the interest rate is typically small and thus the bias of the approximation tends to be rather small also.

The STP decision problem consists of real airlines, where possible, and is filled up with virtual airlines as is necessary. The reason for this approach is that the STP decision problem predominantly models near-term competition between actual airlines, whilst the LTP decision problem consists only of virtual airlines: its purpose is to model the long-term strategic position in airline competition. To link the STP decision problem with the LTP decision problem, we require each airline's decision in the last period of the STP decision problem (period 2) and the first period of the LTP decision problem (period *t*) to be identical, thus preventing reputation milking in the last period of the STP decision problem.

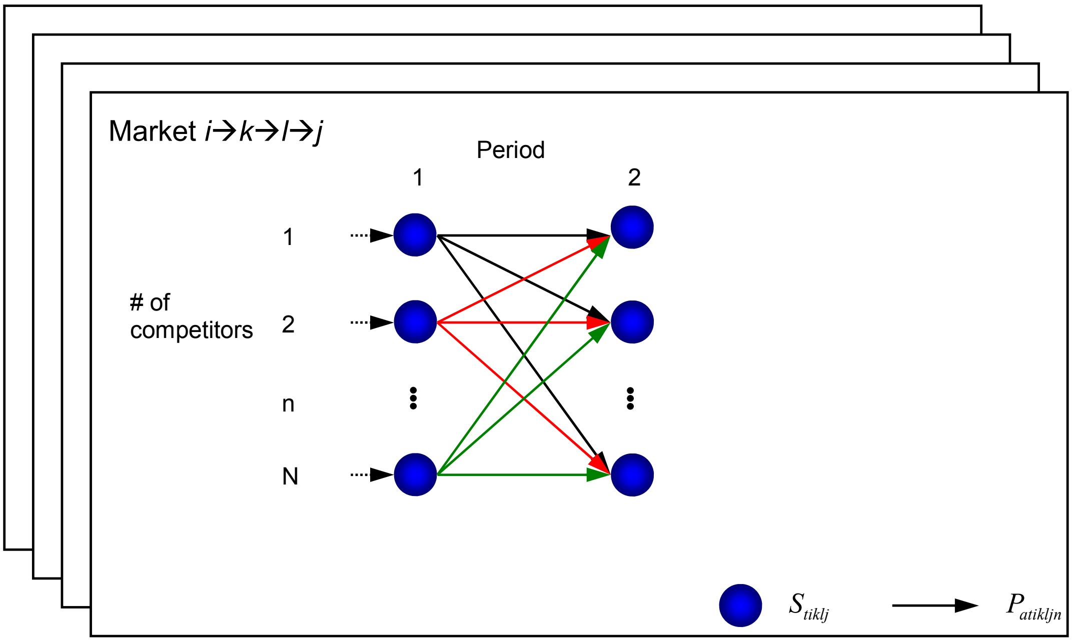

Figure 2: Modelling airlines' short-term payoffs

Therefore, the STP decision problem of an airline *a* is represented as:

(34) $$Max\ \pi_a^{stp}\left(x_a, y_a\right) = \sum_{iklj \in V, n, m} \left(P_{a1ikljn} * \left(\pi_{a1ikljn} + \frac{1}{1+i} * P_{a2ikljm} * \pi_{a2ikljm} \right) \right)$$

Subject to: (7) – (14)

(35) $$x_{a2} = x_{at} \qquad \forall t > 2$$

$P_{a1ikljn}$ are initial probabilities of the different states in period 1 and correspond to the dotted arrows in Figure 2. Remember, $P_{a2ikljm}$ depend on the profits achievable in period 1 and thus on the number of competitors in that period. For clear arrangement, we have disaggregated one-period profits of an airline by flight routes (*iklj*) compared to (3) – (6). Each airlines' ERFs are included in $\pi_{atikljn}$ and, in a Nash equilibrium, each airline maximises (34), subject to the constraints (7) – (14) and (35).

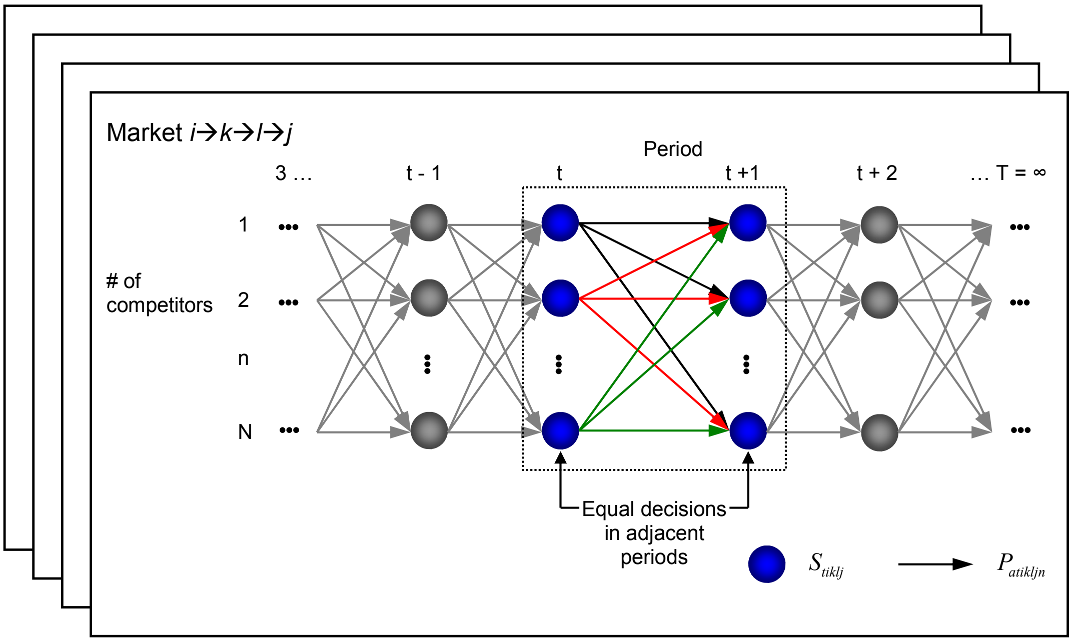

Figure 3: Modelling airlines' long-term payoffs

The LTP decision problem of an airline *a* is defined as:

$$(36) \qquad Max\ \pi_a^{ltp}\left(x_a, y_a\right) = \frac{1}{i\left(1+i\right)^2} \sum_{iklj \in V, n, m} \left(P_{atikljn} * \left(\pi_{atikljn} + \frac{1}{1+i} * P_{a(t+1)ikljm} * \pi_{a(t+1)ikljm} \right) \right)$$

Subject to: (7) – (14) and (35)

$P_{atikljn}$ are initial probabilities of the different states in period *t*. They are initialised by some estimates; for example, on the basis of the STP decision problem, and they are updated during each new planning cycle, according to the observed frequencies of the actual number of competitors in each period through exponential smoothing (see (26) – (28)). In common with the STP decision problem, $P_{a(t+1)ikljm}$ depends on the profits achievable in period *t* and thus on the number of competitors in that period. Payoffs of period *(t+1)* are discounted to the present value of period *t* and then the present value of the infinite sum of stage game payoffs from period 3 to infinity is computed.

The complete decision problem to approximate the net present value maximisation consists of the sum of (34) and (36), subject to (7) – (14) and (35) (at which (35) is eliminated by insertion). There are, essentially, only two different periods for which decisions are to be made, as we require decisions of period 2, *t* and *(t+1)* to be identical. This drastically reduces the number of strategic decision variables and thus model complexity, compared to a case with different decisions for each period in an infinity horizon decision problem (or, at least, a finite horizon decision problem with an ample number of periods). However, depending on the case, the STP decision problem can be extended to more than two periods, if the model remains manageable, and thus the accuracy of the approximation is increased. However, it is typically very expensive (computationally) to increase the number of periods of the STP decision problem so much so that the reputation milking problem sufficiently disappears and we can do without the LTP decision problem. However, the general approach does not change and the flow chart in figure 4 illustrates the complete model's course of action.

To obtain good initial starting values, the model is run through a number of periods; however, the results of the first few periods heavily depend on the starting values and therefore, should not be overestimated. The model may converge after a number of periods to a stable market structure, which we subsequently call a 'long-term equilibrium' as in our model players' decisions of each period represent, by definition, a Nash equilibrium, given their information status at the beginning of each period. The iterative approach, which we have primarily chosen to model learning effects and the temporal development of air transport markets, is also employed as a method for searching for Nash equilibrium in static models of air transport markets (for example Adler, 2005, p.64; Evans et al., 2008, p.2; Evans and Schäfer, 2009, p.2; Hansen, 1990, p.38). However, Adler (2005, p.64) and Hong and Harker (1992, p.317) note that such a long-term Nash equilibrium may not exist or that it is not unique, if one does exist. Sufficient conditions for the existence of a unique Nash equilibrium (Adler, 2005, p.64) are:

- The strategy set of each player is bounded, convex and closed.
- The profit function for each player is concave, with respect to the player's strategy set assuming fixed competitors' strategies.

- All profit functions are continuous over the strategy sets of all players.

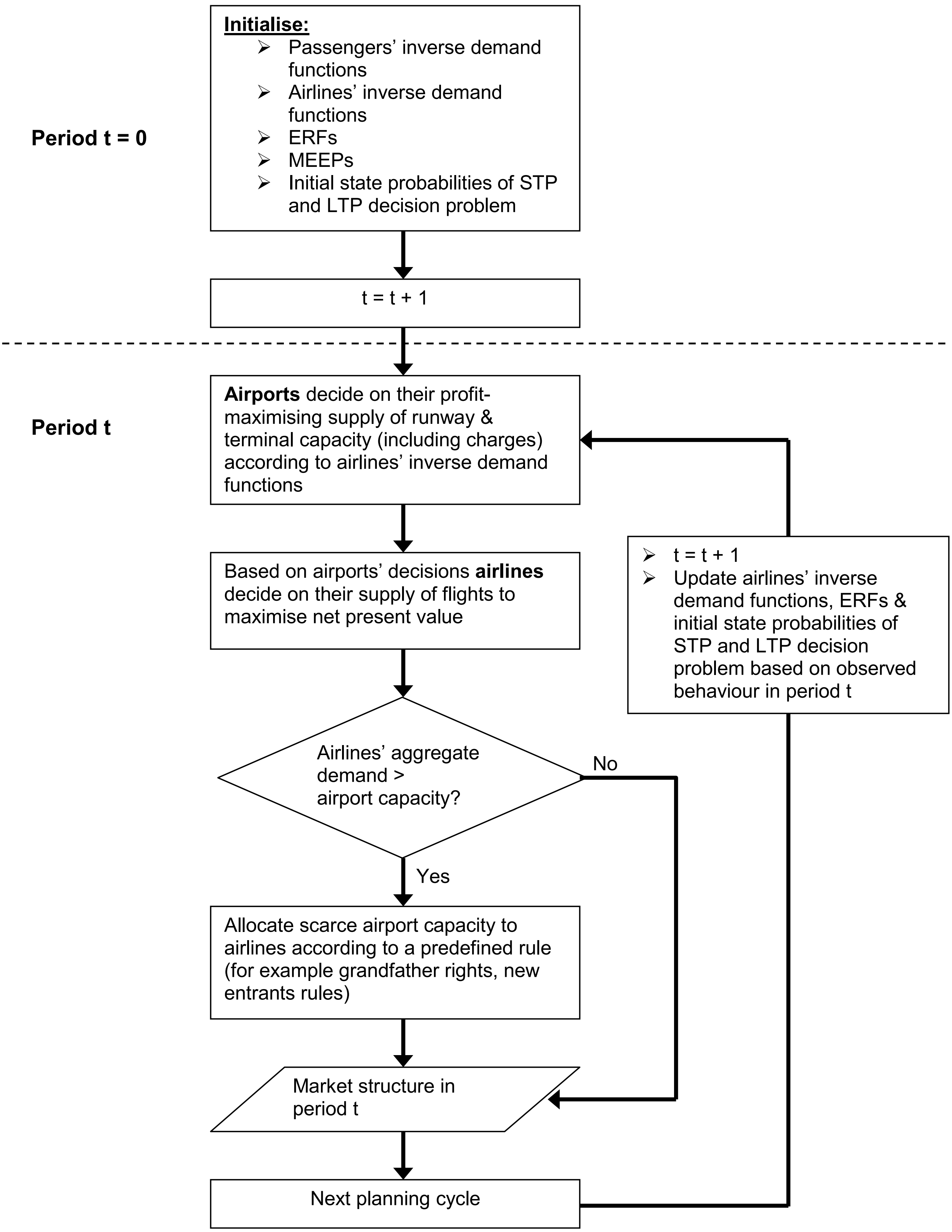

Figure 4: Model flow chart

Typically, the profit functions cannot be guaranteed to be concave. Adler (2005, p.64) and Hansen (1990, p.39) call situations in which the model cycles around two or more possible solutions without convergence or in which the majority of decision variables achieve convergence, with only a few remaining divergent, a 'quasi-equilibrium'. One reason for the non-existence of a stable equilibrium may be an empty core (Button and Nijkamp, 1998, pp.13; Button, 2003, pp.5; Gillen and Morrison, 2005, p.170). An allocation is said to be 'in the core' when there is no group of market participants within the economy that could be better off by trading amongst themselves; i.e., no further gains from trade are possible for any group or subgroup (Button, 2003, p.7). Occurrences that support an empty core and are of particular interest in this paper are (Button and Nijkamp, 1998, pp.21; Button, 2003, p.9):

- The existence of fixed costs and a low variation in suppliers' minimum average costs
- Low elasticity of demand
- Large capacity of a supplier, relative to market size

However, as the model's main objective is to describe market developments over time, the non-existence of such a long-term equilibrium does not limit the scope of the model.

4. Summary and discussion

In this paper, we have presented an approach to model competition in air transport markets, in which we focus on airlines, airports and air passengers. We assume that airlines and airports maximise their profits and air passengers maximise their utility and, compared to related models that mainly focus on what we call a 'long-run equilibrium', this model is primarily aimed at explaining temporal market developments and learning effects, in which a long-run equilibrium is not indispensable. The approach we have chosen is a mixture of decision theory and game theory: we introduce the concepts of a so-called empirical reaction function and market entry/exit probability function, in order to model opponents' reactions on own actions and the market entry and exit of competitors. However, an essential element of this approach is that airlines and airports learn about the behaviour of individual competitors and the market in general, over time, on the basis of observed past actions, which influences future actions. Airlines care about the number of future competitors in the market and thus maximise their net present value to account for the influence of their current strategic position in the competition on future profits. Airlines calculate the benefits of their own market entry, in addition to entry deterrence and market exit of competitors. However, these issues substantially increase model complexity and, therefore, a heuristic equilibrium concept is presented, in order to find a sound approximate model solution for each period.

In the context of market entry and entry deterrence, Selten's chain store paradox (Selten, 1978, pp.127) has received much attention: in a multi-period game, with perfect information about each other's payoffs, entry deterrence is not a perfect equilibrium and thus not rational. The key factor driving this conclusion is that it is common knowledge (Aumann, 1976, pp.1236; Milgrom, 1981, pp.219) that accommodation is the best response to entry and vice-versa. Therefore, reputational effects play no role in this model (Milgrom and Roberts, 1982, pp.282). However, Kreps and Wilson (1981, p.226) note that the common knowledge assumption regarding the monopolist's payoff is very strong in real-life contexts.

The models of Kreps and Wilson (1982, pp.253) and Milgrom and Roberts (1982, pp.280) attempt to explain Selten's chain-store paradox, by introducing incomplete information about the nature of the incumbent into the game.

Kreps and Wilson introduce a small probability p that, at any given stage of the game, a predatory response is directly more profitable for the incumbent than sharing the market. Information about actual payoffs of the incumbent is incomplete from the point of view of an entrant, which is an important difference to Selten's model. Probability p is updated each round, on the basis of observed behaviour. However, the results of the model depend on choosing the nature of p and thus the information of the entrants about the payoffs of the incumbent which is, to some degree, an ad-hoc assumption.

In the Milgrom and Roberts model, there is incomplete information about the nature of the incumbent: There is, in each case, a small probability that the incumbent always reacts aggressively and cooperatively, when market entry occurs. These two probabilities reflect the doubts that entrants have, regarding whether their modelling of the incumbent's behaviour is correct.

In our approach, each player's incomplete information about the nature of his opponents is modelled by means of ERFs, which are updated each period on the basis of observed behaviour, and MEEPs. In the models of Kreps and Wilson and Milgrom and Roberts, the probability of entry deterrence by the incumbent depends on the entire history of the game and, once the incumbent has failed to prey, future preying has no effect and entry occurs. However, in this paper, whether market entry occurs or not in a particular period depends solely on the state and the decisions of the players of the previous period, rather than on the complete history. If cooperative behaviour has occurred in the past (maybe as a 'mistake', as players have only incomplete information and limited rationality), entry deterrence is still possible in the future. Here, players' information structure is founded on observed past behaviour (→ ERFs) and econometric functions (→ MEEPs).

References

Adler, N. (2001), “Competition in a deregulated air transport market”, European Journal of Operational Research, 129(2), 337-345.

Adler, N. (2005), “Hub-spoke network choice under competition with an application to western Europe”, Transportation Science, 39(1), 58-72.

Adler, N. and Berechman, J. (2001), “Evaluating optimal multi-hub networks in a deregulated aviation market with an application to western Europe”, Transportation Research Part A, 35(5), 373-390.

Aguirregabiria, V. and Ho, C.-Y. (2009), “A dynamic game of airline network competition: hub-and-spoke networks and entry deterrence”, Working paper, University of Toronto.

Alamdari, F.E. and Black, I.G. (1992), “Passengers’ choice of airline under competition: the use of the logit model”, Transport Reviews, 12(2), 153-170.

Aumann, R. (1976), “Agreeing to disagree”, Annals of Statistics, 4(6), 1236-1239.

Ben-Akiva, M. and Lerman, S. (1985), “*Discrete choice analysis: theory and applications to travel demand*”, Cambridge University Press, Cambridge, MA, 1985.

Berechman, J. and Shy, O. (1996), “The structure of airline equilibrium networks”, in: van den Bergh, J.C.J.M., Nijkamp, P. and Rietveld, P., editors, (1996): Recent advances in spatial equilibrium modelling, 138-155, Springer, Berlin.

Bieger, T., Wittmer, A. and Laesser, C. (2007), “What is driving the continued growth in demand for air travel? Customer value of air transport”, Journal of Air Transport Management, 13(1), 31-36.

Brown, G.W. (1951), “Iterative solutions of games by fictitious play”, in: Koopmans, T.C., editor, (1951): activity analysis of production and allocation, 374-376, Wiley, New York.

Brueckner, J.K. and Zhang, Y. (2001), “A model of scheduling in airline networks – how a hub-and-spoke system affects flight frequency, fares and welfare”, Journal of Transport Economics and Policy, 35(2), 195-222.

Button, K. (2003), “Does the theory of the “core” explain why airlines fail to cover their long-run costs of capital?”, Journal of Air Transport Management, 9(1), 5-14.

Button, K. and Nijkamp, P. (1998), “Economic stability in network industries”, Transportation Research Part E, 34(1), 13-24.

Dennis, N. (2007), “End of the free lunch? The response of traditional European airlines to the low-cost carrier threat”, Journal of Air Transport Management, 13(5), 311-321.

De Vany, A. (1974), “The revealed value of time in air travel”, Review of Economics and Statistics, 56(1), 77-82.

Dobson, G. and Lederer, P.J. (1993), “Airline scheduling and routing in a hub-and-spoke system”, Transportation Science, 27(3), 281-297.

Douglas, G. and Miller, J. (1974), “*Economic regulation of domestic air transportation: Theory and policy*”, The Brookings Institution, Washington, DC.

Ehmer, H., Berster, P., Basedow, J. and Jung, C. (2002), “*Liberalisierung im Luftverkehr Deutschlands – Analyse und wettbewerbspolitische Empfehlungen*“, German Aerospace Center (DLR), Cologne, 2002.

Evans, A.D. and Schäfer, A. (2009), “Simulating flight routing network responses to airport capacity constraints in the US”, 9th AIAA Aviation Technology, Integrations and Operations Conference, 1-13, Hilton Head, South Carolina, 21-23 September 2009.

Evans, A.D., Schäfer, A. and Dray, L. (2008), "Modelling airline network routing and scheduling under airport capacity constraints", 8th AIAA Aviation Technology, Integration and Operations Conference, 1-13, Anchorage, Alaska, 14-19 September 2008.

Fudenberg, D. and Kreps, D.M. (1993), "Learning mixed equilibria", Games and Economic Behavior, 5(3), 320-367.

Fudenberg, D. and Levine, D.K. (1998), "*The theory of learning in games*", MIT Press, Cambridge, MA, 1998.

Gillen, D. and Morrison, W.G. (2005), "Regulation, competition and network evolution in aviation", Journal of Air Transport Management, 11(3), 161-174.

Goetz, A.R. and Vowles, T.M. (2009), "The good, the bad, and the ugly: 30 years of US airline regulation", Journal of Transport Geography, 17(4), 251-263.

Gordon, S. (1974), "*Relationships between economics of scale and the shape of air transportation networks*", PhD dissertation, Cambridge, MA, 1974.

Hansen, M. (1990), "Airline competition in a hub-dominated environment: an application of noncooperative game theory", Transportation Research Part B, 24(1), 27-43.

Hansen, M. and Kanafani, A. (1990), "Airline hubbing and airport economics in the pacific market", Transportation Research Part A, 24(3), 217-230.

Hendricks, K., Piccione, M. and Tan, G. (1995), "The economics of hubs: the case of monopoly", Review of Economic Studies, 62(1), 83-99.

Hendricks, K., Piccione, M. and Tan, G. (1997), “Entry and exit in hub-spoke networks”, RAND Journal of Economics, 28(2), 291-303.

Hendricks, K., Piccione, M. and Tan, G. (1999), “Equilibria in networks”, Econometrica, 67(6), 1407-1434.

Hess, S., Adler, T. and Polak, J.W. (2007), “Modelling airport and airline choice behaviour with the use of stated preference survey data”, Transportation Research Part E, 43(3), 221-233.

Hong, S. and Harker, P.T. (1992), “Air traffic network equilibrium: toward frequency, price and slot priority analysis”, Transportation Research Part B, 26(4), 307-323.

Hooper, P. (1998), “Airline competition and deregulation in developed and developing country contexts – Australia and India”, Journal of Transport Geography, 6(2), 105-116.

Jacquemin, M. (2006), “*Netzmanagement im Luftverkehr – Statische und dynamische Planungsmodelle zur Gestaltung von Hub&Spoke-Flugnetzwerken*”, PhD dissertation, Wiesbaden, 2006.

Jeng, C.Y. (1987), “*Routing strategies for an idealized airline network*”, PhD dissertation, Berkeley, CA, 1987.

Kanafani, A. and Ghobrial, A.A. (1985), “Airline hubbing – some implications for airport economics”, Transportation Research Part A, 19(1), 15-27.

Kreps, D.M. and Wilson, R. (1982), “Reputation and imperfect information”, Journal of Economic Theory, 27(2), 253-279.

Lancaster, K.J. (1966), “A new approach to consumer theory”, The Journal of Political Economy, 74(2), 132-157.

Maier, G. and Weiss, P. (1990), "*Modelle diskreter Entscheidungen – Theorie und Anwendung in den Sozial- und Wirtschaftswissenschaften*", Springer, Wien, 1990.

Mailath, G.J. and Samuelson, L. (2006), "*Repeated games and reputations*", Oxford University Press, Oxford, 2006.

Manski, C.F. (1977), "The structure of random utility models", Theory and Decision 8(3), 229-254.

Mason, K.J. (2001), "Marketing low-cost airline services to business travelers", Journal of Air Transport Management, 7(2), 103-109.

Mason, K.J. and Alamdari, F. (2007), "EU network carriers, low cost carriers and consumer behaviour: A Delphi study of future trends", Journal of Air Transport Management, 13(5), 299-310.

McFadden, D. (1974), "Conditional logit analysis", in: Zarembka, P., editor, (1974): frontiers in econometrics, 105-142, Academic Press, New York.

Milgrom, P. (1981), "An axiomatic characterization of common knowledge", Econometrica, 49(1), 219-222.

Milgrom, P. and Roberts, J. (1982), "Predation, reputation, and entry deterrence", Journal of Economic Theory, 27(2), 280-312.

Milgrom, P. and Roberts, J. (1991), "Adaptive and sophisticated learning in normal form games", Games and Economic Behavior, 3(1), 82-100.

Miller, R. (1963), "*Domestic airline efficiency*", MIT-Press, Cambridge, MA, 1963.

Nash, J.R. (1950), "*Non-cooperative games*", PhD dissertation, Princeton, NJ, 1950.

Osborne, M.J. and Rubinstein, A. (1994), "*A course in game theory*", MIT Press, Cambridge, MA, 1994.

Oum, T.H., Zhang, A. and Zhang, Y. (1995), “Airline network rivalry”, The Canadian Journal of Economics, 28(4a), 836-857.

Panzar, J.C. (1979), “Equilibrium and welfare in unregulated airline markets”, The American Economic Review, 69(2), 92-95.

Panzar, J.C. (1980), “Regulation, deregulation, and economic efficiency: the case of the CAB”, The American Economic Review, 70(2), 311-315.

Pels, E. (2009), “Network competition in the open aviation area”, Journal of Air Transport Management, 15(2), 83-89.

Pels, E., Nijkamp, P. and Rietveld, P. (2000), “A note on the optimality of airline networks”, Economic Letters, 69(3), 429-434.

Powell, W.B. (2007), “*Approximate dynamic programming – solving the curses of dimensionality*”, Wiley, Hoboken, NJ, 2007.

Robinson, J. (1951), “An iterative method of solving a game”, Annals of Mathematics, 54(2), 296-301.

Rosenthal, R.W. (1981), “Games of perfect information, predatory pricing and the chain-store paradox”, Journal of Economic Theory, 25(1), 92-100.

Schipper, Y., Rietveld, P. and Nijkamp, P. (2003), “Airline deregulation and external costs: a welfare analysis”, Transportation Research Part B, 37(8), 699-718.

Schrage, L. (2006), “*Optimization modeling with LINGO*”, Lindo Systems, Chicago, IL, 2006.

Selten, R. (1978), “The chain-store paradox”, Theory and Decision, 9(2), 127-159.

Shoham, Y. and Leyton-Brown, K. (2009), "*Multiagent systems – algorithmic, game-theoretic, and logical foundations*", Cambridge University Press, Cambridge, MA, 2009.

Takebayashi, M. (2009), "Airlines' behavior for choice of aircraft size and network design: bi-level model", Proceedings of the 13th Air Transport Research Society World Conference 2009, 1-12, Abu Dhabi, VAE, 27-30 June 2009.

Takebayashi, M. and Kanafani, A. (2005), "Network competition in air transportation markets: bi-level approach", Research in Transportation Economics, 13, 101-119.

Train, K.E. (2003), "*Discrete choice methods with simulation*", Cambridge University Press, Cambridge, MA, 2003.

Vowles, T.M. (2001), "The "Southwest Effect" in multi-airport regions", Journal of Air Transport Management, 7(4), 251-258.

Zhang, A. (1996), "An analysis of fortress hubs in airline networks", Journal of Transport Economics and Policy, 30(3), 293-307.