Least Squares Estimation of PSI Networks for Large Scenes with Multithreaded Singular Value Decomposition

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Abstract

In the Persistent Scatterer Interferometry relative height and deformation updates are estimated between nearby persistent scatterers (PSs). In practice, a reference network is established and the absolute values are estimated by integration. However, PSs can be mis-detected and relative measurements mis-estimated. As a consequence, the integration result depends on the integration path. Therefore, inconsistent measurements need to be eliminated. This paper presents our implemented integration technique which is based on a multithreaded Singular Value Decomposition. An advantage of our method is the straight forward error propagation assessment. As a result, the actual network can be improved by removing inaccurate point scatterers.

Keywords: Persistent Scatterer Interferometry, Singular Value Decomposition, Multithreading, Jacobi Transformation, Least Square Estimation

1 Introduction

1.1 Description of the Problem

In Persistent Scatterer Interferometry (PSI) the subject is to retrieve the displacement and height of long time stable scatterers using a stack of SAR interferograms. After data calibration, PS detection, InSAR- and D-InSAR processing and a relative time series analysis, the final processing step is the estimation of the absolute displacement and height values of Persistent Scatterers (PSs) [1]. For each PS, a differential phase is measured, which has contributions from topography, deformation, atmosphere, orbit error and noise. Practically, a reference network of closely located PSs is created. In order to reduce the influence of atmosphere and orbit errors, we only estimate relative DEM and deformation updates between connected points. Finally, the relative measurements in the network are integrated by least squares adjustment. After establishing a consistent network over the whole scene, more points can be connected to the reference points. The creation and integration of the reference network are a challenge for large scenes. Both issues are presented this paper.

1.2 Previous Processing and Problems

In the current operational PSI-GENESIS processor, the reference network is estimated by a least square estimation and the result is tested against an alternative hypothesis: whether there is a better solution with different arcs and different points. For each hypothesis test, an inversion of the complete liner system of equations is necessary. For large scenes, this takes a lot of time and it is difficult to find an appropriate solution at all. Often, incorrect arcs result due to:

- erroneous phase unwrapping in time or
- a wrong deformation model

or incorrect points are utilized due to:

- misdetection of PSs or
- more than a single PS inside the resolution cell.

Such arcs and points need to be identified and eliminated before inverting the linear system.

2 Solving Linear Equations with Singular Value Decomposition

For every m×n matrix A there is a factorization of the form

$$A = USV^T, \qquad (1)$$

which is called Singular Value Decomposition (SVD). U is an orthogonal m×n matrix, V is an orthogonal n×n matrix and S is a diagonal n×n matrix with the singular values on the diagonal. In this section is shown how linear systems of equations can be solved with SVD. Furthermore, we report on a highly efficient algorithm to calculate the SVD which can be implemented supporting multithreading.

Here, we consider matrices A with m>n, which means the corresponding system of liner equations

$$Ax = b \quad (2)$$

(x: vector with n unknowns, b: vector with m observations) that we want to solve with SVD has more observations than unknowns (least squares problem). Since the inversion of an orthogonal matrix ($M^{-1} = M^T$) and a diagonal matrix ($d_i = 1/d_i$ for the diagonal elements d_i) is very easy to calculate, the inverse matrix A^{-1} is practically computed by SVD

$$A^{-1} = (USV^T)^{-1} = VS^{-1}U^T. \quad (3)$$

With the inverse matrix (for rectangular matrices it is called Moore-Penrose-pseudoinverse) the system of linear equations can be solved

$$\hat{x} = A^{-1}b = VS^{-1}U^T b. \quad (4)$$

If a cofactor matrix Q_{bb} with accuracies of the observations b is given, we can scale equation (2) with the weighting matrix $\sqrt{P}$ ($P = Q_{bb}^{-1}$) and a weighted SVD least square estimation can be formulated as

$$\sqrt{P}Ax = \sqrt{P}b\,, \quad (5)$$

which can also be solved with SVD of $\sqrt{P}A$

$$\sqrt{P}A = USV^T\,. \quad (6)$$

The cofactor matrix $Q_{\hat{x}\hat{x}}$ describes the accuracy relations between the estimated unknowns which are calculated with the network geometry and the a priori accuracies of the measurements:

$$Q_{xx} = VS^{-2}V^T. \quad (7)$$

The variance factor a posteriori which describes the quality of the estimation is given by

$$\hat{\sigma}^2 = \frac{v^T P v}{m-n}\,, \quad (8)$$

where $v = A\hat{x} - b$ are the corrections on the observations after the adjustment, m is the number of observations and n is the number of unknowns. Scaling the cofactor matrix with the variance factor yields the covariance matrix

$$\hat{K}_{\hat{x}\hat{x}} = \hat{\sigma}^2 Q_{\hat{x}\hat{x}} \quad (9)$$

which is a measure for the quality of the estimation. For well-conditioned matrices, the pseudo inverse is often calculated by the normal equation ($A^TAx = A^Tb$). In contrast the SVD is a numerically more stable and robust method to calculate it even for ill-conditioned matrices.

The computational complexity of matrix inversion is well known to be high and depends on the matrix dimension N. Typically, $O(N^{2.7})$ operations are required. For large scenes, the reference network can result in matrix dimensions of 5 000 x 30 000. In order to cope with the computation time, we have implemented a multithreaded SVD that is highly adequate and efficient for PSI integration. The algorithm is explained more in detail in [6].

3 Preparation of the Reference Network for Least Squares adjustment

The problems described in section 1.2 require an alternative estimation approach of the absolute values at the network points. Firstly, the relative estimations on the arcs are evaluated. Then, those arcs which are detected to be wrong are rejected. Finally, processing based on our efficient SVD is performed only on the derived stable network. By doing so, processing time and error propagation is notably reduced.

3.1 Rejecting inconsistent arcs

It is the subject to find a conservative field of the arcs. In other words, the integration along the arcs between two points provides the same result for different integration paths.

Because of the estimation errors, “good” arcs have to be selected and “bad” arcs rejected. Therefore, a very dense network with redundant (~10) arcs per points should be created in order to establish multiple closed integration paths. Then, for all cycles with three arcs (see figure 1) the integration residuals for both observations i.e. deformation and topography are determined.

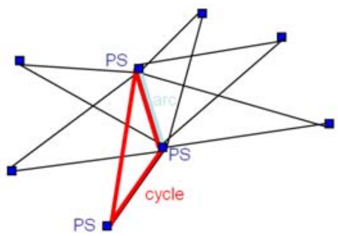

Figure 1: For each arc we determine all three sided cycles which contain it and integrate deformation and topography in that loop. As quality criteria we analyze the integration residuals.

In order to evaluate the quality of an arc, we select all of the three sided cycles that contain it. If the integration along one of those cycles is smaller than a threshold *max_res* that we have fixed (~1m for topography and ~1 mm/year for deformation), then we accept the cycle for this arc. If not, then we reject it. For each arc a ratio of corresponding accepted and not accepted cycles is calculated. If all cycles corresponding to an arc are not accepted, then the arc has clearly to be rejected. If there are both accepted and not accepted cycles to an arc, then we go on with the following iterative process:

1. Calculate the new ratios for the existing arcs and their corresponding cycles
2. Reject all arcs which have the lowest acceptance ratio for both observations
3. Reject all arcs which are controlled in less than two cycles for stability purposes

This procedure is iterated until there are no more arcs to be reject and all left cycles have a residual below the given threshold. With the remaining arcs, we can build a consistent linear system of equations for the least squares estimation.

This procedure tests the arcs only locally but not globally over the whole network, what is sufficient for most PSI networks. If there are still inconsistencies it can be adapted after the adjustment using the resulting covariance matrix.

In large sparse areas the network may be divided into non connected parts during this iterative procedure. In such cases, more separated reference networks and the corresponding independent linear systems of equations are created.

In order to fix the integration constant, a reference point has to be chosen where topography and deformation is known. In the case of more independent networks, one has to be set in each.

3.2 Estimation of Absolute Values with SVD

After setting up the linear system of equations, the absolute values can be estimated by the SVD as described in section 2. By estimating the covariance matrix and analyzing the error propagation, it can be decided whether all points are used for the further processing or some are rejected aposteriori.

For regularization of inverse problems, the small singular values in equation (4) are typically neglected or weighted. These can be related to noise and error propagation. Since we already evaluated the measurements with the algorithm described in section 3.1 and it is assured that the build linear equation (condition number ~300) do not have configuration defects (singularities) we process in our case the data with all singular values and access the accuracy by using the Covariance Matrix $\hat{K}_{\hat{x}\hat{x}}$ of equation (9).

As described above, in our PSI technique, we first establish a reference network covering the whole scene and afterwards we estimate locally more points relative to the reference network. If the reference network is separated in independent clusters it is possible, to merge close adjacent networks by this relative estimation. Therefore, we are looking for points which are not part of the reference network and can be estimated relatively from both of the two networks we want to merge. By analysing only the most reliable of these points we can determine the relative deformation and topography between these networks and connect them to a single one. By applying iteratively this procedure, we connect the network for the whole scene.

4 Results

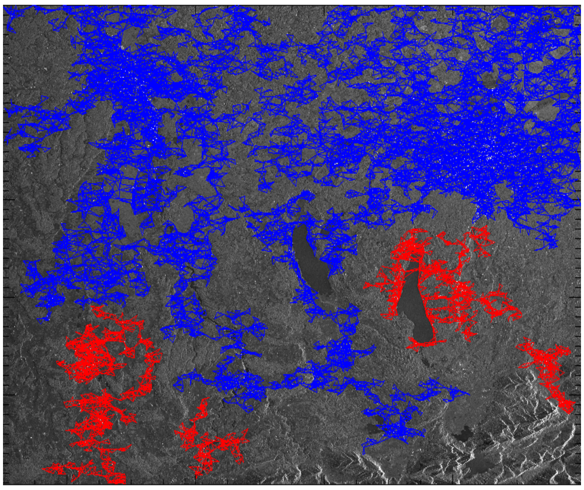

Figure 2: Creation of the reference network. By controlling the integration residuals (3 m for topography and 3 mm/year for deformation) in three sided cycles we reject wrong relative measurements. The network is separated in more not connected regions (blue largest network, red independent networks).

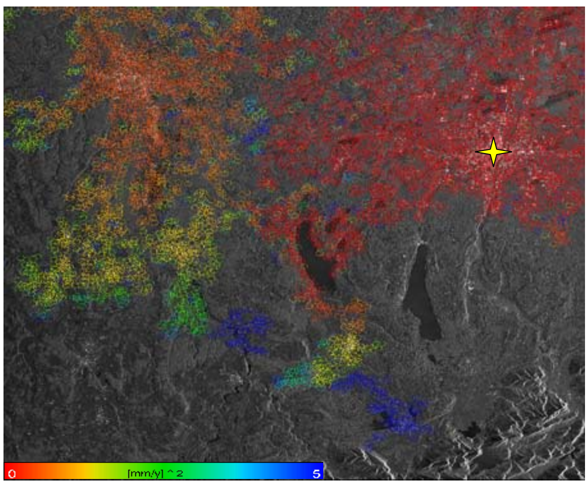

Figure 3: Executing least square estimation with SVD we get the absolute values at the PSs and their variance (colour coded in the figure). Clearly visible is the error propagation: Variance is increasing with distance from the reference point (yellow cross).

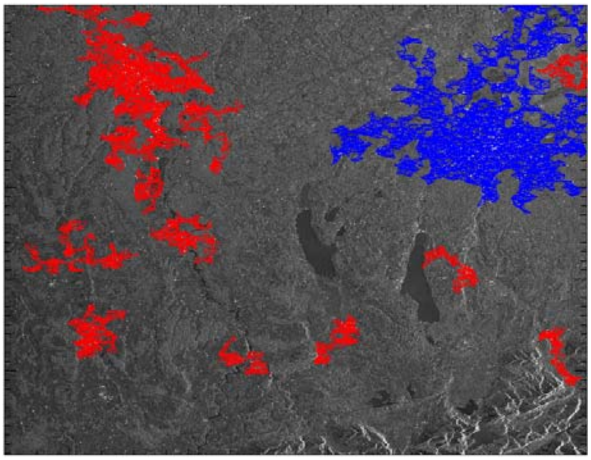

Figure 4: Creation of a more reliable reference network compared to figure 2. Maximum residuals in three sided cycles: 1m for topography and 1mm/year for deformation.

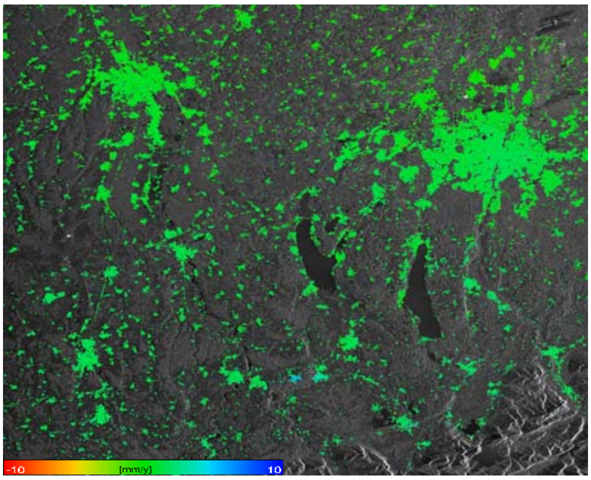

Figure 5: Final result for deformation after processing all independent networks and following merging by processing all available points.

In figure 2, a large network (100km × 100km area in south Bavaria) is shown. It has been processed with the algorithm presented section 3.1. The difficulty pointed out needs to be faced: A separated network with the largest cluster in blue and some smaller networks in red result. Obviously, we get a dense network in city areas and a sparse network in rural areas. In this example we used a relaxed threshold *max_res* for the residual in one cycle (3 m for topography and 3 mm/year for deformation) with a risk of a not reliable result after the least square adjustment with the weighted SVD algorithm.

By checking the variance of the estimated absolute values for each PS (shown colour coded in figure 3) we can see that the precision of the estimates decreases with distance to the reference point (yellow cross). In order to maximise the overall quality, such point is chosen in the geometrical centre of the reference network. In this example test site, it is recommended to accept only the points with a variance smaller than 1 m^2 (1 mm/y^2) and to reject the bad estimated ones (green/blue in figure 3) for the further processing of more points relative to the reference network.

We get a more reliable procedure if we start the processing with a smaller threshold *max_res* (1 m for topography and 1 mm/year for deformation) to build smaller but more stable networks (figure 4). Each single network is independently processed using the SVD algorithm from section 2 and we get for all points an acceptable variance below 1 m^2 respectively 1mm/y^2. Since the networks are located quite close to each other we can merge them with the procedure described in section 3.3 and get finally an estimation for the whole scene (final result for deformation is shown in figure 5).

5 Summary

An efficient and robust PSI integration algorithm suitable for wide areas has been developed. The new algorithm includes the selection of reliable arcs and the rejection of wrong arcs. In a first step, it creates a consistent reference network. Afterwards, estimating the absolute height and deformation values for the PSs can be efficiently performed by a multithreaded SVD. An advantage of the implemented method is the straight forward error propagation assessment. Consequently, the estimation quality is made available.

6 References

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