Two-lane traffic rules for cellular automata: A systematic approach

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Microscopic modeling of multilane traffic is usually done by applying heuristic lane changing rules and often with unsatisfying results. Recently, a cellular automaton model for two-lane traffic was able to overcome some of these problems and to generate the density inversion found in many European countries at densities somewhat below the maximum flow density. In this paper we summarize different approaches to lane changing and their results and propose a general scheme, according to which realistic lane changing rules can be developed. We test this scheme by applying it to several different lane changing rules, which, in spite of their differences, generate similar and realistic results. We thus conclude that, for producing realistic results, the logical structure of the lane changing rules, as proposed here, is at least as important as the microscopic details of the rules. [S1063-651X(98)03008-6]

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I. INTRODUCTION

Much progress has been made in understanding single-lane traffic by using simple models (e.g., [1,2]). Although one could claim that these models also explain homogeneous multilane traffic, they definitely fail when traffic in different lanes behaves differently. If one wants to investigate lane specific dynamics, one has to address the question of how vehicles change from one lane to another. Here we propose an elementary scheme to develop such rules and compare the simulation results of different realizations of this scheme with empirical data from the German highway.

The preferred approach in science is to start from first principles and then to derive macroscopic (emergent) relationships. In sciences that involve human beings this is hopeless: The gap between first principles and human behavior is too big. One alternative is to search heuristically for microscopically minimal ‘‘plausible’’ models that generate observed behavior on the macroscopic level. It is this approach that has often been used successfully when physics methods have been applied in the area of socioeconomic systems (see, e.g., [3] and references therein). In this paper we want to go one step beyond that and look for systematic logical structures in the rule sets for lane changing.

There are currently two major methods of how to get from the microscopic to the macroscopic relations: computational and analytical. This paper concentrates entirely on computational approaches; analytical approaches to the same problem can be found, e.g., in [4–8].

Often, the analytical approaches are logically somewhat more satisfying, whereas the computational approaches are more flexible with respect to what kind of microscropic structure they accept while remaining feasible.

We start out from real world data (Sec. II), followed by a short review of traditional approaches to this problem in traffic science (Sec. III). Section IV outlines our approach. In Secs. V–VII we describe simulation results with different rules. Section VIII looks closer into the mechanism of flow breakdown near maximum flow in the two-lane models. Section IX is a discussion of our work, followed by a section showing how other multilane models for cellular automata fit into our scheme (Sec. X). The paper concludes with a short summary (Sec. XII).

II. REAL WORLD MEASUREMENTS

As stated above, we are interested in *macroscopic* observations of traffic flow quantities related to lane changing behavior. A typical such measurement can look like Fig. 1. It contains measurements of density [in vehicles (veh)/km/2 lanes], flow (in veh/h/2 lanes), velocity (in km/h), and lane usage (in %), all averaged over 1-min intervals. The left column shows velocity and lane usage as functions of flow; the right column shows flow, velocity, and lane usage as functions of density. For theoretical purposes, using flow as the control parameter has the disadvantage that for the same flow value one has two different regimes: at high density and at low density. For example, in the lane usage plot, one cannot distinguish which data points belong to which regime. We will therefore concentrate on plots where density is the control parameter.

The top right plot shows the typical flow-density diagram. Flow first increases nearly linearly with density, until it reaches a maximum at $\rho \approx 40$ veh/km/2 lanes and $q \approx 3500$ veh/h/2 lanes. From there, flow decreases with increasing density and the scatter of the values is much larger than before. The currently best explanation for this [10–12] (but see also [13,14]) is that, for low densities, traffic is roughly laminar and jams are short lived. As a consequence, the addition of vehicles does not change the average velocity much

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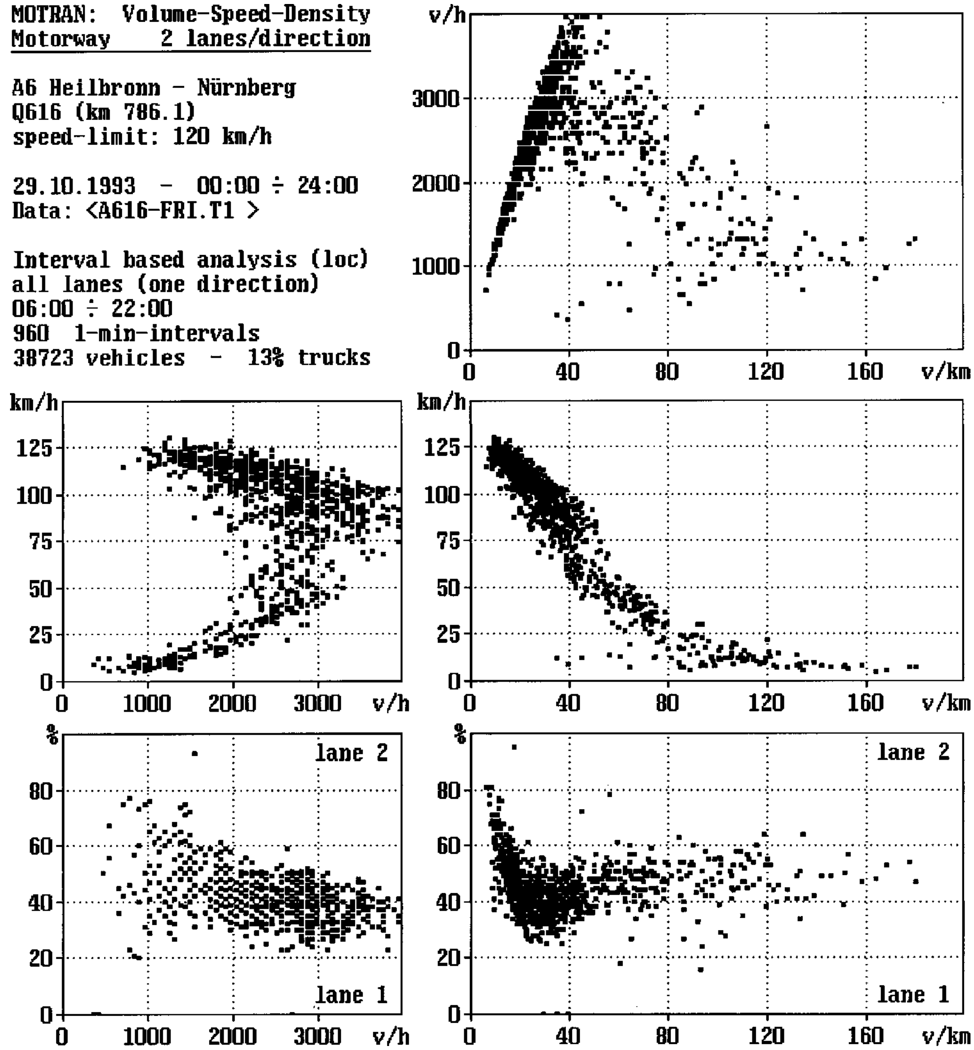

FIG. 1. Traffic measurements in reality as a function of flow (left column) and as a function of density (right column). Top, flow; middle, velocity; bottom, lane usage fraction of vehicles in right lane. The units for density are vehicles per kilometer per two lanes and for flow they are vehicles per hour per two lanes. Each data point corresponds to a 1-min average. Data are from Wiedemann; see [9] for further information.

and flow is a linear function of density: $q=\rho v$. For high densities, traffic is an irregular composition of jam waves and laminar outflow traffic between jams. Here data points are arbitrary averages over these regimes, leading to a much larger variability in the measurements.

The plot of the velocity vs density confirms this: There is an abrupt drop in the average velocity at $\rho\approx 40$ veh/km/2 lanes. Yet, velocity is also not constant at lower densities, leading indeed to a curvature of the flow-vs-density curve below the value $\rho\approx 40$ veh/km/2 lanes, which can be explained by the increasing influence of the slower vehicles in multilane traffic.

The lane usage shows a peculiarity that is particularly strong in Germany. As should be expected, at very low densities all traffic is in the right lane. (For countries such as Great Britain or Australia, left and right have to be interchanged.) However, with increasing density, eventually more than half of the traffic is in the left lane. Only at densities above the maximum flow point does this revert to an equal distribution of densities between lanes.

Figure 1 does not show the flows of the individual lanes. Reference [15] contains such plots. They show that the pointed peak of the overall flow is caused by a pointed peak in the flow of the left lane; flow on the right lane remains constant over a large density range. All this suggests the interpretation that the flow breakdown mechanism on German autobahns is complicated, with flow breaking down in the left lane first and thus not allowing the right lane to reach its possible full capacity [16].

III. TRADITIONAL APPROACHES

Sparmann [15] discusses a lane changing implementation for the microscopic Wiedemann model [17]. Following Wiedemann's proposition, he distinguishes between the wish to change lanes and the decision to change lanes. For a lane change from right to left, these two parts are a *wish* to change lanes if in any of the two lanes there is another vehicle ahead and obstructing, and a *decision* to actually change lanes if there is enough space in the other lane. Conversely, for changing from left to right there is a *wish* to change lanes if in both lanes there is nobody ahead and obstructing, and a *decision* to actually change lanes if there is enough space in the other lane. According to the philosophy of the Wiedemann approach, ''obstructing'' is defined in terms of so-called psychophysiological thresholds, which depend mostly on speed difference and distance and allow three outcomes: no obstruction, light obstruction, and severe obstruction. Gipps [18] reports a similar model. The results are reported to be satisfying, yet unrealistic in at least one respect: The density inversion between right and left lanes near maximum flow is not reproduced.

The Wiedemann approach is a time-discrete formulation of a stochastic differential equation and therefore continuous in space. Some recent work in traffic has used a cellular automata approach, which is coarse-grained discrete in both time and space. Early lane changing rules in the context of cellular automata models for traffic flow are due to Cremer and co-workers [19,20]. Following Sparmann, they implemented lane changes in the following way: Lanes are changed to the left if a slower vehicle is less than l_l cells ahead and if a gap of size Δx exists on the left lane; lanes are changed to the right if, *in the right lane*, there is no slower vehicle less than l_r cells ahead and there is a gap of size Δx in the right lane. Again, they failed to reproduce the density inversion in the lane usage.

IV. OUR APPROACH

Which contribution can statistical physics make in such a situation? The strength of statistical physics is to explain how microscopic relationships *generate* macroscopic behavior. Thus the contribution of statistical physics in traffic science (or in socioeconomic systems in general) will be to investigate which microscopic rules contribute to certain aspects of macroscopic behavior and how.

Since current psychological knowledge does not allow us to define beyond doubt the set of microscopic rules involved in lane changing, we propose to construct these rules according to certain symmetries inherent in the problem. As we will point out, these symmetries simplify considerably the construction of consistent lane changing rules.

Now, in spite of the absence of ''first principles,'' it certainly still makes sense to have a ''plausible'' starting point. We thus state here what we will use as the elementary laws and later how we derive algorithmic rules from them. Similar to Ref. [15], we propose that the basic ingredients are security, legal constraints, and travel time minimization. Security requires one to leave enough space between all vehicles. The legal constraints depend on the country. Travel time minimization means that one chooses the optimal lane under these constraints.

Let us start with security. Security means that one leaves enough space in front of and behind oneself. As long as one stays in one lane, this is ensured by single-lane driving rules as given, e.g., by the rules in Refs. [21,22]. In the context of changing a lane this means that there must be enough space on the target lane. Technically, one can say that there must be a gap of size $\mathrm{gap}_- + 1 + \mathrm{gap}_+$. The label $+$ $(-)$ belongs to the gap on the target lane in front of (behind) the vehicle that wants to change lanes. In the following we characterize the security criterion by the boundaries $[-\mathrm{gap}_-, \mathrm{gap}_+]$ of the required gap on the target lane relative to the current position of the vehicle considered for changing lanes.

Different choices for both parameters are possible. Throughout this paper we use $\mathrm{gap}_+ = v$ and $\mathrm{gap}_- = v_{max}$ (i.e., $[-v_{max}, v]$), where v is the speed of the vehicle that changes lanes and v_{max} is the maximum velocity allowed in the cellular automaton.

Let us now go to legal constraints. For example, in Germany, lane usage is regulated essentially by two laws: The right lane has to be used by default and passing has to be on the left. In the United States, the second law is considerably relaxed. In this paper we will use ''Germany'' and ''United States'' as placeholders for two somewhat extreme cases. We expect that the behavior of many other countries will be found somewhere in between.

Travel time optimization means that lane changes to the left are triggered by a slow vehicle in the same lane ahead and when the target lane is more attractive (because of optimization). In this context, ''slow'' means a velocity smaller than or equal to the one of the car behind. Here we give two examples, first for changing to the left.

(a) Hypothetical German criterion. In Germany passing is *not allowed* on the right. Hence, if there is a slow vehicle in the *left* lane, one has to change to the left, behind that slow vehicle. Thus one changes to the left if there is a slow car ahead in the same lane *or* in the left:

$$v_r \leqslant v \,.\mathrm{OR}.\, v_l \leqslant v. \tag{1}$$

v_r, v_l are taken within a certain distance one looks ahead d, which is a free parameter. If there is no vehicle within this distance, the respective velocity is set to ∞.

(b) Hypothetical American criterion I (asymmetric). In contrast, in America passing on the right is not explicitly forbidden. The left lane is only more attractive if the traffic there is faster than in one's own lane. Thus one changes from the right to the left if there is a slower car ahead in the same lane and if the next car in the left lane is faster than the car ahead:

$$v_r \leqslant v \,.\mathrm{AND}.\, v_r \leqslant v_l. \tag{2}$$

The easiest implementation of the law to use the right lane by default is to make the criterion for changing back to the right lane the logical negation of the criterion to change to the left lane; i.e., whenever the reason to change to the left lane ceases to exist, one changes back.

(a′) This means for Germany that a change back to the right lane is tried as soon as the velocities of the cars ahead in both lanes are sufficiently large:

$$v_r > v \,.\mathrm{AND}.\, v_l > v. \tag{3}$$

(b′) In America, the rule would mean that one tries to change back if there is a faster car than oneself (or no car at all) in the right lane or if traffic in the right lane is running faster than in the left lane:

$$v_r > v \,.\mathrm{OR}.\, v_r > v_l. \tag{4}$$

In summary, a lane is changed if two criteria are fulfilled: (i) the security criterion, $[-v_{max}, v]$ empty in target lane, and (ii) the incentive criterion, i.e., is there a good reason to change lanes? The examples above illustrate that the wish to change from right to left in general depends on both lanes. If the right lane is used by default, the criterion to change from left back to right is that the reason to change from right to left is no longer given, which is the negation of the former criterion.

Note that these considerations can easily be extended to multilane traffic. Also note that our paper treats only unidirectional traffic, i.e., all vehicles are headed into the same direction. References [20, 23] are examples for the treatment of bidirectional traffic by cellular automata.

V. SYMMETRIC LANE CHANGING RULES

If the right lane is *not* used by default, it is natural to consider symmetric incentive criteria: The return to the right lane then depends on the same criterion as the transition to the left lane, with ''left'' and ''right'' interchanged. The simplest example involves only one lane. One changes lanes only when a slow vehicle is ahead: The criterion for change from right to left is $v_r \leq v$ and the Criterion for change from left to right is $v_l \leq v$. This implies that vehicles stay in the left lane even when the right lane is completely empty.

On multilane freeways, American drivers often do not use the rightmost lane in order to avoid the repeated disturbances due to slow vehicles coming from on-ramps. That is, when these drivers encounter *one* slow vehicle from an on-ramp, they switch to the left lane and stay there until they run into a slower vehicle in that lane or until they want to get off the freeway. This implies that symmetric rules may be more useful to describe *actual* American driving behavior than the asymmetric ''Hypothetical American criterion I'' above [Eq. 2]. For that reason, TRANSIMS [24,25], in its current microsimulation, uses a totally symmetric lane changing rule set. This paper will concentrate on asymmetric lane changing rules; see Refs. [26–28] for symmetric lane changing rules.

VI. COMPUTER SIMULATIONS OF THE BASIC VELOCITY RULES

We now proceed to present computer simulations of the German rule set [Eqs. (1) and (3)] to illustrate the above principles. Following Refs. [29,27,28], an update step of the whole system is divided into two major substeps: (i) lane changing and (ii) forward movement.

A. Lane changing

Lane changing here is implemented as a pure sideways movement. One should, though, better look at the overall result after the whole time step is completed; by then, lane-changing vehicles usually will have moved forward. Still, the algorithm underestimates the time vehicles usually need to change lanes: One cellular automation iteration roughly corresponds to 1 sec; lane changes in reality need about 3 sec [15].

More specifically, the lane changing algorithm is an implementation of the following. In *even* time steps, perform lane changes from right to left. [We separate changes from left to right and changes from right to left in anticipation of three-lane traffic. In three-lane traffic, in a simultaneous update it is possible that a vehicle from the left lane and a vehicle from the right lane want to go to the same cell in the middle lane. From a conceptual viewpoint of simulation, this may be called a scheduling conflict. Such conflicts can be resolved by, e.g., different update schedulings (such as here) [30,31].] All vehicles in the right lane for which the incentive criterion ($v_r \leq v$.OR.$v_l \leq v$) and the security criterion ($[-v_{max}, v]$) are fulfilled are simultaneously moved to the left. In *odd* time steps, perform lane changes from left to right. All vehicles in the left lane for which the incentive criterion ($v_r > v$.AND.$v_l > v$) and the security criterion ($[-v_{max}, v]$) are fulfilled are simultaneously moved to the right.

The number of sites one looks ahead for the incentive criterion *d* plays a critical role. Quite obviously, if one looks far ahead, one has a tendency to go to the left lane already far away from an obstructing vehicle, thus leading to a strong density inversion at low densities. Thus this parameter can be used to adjust the density inversion. The results described below were obtained with a look ahead of $d = 16$ sites, that is, if no vehicle was detected in that range on that lane, the corresponding velocity v_r or v_l was set to ∞.

B. Forward movement

The vehicle movement rules (ii) are taken as the single-lane rules from Nagel and Schreckenberg [21,22], which are by now fairly well understood [10,32,33]. For completeness, we mention the single-lane rules here. They are IF ($v < v_{max}$) THEN $v := v+1$ (accelerate if you can); IF ($v > gap$) THEN $v := gap$ (slow down if you must); IF ($v \geq 1$) THEN WITH PROBABILITY p DO $v := v-1$ (sometimes be not as fast as you can for no reason). These rules for forward movement will be used throughout the paper, with $p = 0.25$. All simulations are performed in a circle of length $L = 10\,000$. The maximum velocity is $v_{max} = 5$. In order to compare simulation results to field measurements, the length of a cell was taken as 7.5 m and a time step as 1 sec. This means, for example, that $v_{max} = 5$ cells/update corresponds to 135 km/h.

C. Results

As shown in Fig. 2, these rules generate reasonable relations between flow, density, and velocity. More importantly, they generate the density inversion below maximum flow, which is an important aspect of the dynamics on German freeways. Note that, maybe contrary to intuition, it is not necessary to have slow vehicles in the simulations in order to obtain the density inversion.

VII. COMPUTER SIMULATIONS OF GAP RULES

For comparison, we also simulated a version of Wagner's ''gap rules'' [34,35], which is adapted to our classification scheme above. The reason to change to the left then becomes

$$\mathrm{gap}_r < v_{max}.\mathrm{OR}.gap_l < v_{max}, \tag{5}$$

i.e., one has a reason to change to the left when there is not enough space ahead either in the right or in the left lane.

As stated above, as reason to change to the right we take the negation, although we allow for some ''slack'' Δ:

$$\mathrm{gap}_r \geq v_{max} + \Delta.\mathrm{AND}.\mathrm{gap}_l \geq v_{max} + \Delta, \tag{6}$$

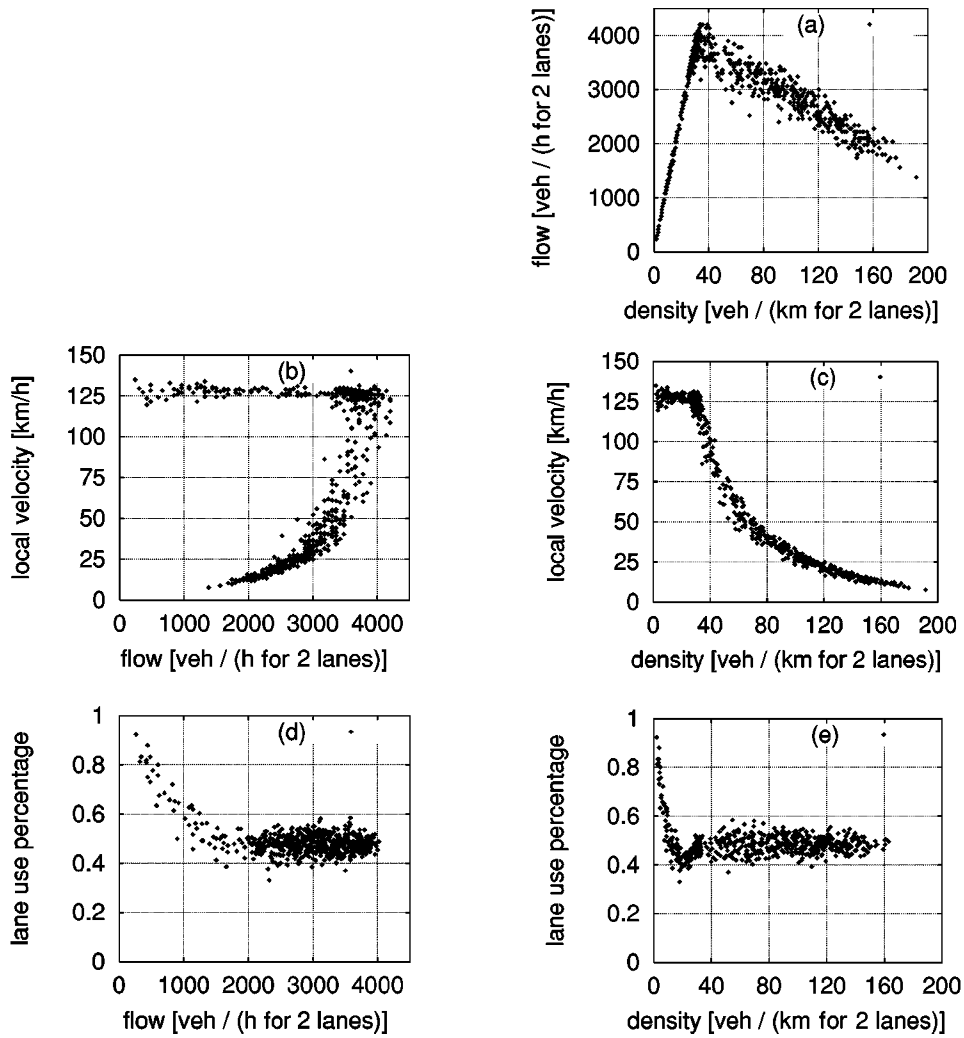

FIG. 2. Simulation results for the basic version of the velocity-based lane changing rules [Eqs. (1) and (3)]. Same type of plots as in Fig. 1. Each data point is a 1-min average, except for lane usage, where each data point is a 3-min average.

i.e., one changes from left to right if in both lanes there is enough space ahead.

The slack parameter Δ has been introduced in Ref. [34]. The larger it is the less inclined the driver is to change back to the right lane and hence the more pronounced the lane inversion is. In this sense the parameter Δ plays a role similar to these gap rules as the look-ahead distance d in the basic velocity rules discussed before. We will use $\Delta=9$, the same value as in Ref. [35].

Figure 3 shows the results of simulations with these rules. One immediately notes that these rules both qualitatively and quantitatively generate the correct density inversion at maximum flow, i.e., at $\rho \approx 38$ veh/km/2 lanes, but from there on with further increasing density the density inversion increases further, contrary to reality. Reference [35] uses rules that (i) prohibit passing on the right and (ii) symmetrize traffic at very high densities; as a result, lane usage becomes much more symmetric above the density of maximum flow.

VIII. EXTENSIONS FOR REALITY

After having shown that both velocity-based and gap-based lane changing rules, based on the introduced logical scheme, can generate the density inversion effect, we now proceed to include more realism to bring the result closer to Wiedemann's data (Fig. 1).

A. Slack

With the basic velocity-based rules, one can adjust the density inversion to the correct lane use percentage, but the maximum inversion is reached at too low densities (at approximately 16 veh/km/2 lanes compared to approximately 28 veh/km/2 lanes in reality). One possibility to improve this is to introduce some slack $\Delta=3$ into the rules similar to the slack in the gap-based rules, i.e., vehicles change to the left according to the same rules as before, but the incentive criterion for changing back is not the inversion of this. Instead, it now reads

$$v_r > v + \Delta .\mathrm{AND}. v_l > v + \Delta \tag{7}$$

(and still

$$v_r \leq v .\mathrm{OR}. v_l \leq v \tag{8}$$

for changing from right to left). Since these rules tend to produce a stronger density inversion than before, we reduced

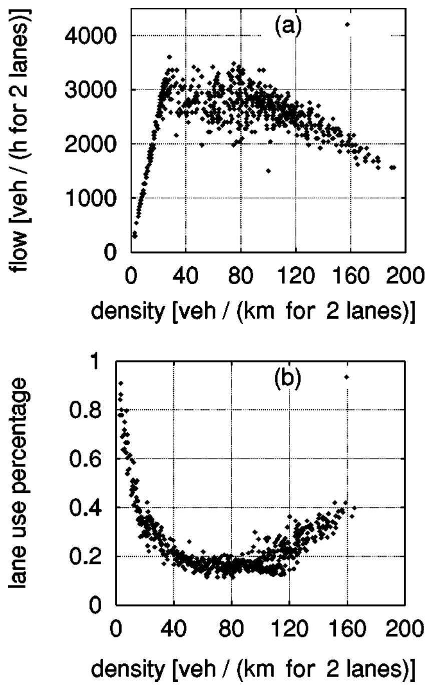

FIG. 3. Simulation results for gap-based lane changing rules (see Sec. VII): (a) flow vs density and (b) lane usage vs density.

the look-ahead value d to 7 to obtain realistic lane usage values. The results are shown in Fig. 4.

B. Slack plus symmetry at high densities and low velocities

In order to be able to tune the onset density as well as the amount of lane inversion, the second parameter slack had

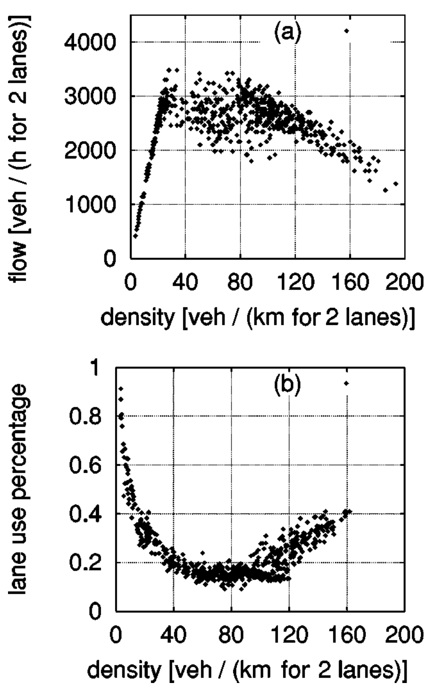

FIG. 4. Simulation results for velocity-based lane changing rules with slack [i.e., there is some ''slack'' in the incentive criterion for changing to the right compared to the one for changing to the left; see Eqs. (7) and (8)]: (a) flow vs density and (b) lane usage vs density.

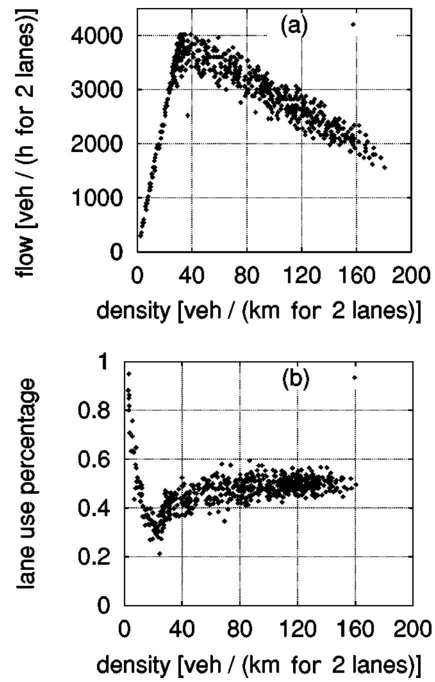

FIG. 5. Plots when slack is used and symmetry at low velocities included; see Sec. VIII B: (a) flow vs density and (b) lane usage vs density.

been introduced in addition to the look ahead. This, however, has the side effect that traffic never reverts to an equal lane usage, even at very high densities, similar to what we obtained with the gap rules above. In order to improve this, we make the rule set symmetric at zero speed. In technical terms, this means that a vehicle at speed zero only checks if the speed in the other lane is higher than in its own lane, and if so, attempts to change lanes (restricted by the security criterion). Other solutions are possible to achieve this [see,

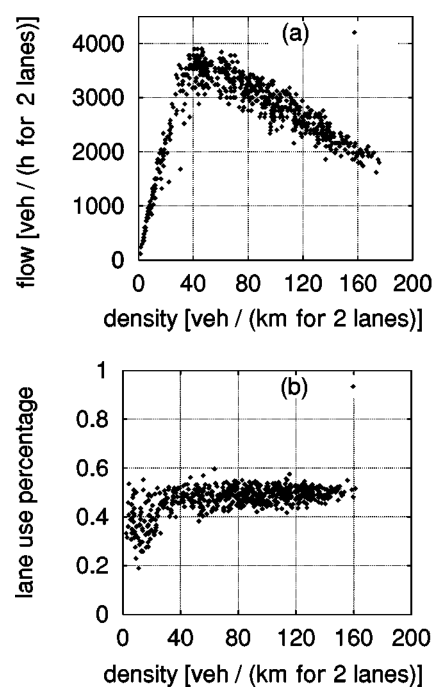

FIG. 6. Plots when slow vehicles are included (see Sec. VIII C): (a) flow vs density and (b) lane usage vs density.

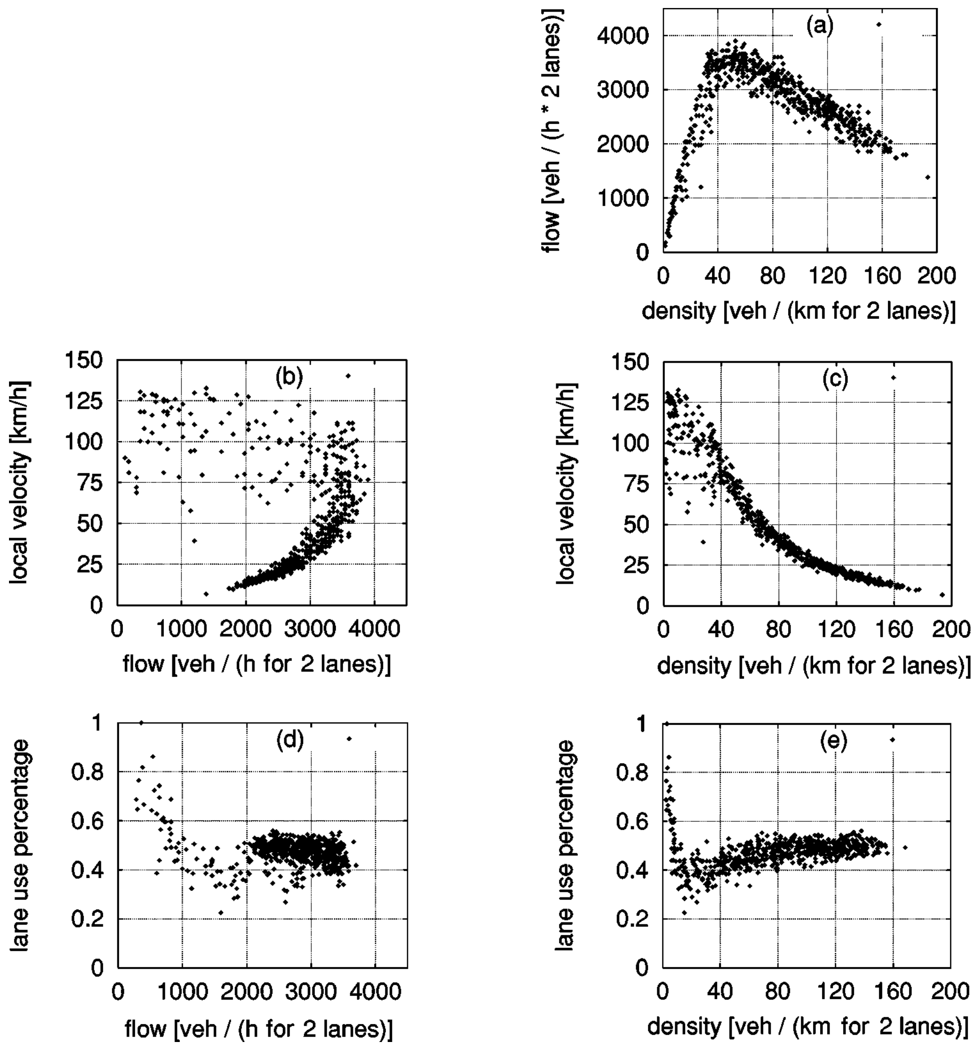

FIG. 7. All three extensions of the basic rule set [trucks, symmetry at high density, and slack, i.e., Eqs. (7) and (8) plus the additions of Secs. VIII B and VIII C] are included. Compare this figure to Fig. 1.

e.g., Ref. [35]; or one could attempt to make the look-ahead distance d a function of the velocity, for example, $d=d(v)\propto v$, instead of $d=\text{const}$ as before]. Figure 5 shows that our approach indeed works, i.e., the lane usage at high densities now goes indeed to approximately 50% for each lane. [Note that the finer points of this are subtle. Some measurements do not show a significant deviation from equal lane usage at high densities; see, e.g., our Fig. 1. Other measurements indicate that densities on the left lane can be higher than on the right lane for densities far above maximum flow (see, e.g., [4] for field results from the Netherlands). Also, due to the lack of a good theoretical idea, it is unclear how to account for the presence of trucks in these cases: Does one count them once, or as multiple passenger cars as often done in the field; or does one measure ''occupancy'' (fraction of time a sensor senses a vehicle), which is a related but different quantity?]

C. Slow vehicles

Wiedemann's data includes 10 % trucks. We model the effect of trucks by giving 10 % of the vehicles a lower maximum velocity [27,29,36]. Note that this models only the lower speed limit, which is in effect for trucks in most European countries, but not the lower acceleration capabilities. The result for the flow-density curve and for the lane usage is shown in Fig. 6. The main difference from before is that the maximum flow is shifted towards higher densities and there are more fluctuations in that region [27].

D. Combination of all extensions

Finally, we show simulation results where all the above improvements (trucks, symmetry at high densities, and slack, i.e., Secs. VIII B and VIII C) are used simultaneously (Fig. 7). Indeed, the results are now close to reality (cf. Fig. 1).

IX. THE FLOW BREAKDOWN MECHANISM NEAR MAXIMUM FLOW

One of the questions behind this research was to investigate if, in highly asymmetric two-lane systems, flow breakdown is indeed triggered by a single-lane flow breakdown in the left lane. In order to address this question, we will, in the following, study space-time plots of the respective traffic dynamics as well as fundamental diagrams by lane. Since it turns out that traffic without slow vehicles is fundamentally different from traffic with slow vehicles, we will treat the two situations separately.

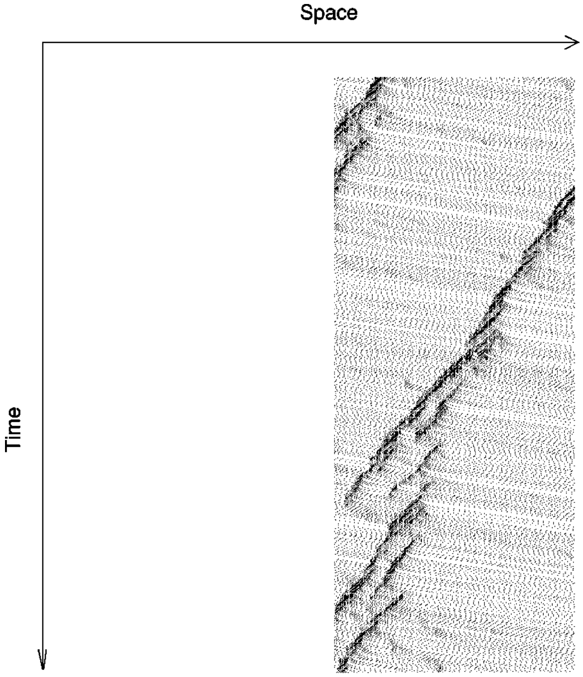

FIG. 8. Space-time plot of one-lane traffic without slow vehicles.

A. Maximum flow without slow vehicles

Figures 8 and 9 compare space-time plots from a one-lane situation with the two-lane situation using the ''basic'' velocity-based lane changing rules, in both cases approximately at maximum flow. Not much difference in the dynamics is detectable except that maybe the two-lane plot shows more small fluctuations instead of fully developed jams. This is confirmed by the single-lane fundamental diagrams for the systems (Fig. 10): The fundamental diagram for the left lane of basic velocity-based lane changing rules looks very similar to the corresponding one-lane diagram and also the right lane does not look much different. Also, the density inversion has reverted to 50:50 at maximum flow (Fig. 2).

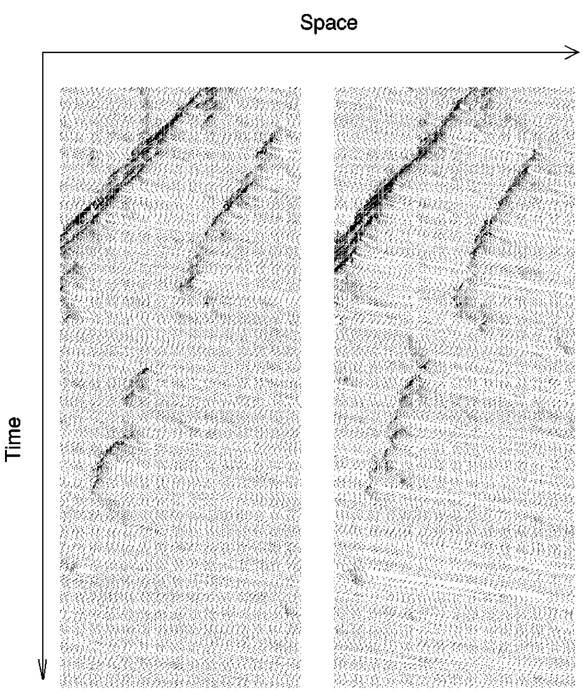

FIG. 9. Space-time plot of two-lane traffic with the ''basic'' lane changing rules (1) and (3) (i.e., without slow vehicles): left, left lane; right, right lane.

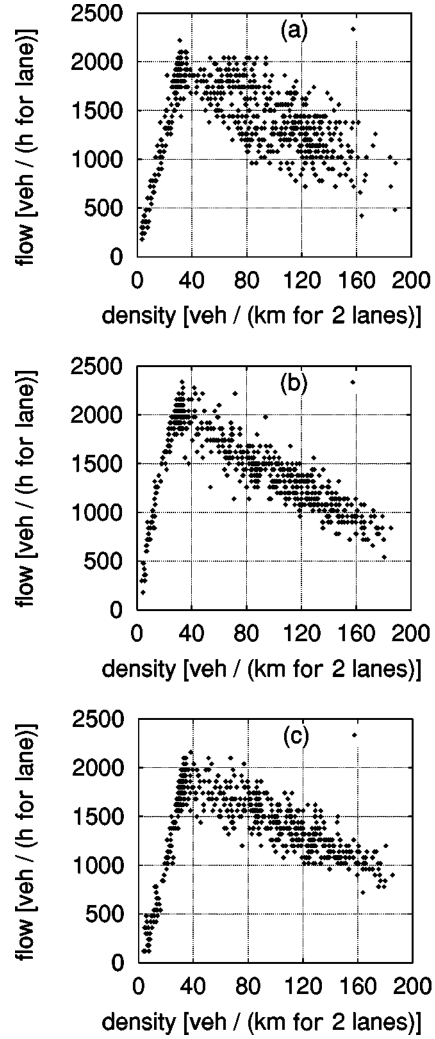

FIG. 10. (a) Fundamental diagram for single-lane rules. (b) Fundamental diagram, i.e., plotting flow on the left lane vs density on both lanes for 1-min averages, for the left lane of basic velocity two-lane rules [Eqs. (1) and (3)]. (c) Fundamental diagram for the right lane of basic velocity two-lane rules.

Thus the approach to maximum flow via increasing density is better described in the way that the left lane reaches maximum flow earlier than the right lane and from then on all additional density is squeezed into the right lane. Only when the combined density of both lanes is above the maximum flow density, flow breakdown happens. This argument is confirmed by the observation that there are many measurement points near maximum flow in all fundamental diagrams, whereas at densities slightly higher than this significantly fewer data points exist. This should be compared to the situation that includes slower vehicles, which will be explained next.

B. Maximum flow with slow vehicles

The situation when slow vehicles are present is markedly different. The two-lane situation with slow vehicles (Fig. 12)

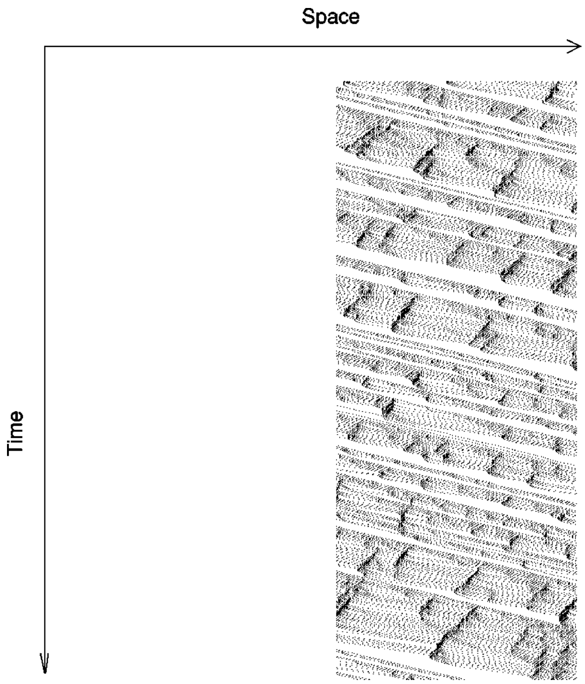

FIG. 11. Space-time plot of one-lane traffic near maximum flow including 10% slow vehicles.

looks more like the one-lane situation with slow vehicles (Fig. 11) than like the two-lane situation without slow vehicles (Fig. 9). This means that the presence of slow vehicles has a stronger influence on the dynamics than the difference between one-lane and two-lane traffic. The dominating feature is that fast vehicles jam up behind slow vehicles and get involved in a start-stop dynamics that gets worse with in-

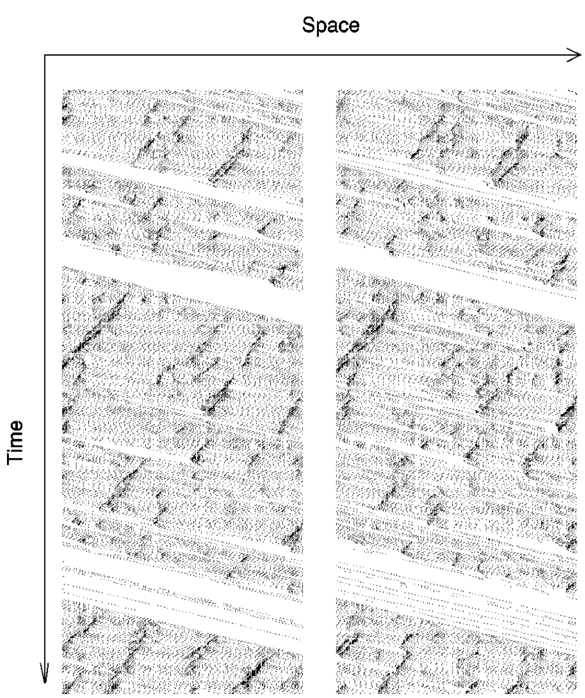

FIG. 12. Space-time plot of two-lane traffic near maximum flow including 10% slow vehicles using the basic velocity-based lane changing rules of this paper [Eqs. (1) and (3) plus ''slow'' vehicles (Sec. VIII C)]: left, left lane; right, right lane.

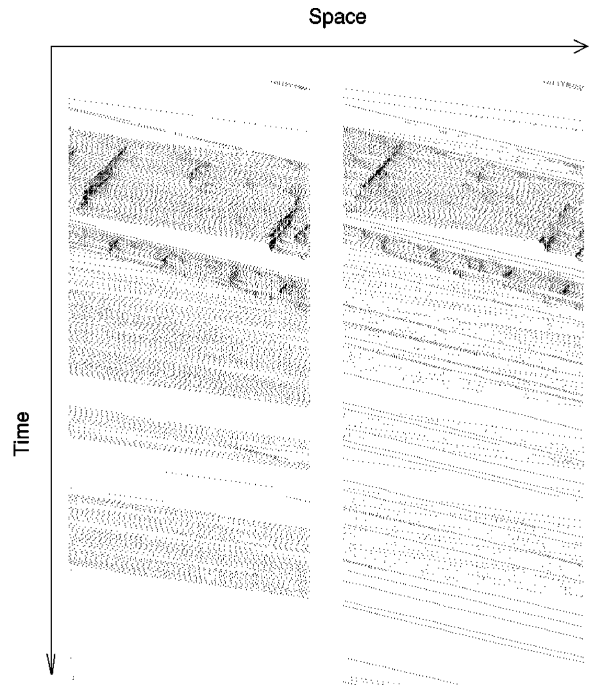

FIG. 13. Space-time plot of two-lane traffic at about half the density of maximum flow, including 10% slow vehicles, using the basic lane changing rules of this paper. Same as Fig. 12, except for the lower density: left, left lane; right, right lane.

creasing distance from the leading slow vehicle. In the two-lane situation, these ''plugs'' are caused by two slow vehicles side by side, a situation which is empirically known to happen regularly.

For the basic lane changing rules, the queues behind the plugs have similar length in both lanes, both near the density of maximum flow (Fig. 12) and at lower densities (Fig. 13). In contrast, when using the lane changing rules with slack and symmetrization, then in the same situation, there are more vehicles behind the truck in the left than there are behind the truck in the right (Fig. 14). Experience seems to indicate that the more complicated rule set is the more realistic one here.

The lane-based fundamental diagrams (Fig. 15) confirm the observation that slow vehicles change the dynamics. The marked peak and the accumulation of data points near maximum flow are both gone; maximum flow is found over a wider density range than before. The flow in the left lane generally reaches higher values than flow on the right lane and single-lane traffic flow.

Space-time plots (Figs. 12 and 13) show why this is the case. Traffic in this situation is composed of two regimes: (i) plugs of slow vehicles side by side and faster vehicles queued up behind them and (ii) ''free flow'' regions, where the slow vehicles stay on the right and the fast vehicles are mostly on the left. At low density, there are mostly free flow regions and a couple of plugs with queues behind them. With increasing density, the share of the free flow regions *de*creases while the share of the queueing regions *in*creases. Eventually, the free flow regions get absorbed by the queueing regions, a two-lane variant of the mechanism described in Refs. [37,38].

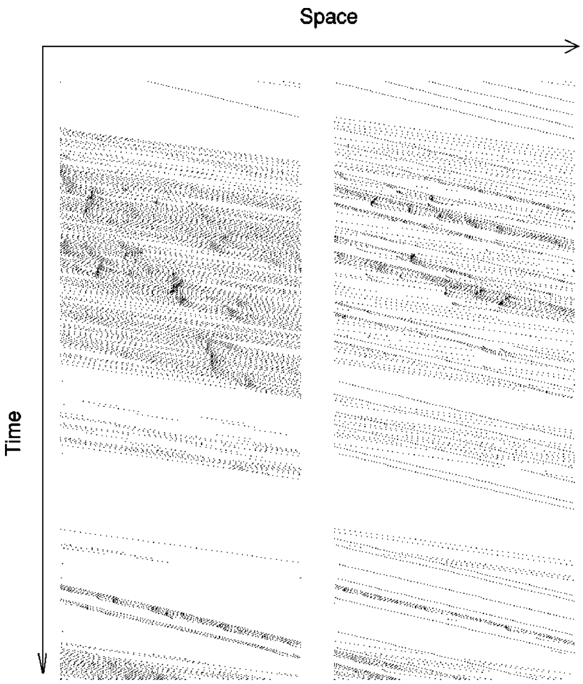

FIG. 14. Space-time plot of two-lane traffic at about half the density of maximum flow, including 10% slow vehicles, using the lane changing rules with slack and symmetrization [i.e., Eqs. (7) and (8) plus Sec. VIII B]: left, left lane; right, right lane. The trajectories of fast vehicles (less steep slope) in the right lane that seem to go ''through'' the slow vehicles (steep slope) are actually interrupted and go to the left lane for short times.

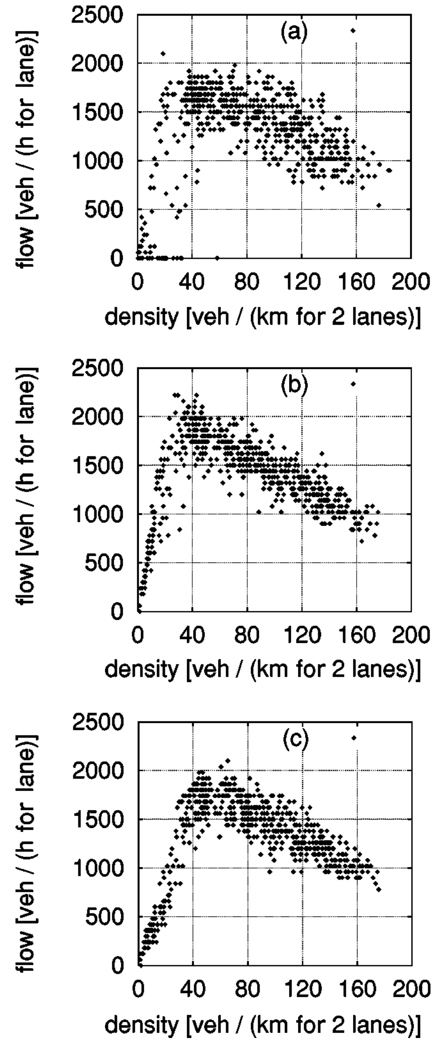

FIG. 15. Simulation results for traffic including 10% slow vehicles (Sec. VIII C). (a) Fundamental diagram for single-lane rules. (b) Fundamental diagram, i.e., plotting flow in the left lane vs density in the left lane for 1-min averages, for left lane of basic velocity-based two-lane rules [Eqs. (1) and (3)]. (c) Fundamental diagram for the right lane of basic velocity two-lane rules.

From visual inspection, it is clear that up to that density (approximately 40 veh/km/2 lanes) the left lane carries a higher flow since it has only fast cars in the free flow regions. Above this density, it is clear that now also the slow vehicles get slowed down by the end of the queue ahead of them.

X. DISCUSSION

In spite of widespread efforts, many earlier models were *not* able to reproduce the lane inversion. Why is that so? The reason is that the lane inversion is a subtle spatial correlation effect: ''I stay in the left if there is somebody ahead on the left.'' Indeed, some of the earlier models [20,29] do not contain this crucial rule. Sparmann [15] contains it, but still does not reproduce the density inversion; so one would speculate that the weight for this rule was not high enough.

Real world traffic seems to be more stable in the laminar regime than our simulated two-lane traffic. This can be seen in the ''overshoot'' (hysteresis; see Ref. [39]) of the low-density branch of the flow-density plot, which is more pronounced in reality than in the results of this paper. The single-lane model [40] looked more realistic here. Yet, recent research shows that the hysteresis effect is actually related to the structure of the braking rules of the single-lane velocity rules [12,41]. More precisely, in models with more refined braking rules the laminar traffic does not break down that easily because small disturbances can be handled by small velocity adjustments.

In this context, it should be stressed that, as mentioned above, our plots actually show 3-min averages for the lane usage plots, whereas all other plots are generated from 1-min averages. The reason for this is that 1-min averages for lane usage had so much variance that the overall structure was not visible. Yet, in reality, 1-min averages are sufficient also for this quantity. This indicates that our models have, for a given two-lane density, a higher variation in the lane usage than reality has. Also, the plots of velocity vs flow indicate that the range of possible velocities for a given flow is wider in the simulations than in reality, again indicating that for a given regime, our model accepts a wider range of dynamic solutions than reality.

The fact that we needed space-time plots for resolving many of the dynamical questions indicates that the methodology of plotting short time averages for density, flow, and velocity has shortcomings. The reason has been clearly pointed out in recent research [11,10,42]: Traffic operates in distinctively different dynamic regimes, two of them being laminar traffic and jammed traffic. Averaging across time means that often this average will, say, contain some dynam-

TABLE I. Lane changing rules in the literature. The left column gives the ''incentives to change lane'' for the indicated lane change right to left ($R \to L$) or left to right ($L \to R$). The right column gives the security criterion, i.e., the sites on the target lane that need to be empty. Underlined parts need to be added to make the incentive to go right the logical negation of the incentive to go left. The ''look-ahead distance'' is the distance to look ahead. v_{back} is the velocity of the next vehicle behind on the target lane. v_d is the desired speed (i.e., may be smaller than v_{max} to denote a slower vehicle class). $v_{d,r}$ is the desired speed of the next vehicle ahead in the right lane. $v_{d,back}$ is the desired speed of the next vehicle *behind* in the target lane.

Incentives to change lane	Security criterion
$R \to L$: $v_r < v$ (look-ahead distance = 9)[a]	$[-v_{max}+1, v_{max}]$
$L \to R$: $v_r \geq v$ (look-ahead distance = 15)[a]	
$R \to L$: $\mathrm{gap}_r < \min[v+1, v_{max}]\underline{.\mathrm{OR.}\,\mathrm{gap}_l \leq 2\min[v+1, v_{max}]}$[b]	$[-v_{max}; \min(v+1, v_{max})]$
$L \to R$: $\underline{\mathrm{gap}_r \geq \min[v+1, v_{max}]}.\mathrm{AND.}\,\mathrm{gap}_l > 2\min[v+1, v_{max}]$[b]	
$R \to L$: $\mathrm{gap}_r < \min[v+1, v_{max}]\underline{.\mathrm{OR.}\,\mathrm{gap}_l \leq 2v_{max}.\mathrm{OR.}\,v < v_{max}} - 1$[b]	$[-v_{max}, \min(v+1, v_{max})]$
$L \to R$: $\underline{\mathrm{gap}_r \geq \min[v+1, v_{max}]}.\mathrm{AND.}\,\mathrm{gap}_l > 2v_{max}.\mathrm{AND.}\,v \geq v_{max} - 1$[b]	
$R \to L$: $\mathrm{gap}_r < v$[c]	[0,0] (i.e., neighbor cell empty)
$L \to R$: $\mathrm{gap}_r \geq v$[c]	
$R \to L$: $\mathrm{gap}_r < f(v).\mathrm{AND.}\,\mathrm{gap}_l > \mathrm{gap}_r$, $f(v) = v, v+1, v+2, v_{max}$[c]	[0,0], $[-2,0]$
$L \to R$: $\mathrm{gap}_r \geq f(v)\underline{.\mathrm{OR.}\,\mathrm{gap}_l \leq \mathrm{gap}_r}$[c]	
$R \to L$: $\mathrm{gap}_r < v+1$[d]	$[-(v_{max}+1), v+1]$
$L \to R$: $\mathrm{gap}_r \geq v+1$	
$R \to L$: $\mathrm{gap}_r < v_{max}.\mathrm{AND.}\,\mathrm{gap}_l > \mathrm{gap}_r$[e]	$[-(v_{back}+1), 0]$
$L \to R$: $\mathrm{gap}_r \geq v + \Delta'.\mathrm{AND.}\,\mathrm{gap}_l \geq v + \Delta'$[e]	
$R \to L$: $\mathrm{gap}_r < v_{max}\underline{.\mathrm{OR.}\,\mathrm{gap}_l < v_{max}}$[f]	$[-(v_{back}+1), \min(\mathrm{gap}+1, v_{max})]$
$L \to R$: $\mathrm{gap}_r \geq v_{max} + \Delta(v).\mathrm{AND.}\,\mathrm{gap}_l \geq v_{max} + \Delta(v)$, $\Delta(v) = \Delta' - v_{max} + v$[f]	
$R \to L$: $\mathrm{gap}_r < v_{max}.\mathrm{AND.}\,\mathrm{gap}_l \geq \mathrm{gap}_r$[g]	$[-v_{max}, 0]$
$L \to R$: $\mathrm{gap}_r > v_{max} + \Delta'.\mathrm{AND.}\,\mathrm{gap}_l > v_{max} + \Delta'$[g]	
$R \to L$: $\mathrm{gap}_r < v_{max}\underline{.\mathrm{OR.}\,\mathrm{gap}_l < v_{max}}$[h]	$[-v_{max}, \min(\mathrm{gap}, v_{max})]$
$L \to R$: $\mathrm{gap}_r \geq v_{max} + \Delta.\mathrm{AND.}\,\mathrm{gap}_l \geq v_{max} + \Delta$, $\Delta = \Delta' + 1$[h]	
$R \to L$: $\mathrm{gap}_r < v.\mathrm{OR.}\,v_d > v_{d,r}$ (look-ahead distance v_{max})[i]	$[-v_{d,back}, \mathrm{gap}]$
$L \to R$: $\mathrm{gap}_r \geq v.\mathrm{AND.}\,v_d \leq v_{d,r}$ (look-ahead distance v_{max})[i]	$[-v_{d,back}, v]$
$R \to L$: $v_r \leq v.\mathrm{OR.}\,v_l \leq v$ (look-ahead distance = 16)[j]	$[-v_{max}, v]$
$L \to R$: $v_r > v + \Delta.\mathrm{AND.}\,v_l > v + \Delta$ (look-ahead distance = 16)[j]	
$R \to L$: $\mathrm{gap}_r < v_{max}.\mathrm{OR.}\,\mathrm{gap}_l < v_{max}$[k]	$[-v_{max}, v]$
$L \to R$: $\mathrm{gap}_r \geq v_{max} + \Delta.\mathrm{AND.}\,\mathrm{gap}_l \geq v_{max} + \Delta$[k]	

[a]Reference [20].
[b]Reference [27].
[c]Reference [29].
[d]Reference [28].
[e]Reference [34] (rules as stated in the reference)..
[f]Reference [34] (transformed); see the Appendix of the present work.
[g]Reference [35] (rules as stated in the reference)..
[h]Reference [35] (transformed).
[i]Reference [36].
[j]Present work (velocity).
[k]Present work (gap).

ics from the laminar regime and some dynamics from the jammed regime, thus leading to a data point at some intermediate density and flow.

In transportation science, it seems that this problem is empirically known because people are using shorter and shorter time averages (1-min averages instead of 5-min averages used a couple of years ago or 15-min averages used ten or more years ago). It seems that one should try vehicle based quantities. Plotting $v/\Delta x$ as a function of $1/\Delta x$, where Δx is the front-bumper to front-bumper distance between two vehicles, is still a flow-density plot, but now individualized for vehicles. Instead of just plotting data point clouds, one would now have to plot the full distribution (i.e., displaying the number of ''hits'' for each flow-density value).

XI. OTHER TWO-LANE MODELS

It is possible to review earlier lane changing models in the view of the scheme presented in this paper. In general, classifying some of the earlier rules into our scheme is sometimes difficult, but usually possible. For example, when one uses

$$\mathrm{gap}_r < v_{max}.\mathrm{OR.}\,\mathrm{gap}_l < v_{max} \tag{9}$$

as a reason to change to the left, then the negation of that, including slack Δ, would be the reason to change to the right. Let us also use a security criterion as

$$\mathrm{gap}_- = v_{back} + 1 \tag{10}$$

(i.e., the distance to the car behind on the other lane should be larger than its velocity) and

$$\mathrm{gap}_+ = \min[\mathrm{gap}+1, v_{max}] \tag{11}$$

[i.e., the distance to the car ahead on the target lane should be larger than either (i) the distance to the car ahead on the current lane or (ii) the maximum velocity]. With the exception of the addition of the second part of the incentive criterion to change left, these are exactly the same rules as used in Ref. [35].

Note, though, that this is not completely trivial. For example, the incentive to change left ''$\mathrm{gap}_l > \mathrm{gap}_r$'' of Ref. [35] is now in the security criterion. Also, for changes from left to right, the forward part of the security criterion could be left out, at least for the values of Δ that have been used. Quite generally, it can happen that a rule fits into our logical scheme, but part of the rule will never be used, and this part can thus be omitted without changing anything in the results.

Indeed, many asymmetric lane changing rules investigated in the literature can be viewed through our characterization. Table I contains many asymmetric lane changing rules from the traffic cellular automaton literature. The underlined parts have been added to make the rules completely fit into our scheme, i.e., to make the incentive to change to the right the logical negation (sometimes including slack) of the incentive to change to the left. It would be interesting to test whether or not the neglected part of the rules would be used often if they were actually implemented.

XII. SUMMARY

This paper classifies the multitude of possible lane changing rules for freeway traffic. The first part of this follows Sparmann [15]: One can separate the rules into an ''incentive to change lanes'' and a security criterion, which asks if there is enough space available in the target lane. The second part of this is the observation that in countries with a default lane and a passing lane, the incentive to change right is just the logical negation of the incentive to change left, with possibly some slack (inertia).

The security criterion seems to be universal for all reasonable lane changing rules: $[-\mathrm{gap}_-, \mathrm{gap}_+]$ has to be empty on the target lane; the exact values of the parameters gap_- and gap_+ do not seem to matter too much as long as they are reasonably large. We used $\mathrm{gap}_- = v_{max}$ and $\mathrm{gap}_+ = v$. For the incentive criterion we argue that its general structure for highly asymmetric traffic has to be ''change to the left when either in your lane or in the left lane somebody is obstructing you'' and ''change back when this is no longer true.'' Since this usually leads to a generic density inversion at high densities, one has to add a symmetrizing rule for high-density traffic. We simply used a symmetric incentive criterion for vehicles with velocity zero.

Both velocity- and gap-based implementations of this give satisfying results. Further, we showed that most asymmetric lane changing models in the physics literature fit into this scheme.

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APPENDIX: TRANSFORMATION OF WAGNER'S RULES FROM REF. [34]

Finding a correspondance for the rules of Wagner in Ref. [34] is not straightforward. However, at closer inspection, the rules turn out to be inconsistent for certain choices of parameters. The forward part of the incentive criterion is

$$R \to L: \quad \mathrm{gap}_r < v_{max}\,.\mathrm{AND}.\,\mathrm{gap}_l > \mathrm{gap}_r\,, \tag{A1}$$

$$L \to R: \quad \mathrm{gap}_r > v + \Delta'\,.\mathrm{AND}.\,\mathrm{gap}_l > v + \Delta'. \tag{A2}$$

Assume, for example, a case where $\mathrm{gap}_r = 3$, $\mathrm{gap}_l = 4$, $v = 0$, $v_{max} \geqslant 4$, and $\Delta' = 0$. Then the vehicle does not want to be in either lane. This problem gets resolved for $\Delta' \geqslant v_{max} - 1$, and indeed $\Delta' \geqslant 6$ was used.

Now, if one assumes $\Delta' \geqslant v_{max} - 1$, then one can simplify the rule set. One can move the condition $\mathrm{gap}_l > \mathrm{gap}_r$ into the security criterion $\mathrm{gap}_+ \geqslant \min[gap+1, v_{max}]$ and the remaining incentives to change lanes are

$$R \to L: \mathrm{gap}_r < v_{max} \underline{.\mathrm{OR}.\,\mathrm{gap}_l < v_{max}}, \tag{A3}$$

$$L \to R: \mathrm{gap}_r \geqslant v_{max} + \Delta(v)\,.\mathrm{AND}.\,\mathrm{gap}_l \geqslant v_{max} + \Delta(v), \tag{A4}$$

where, as in Table I, the underlined part is added to make the rule fit into the scheme. Note that in this interpretation, the slack now is $\Delta(v) = \Delta' - v_{max} + v$, i.e., a function of the velocity.

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