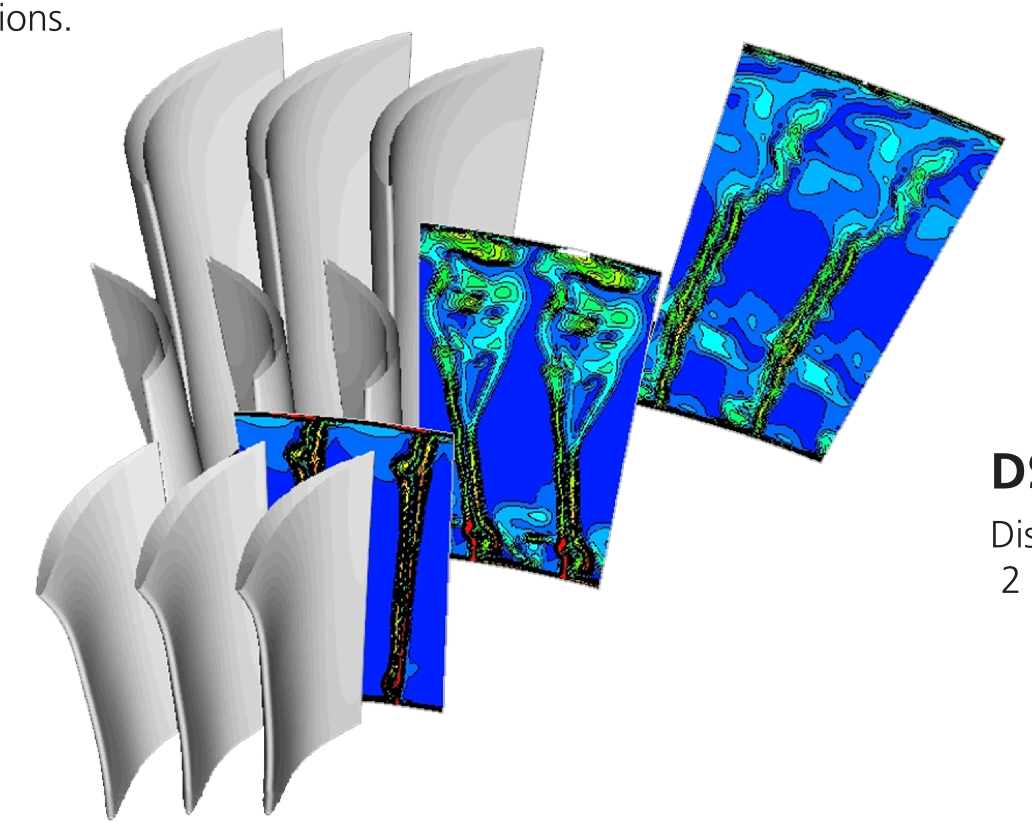
Scalable Distributed Schur Complement Preconditioning for Real and Complex CFD Problems on Many-Core Architectures

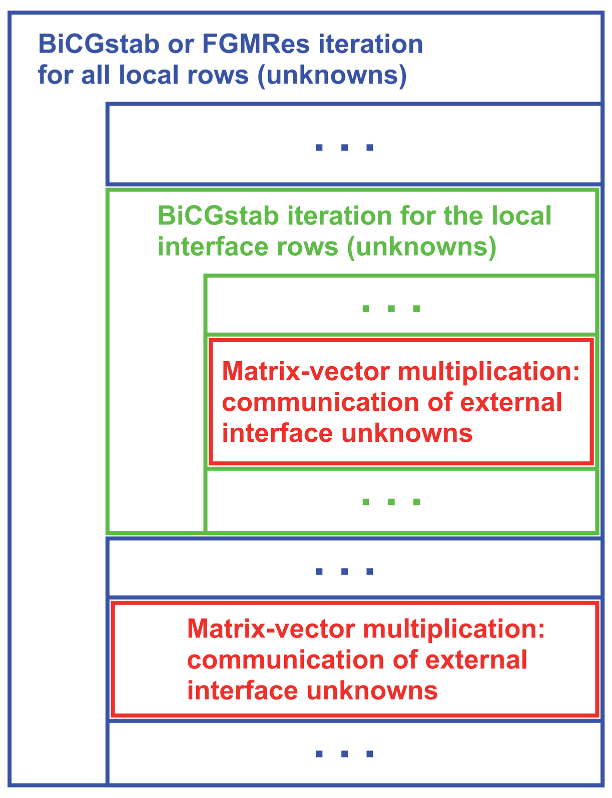
Parallel Simulation System TRACE

- TRACE: Turbo-machinery Research Aerodynamic Computational Environment
- Developed by the Institute for Propulsion Technology of the German Aerospace Center (DLR)
- Calculates internal turbo-machinery flows
- Finite volume method with block-structured grids
- The linearized TRACE modules require the parallel, iterative solution of large, sparse non-symmetric systems of linear equations.



DSC Method: Algorithm

Schematic view on each processor



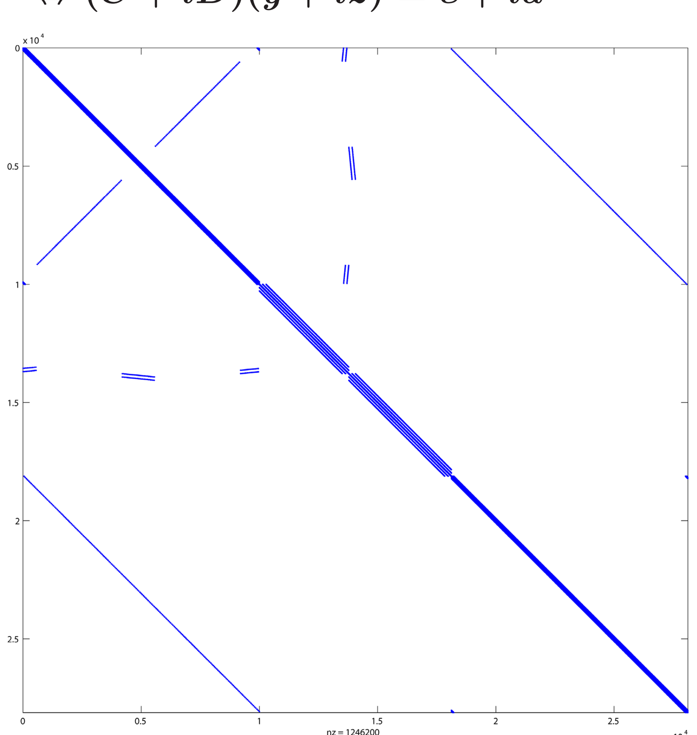
Typical *linearTRACE* Matrix Problem

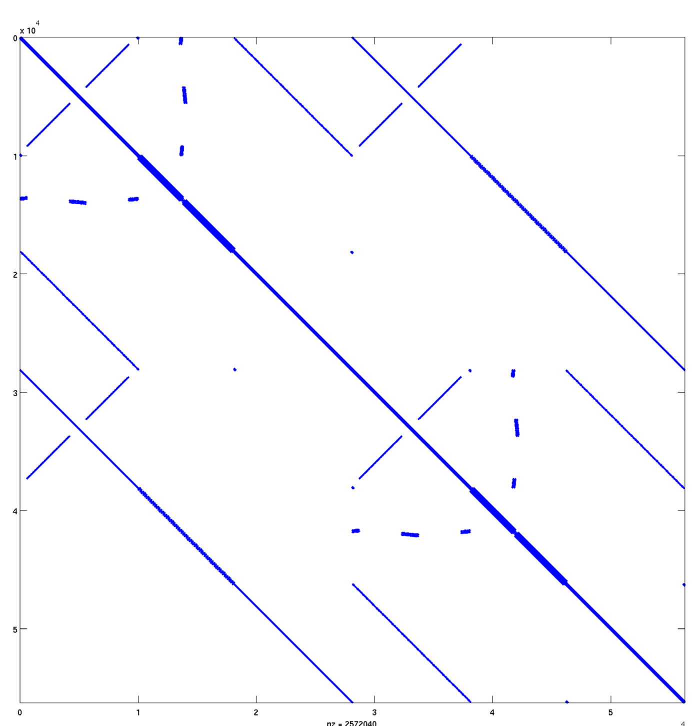
Complex TRACE matrix
n=28,120; nz=1,246,200; condition: 6.7·10[6]

$$Ax = b$$
$$\Leftrightarrow (C + iD)(y + iz) = c + id$$

Real TRACE matrix
n=56,240; nz=2,572,040; condition: 8.4·10[6]

$$\begin{pmatrix} C & -D \\ D & C \end{pmatrix} \begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} c \\ d \end{pmatrix}$$
$$\Leftrightarrow Gw = e$$





Preconditioner for TRACE: Background

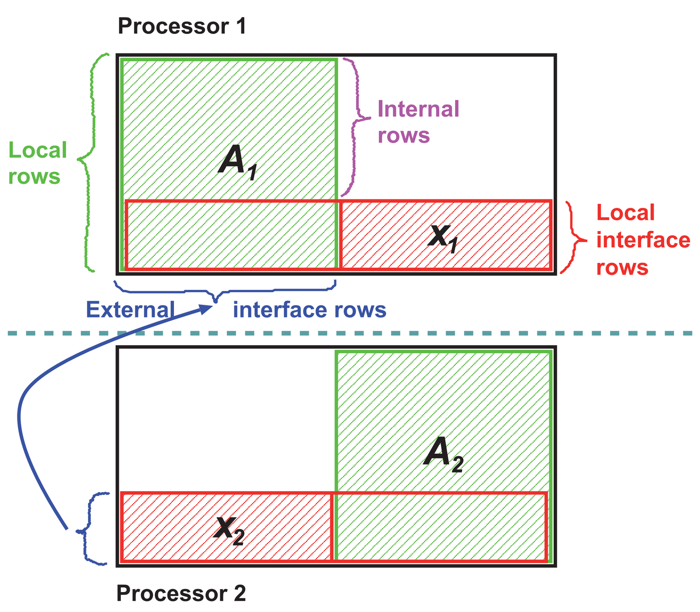
- Modules *linearTRACE* or *adjointTRACE* $Ax = b$
- A non-symmetric, complex or real, sparse
- Parallel iterative solver: (F)GMRes with preconditioning

$$P^{-1}Ax = P^{-1}b$$

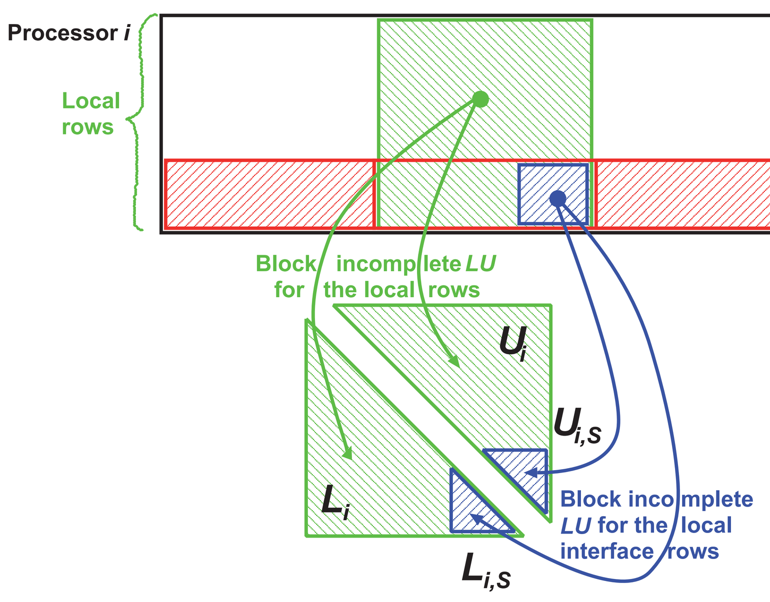
- Distinctly dominates the time behaviour
- Matrix-vector and vector-vector operations
- Preconditioning usually is the most time-consuming operation
  - Crucial for scalability
  - Status: block-local preconditioning
    - ILU, SSOR
    - Scalability limited
- Goal: global, scalable preconditioner
  - Experiments with Distributed Schur Complement (DSC) methods

DSC Method: Definitions

Distributed Matrix, 2 processors



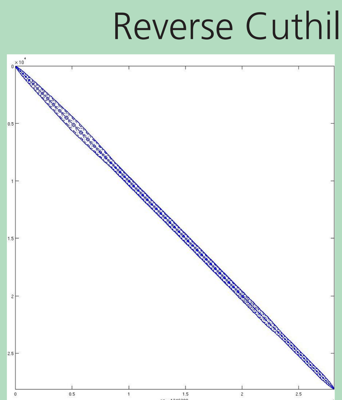
DSC Method: Incomplete LU Factorizations

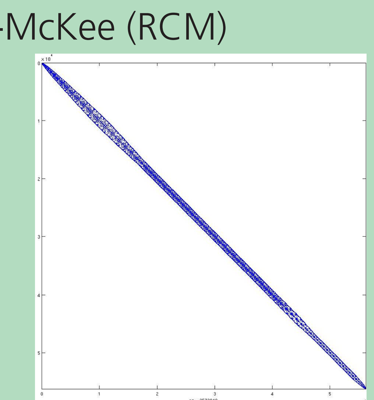


Matrix Permutation for Fill-in Reduction

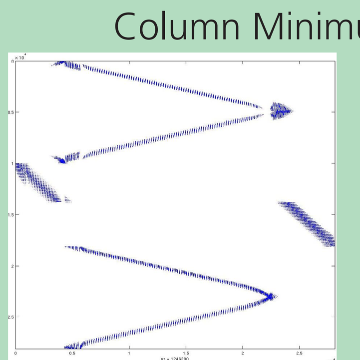
Complex **Real**

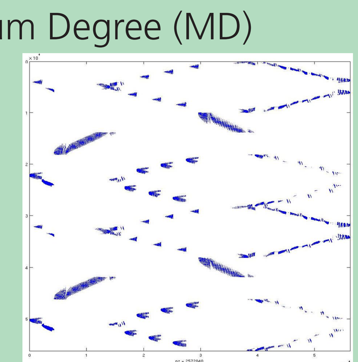
Reverse Cuthill-McKee (RCM)





Column Minimum Degree (MD)





Results of the Performance Analysis for TRACE Matrix Problems

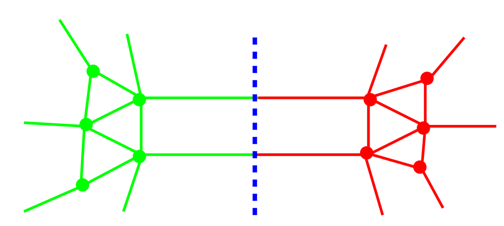
- Matrix permutations are crucial for preconditioner and iterative solver performance.
- The ILU preconditioned iterative solvers for the complex problem formulation distinctly outperform the solvers for the real formulation.
- **Reasons:** Complex formulation results in lower problem order, more advantageous matrix structure, has higher data locality and a better ratio of computation to memory access.
- The DSC method shows an advantageous scaling behavior.

DSC Method and Partitioning

Graph partitioning: *ParMETIS* (University of Minnesota)

Goal:

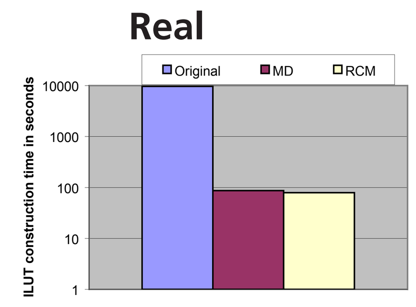
Minimize the number of edges cut ⟷ **number of interface unknowns**



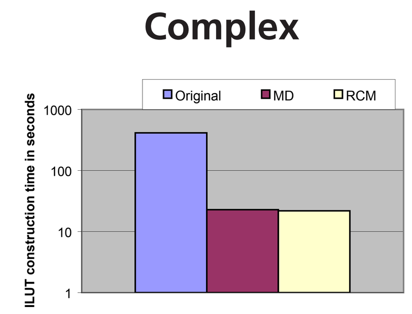
Performance Tests on a Quad-Core Intel Xeon CPU L5420 Workstation (MATLAB)

Effect of different matrix permutations (ILU threshold: 10^{-3})

Real

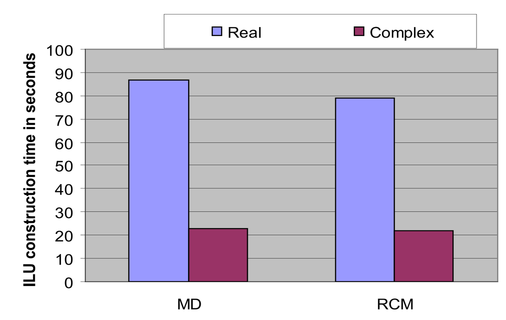


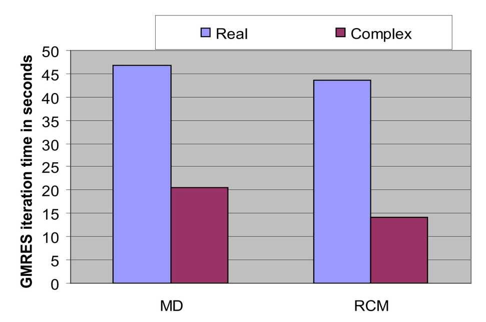
Complex



Matrix permutations significantly reduce fill-in and solution time.

GMRES preconditioned by ILU; complex or real arithmetics?



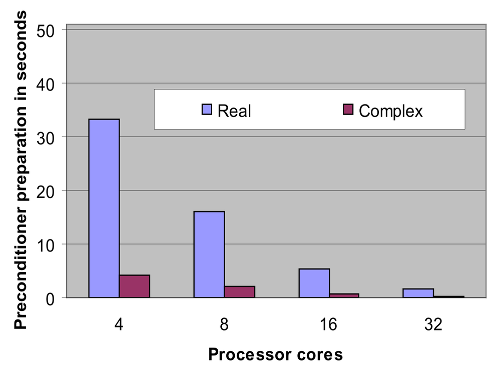


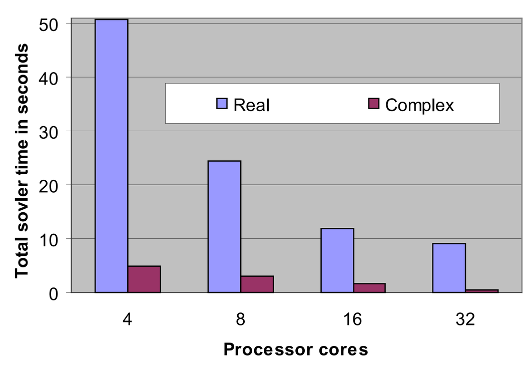
Complex formulation results in distinctly higher performance.

Performance on a Cluster at DLR

Quad-Core Intel Harpertown; dual-processor nodes; 2.83 GHz

Comparison: DSC method, real vs. complex problem formulation

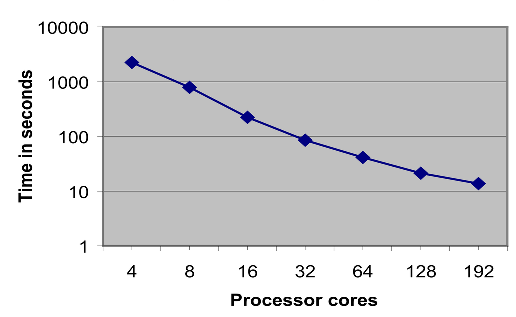




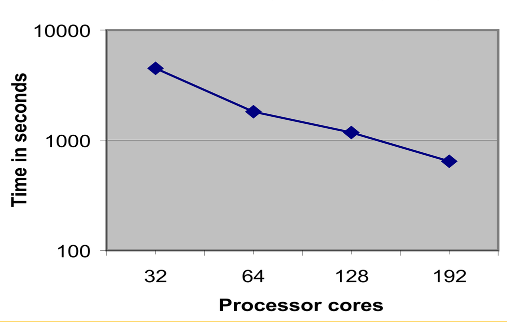
Performance for complex formulation is significantly superior.

Strong scaling for large complex TRACE matrix problems

THD: n=378,400; nz=45,456,500



UHBR: n=4,497,520; nz=552,324,700



The DSC method scales very well for large problems.

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HICFD - Hocheffiziente Implementierung von CFD-Codes für HPC-Many-Core-Architekturen (BMBF-Projekt im Rahmen des „IKT 2020 - Forschung und Innovation")

www.hicfd.de

GEFÖRDERT VOM

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