Deutsches Zentrum
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# AtPATH – Applying the path-following method to a complex railway vehicle model

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# 1 Introduction

The mechanical design of a railway vehicle is essential to provide the safe operation of the vehicle. There are several requirements to the mechanical behaviour of the vehicle. One of them is the running safety and running stability. This means that the vehicle follows the track and that it doesn't damage the track, e.g. by excessive hunting of the wheelsets and the bogies.

Below a certain driving speed, the so-called critical speed, lateral motions of the vehicle die out and the vehicle centres itself within the track. But if the vehicle runs faster than the critical speed, the hunting becomes a permanent motion. In many cases, this permanent hunting includes impacts between the flange of the wheel and the rail head which are related to high lateral forces that can damage the track. Therefore, the critical speed is an important characteristic of the vehicle, because it limits the operational speed.

However, finding the critical speed can be difficult and laborious. Usually, the behaviour of a railway vehicle is analysed on the base of a multibody dynamics model. Since the equations of motion are usually non-linear, they are solved by numerical time-integration. Since the behaviour of the vehicle strongly depends on the driving speed, it is obvious to vary the driving speed and calculate the motions of the vehicle for each value of the speed. However, a simple parameter variation may not be sufficient here. Firstly, it has always to be checked whether a permanent hunting of the vehicle occurs. In some cases, a time-integration over a long interval may be necessary, especially for a complex mechanical system like a railway vehicle, until it turns out, whether the motion only dies out slowly or it is permanent. Secondly, at a nonlinear system like a railway vehicle, the motions strongly depend on the initial conditions, i.e. for certain initial conditions, the motions die out, while for other initial conditions, a permanent oscillation occurs. All this can make the determination of the critical speed a very tedious task.

The question arises whether there are other methods to find the critical speed. One method is the quasi-linearization: Here, the original non-linear system is approximated by a linear system which can be solved very easily. However, this method provides only comparatively rough estimations, especially if the original system contains strong nonlinearities like the wheel-rail contact. Another method is the path-following method. Here, the original nonlinear equations of motions are solved. But in contrary to a ßimple"time-integration, solutions with special characteristics, e.g. periodic solutions, are calculated here.

A few years ago, an algorithm for the path-following has been successfully coupled to the multibody software package SIMPACK by Schupp. To test the capability of the path-following method for industrial applications, the project AtPATH has been launched. Here, the model of a modern double-deck passenger coach was analyzed, using several combinations of profiles for wheels and rails. The central central aspect of the project AtPATH is the question: Is the path-following method an applicable and useful tool for finding the critical speed of a given vehicle and, thereby, for the mechanical design of the vehicle?

# 2 Theoretical background

## 2.1 Nonlinear dynamics

Firstly, some basics of nonlinear dynamics shall be presented which can be applied to any kind of dynamical systems, not only to mechanical systems. Only a short overview shall be given, details can be found in several books on nonlinear dynamics.

An important characteristic of a dynamical system are its attractors. An attractor is a certain state or a certain motion like a permanent vibration that the system tries to reach. A dynamical system can have one or more attractors. Each attractor has a certain basin of attraction. If the initial state of the system lies within the basin of attraction of a certain attractor, the system will try to approach this attractor. The existence of an attractor and its basin of attraction depend on the system's parameters. Of course, the existence, the type and the shape of an attractor can change depending on changes of the parameter. In some cases, there can be several attractors for one set of parameters. This is called "coexistence". It depends on the initial states of the system which attractor the system will approach, i.e. in the basin of attraction of which attractor the initial states of the system lie.

Generally, there are four types of attractors:

1. Fixed points
2. Limit cycles
3. Limit tori
4. Strange attractors

A fixed point characterises a stationary solution of the system, i.e. if the system is approaching a fixed point, all changes of the system's states will die out and eventually the states will be constant. A classical example is a damped mechanical system. Here, the fixed point is its equilibrium.

A limit cycle is a periodic sequence of states. After a period, the states of the system have the same values as before the period. If $\mathbf{z}(t)$ is the state vector of the system and $T$ is the duration of the period, then the periodicity can be expressed by:

$$\mathbf{z}(t) = \mathbf{z}(t+T) \tag{2.1}$$

Since a periodic process can be described by a Fourier series, the limit cycle can be expressed as:

$$\mathbf{z}(t) = \sum_{i=1}^{n} \left[\mathbf{z}_{C,i} \cos\left(k_i \omega t\right) + \mathbf{z}_{S,i} \sin\left(k_i \omega t\right)\right] \ , \ \omega = \frac{2\pi}{T} \ , \ k_i \in \mathbb{N}_0 \tag{2.2}$$

Since all circular frequencies are integral multiples of the basic circular frequency $\omega$, the quotient of two circular frequencies $\omega_i = k_i\omega$ and $\omega_j = k_j\omega$ is always a rational number, i.e.:

$$\frac{\omega_i}{\omega_j} = \frac{k_i}{k_j} \in \mathbb{Q} \tag{2.3}$$

Typical examples for systems showing limit cycles are musical instruments like violins, the periodic self-excitation of the human heartbeat or the electric doorbell.

For a limit torus, the sequence of states can be expressed by a Fourier series:

$$\mathbf{z}(t) = \sum_{i=1}^{n} \left[\mathbf{z}_{C,i} \cos(\omega_i t) + \mathbf{z}_{S,i} \sin(\omega_i t)\right] \tag{2.4}$$

However, in this case the quotient of two circular frequencies is not always rational, i.e. there are at least two circular frequencies $\omega_i$ and $\omega_j$ with $\frac{\omega_i}{\omega_j} \notin \mathbb{Q}$. However, the spectrum of the system still consists of discrete circular frequencies.

A strange attractor is related to chaotic motions, i.e. irregular oscillations.

## 2.2 Application to railway vehicles

In the case of a railway vehicle, an important parameter is the driving velocity of the vehicle, since this parameter changes very often during operation. The attractors of a railway vehicle mainly are a stationary behaviour, i.e. a centred running within the track, and permanent self-excited vibrations. Usually, the stationary behaviour is desired. Permanent vibrations of the vehicle, especially the hunting motion, can cause damage to the track and, thereby, increase the risk of derailment. Therefore, the knowledge where an attractor leading to permanent vibrations exists is essential for the safe operation of the vehicle.

A generic example for the non-linear behaviour of a railway vehicle is shown in 2.1. Here, the amplitude $\hat{y}$ of the lateral motion of the wheelset versus the driving speed $v_0$ is displayed.

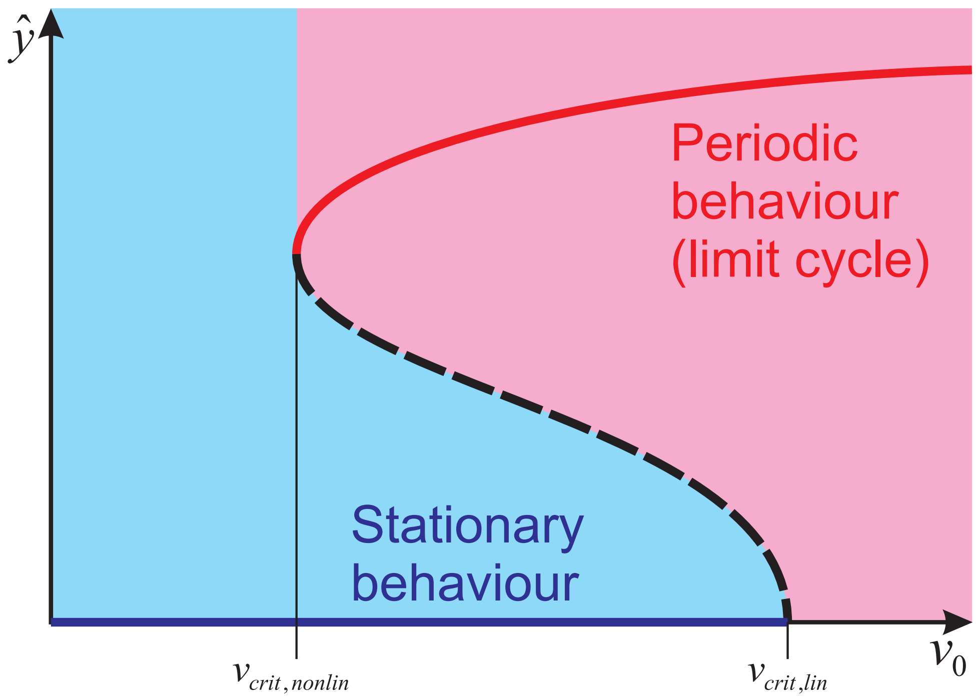


Abbildung 2.1: Attractors for a railway vehicle

There are three ranges of the driving speed, which are limited by the non-linear critical speed $v_{crit,nonlin}$ and the linear critical speed $v_{crit,lin}$. The first attractor of the system is the stationary behaviour (blue line), i.e. the centred position within the track. Its basin of attraction is displayed in light blue. The second attractor is a limit cycle (red line), i.e. a permanent hunting motion. Its basin of attraction is the pink area. In the first range below the non-linear critical speed, only one attractor exists: The stationary behaviour. For any initial lateral displacement, the oscillations will decay and the wheelset will centre itself within the track. In this region, a safe operation of the vehicle is given. Between the non-linear critical speed and the linear critical speed, two attractors are coexisting, namely the stationary behaviour and the permanent hunting. The broken line separates the basins of attractions of these attractors: If the initial displacement lies below the broken line, the lateral displacement will die out and the wheelset will try to reach its centred position; if the initial displacement is above the broken line, a permanent oscillation will occur. Above the linear critical speed, the attractor of the centred position doesn't exist anymore; in every case, a permanent oscillation will occur.

Both attractors are stable in the sense of Lyapunov, i.e. a motion starting in one basin of attraction will not leave the basin. If the attractor is asymptotically stable, then the motion will approach the attractor. However, in railway technology, the terms ïnstabilityänd ünstable runningäre widely used for the permanent hunting, although the related attractor is stable in the mathematical sense.

As mentioned before, it is desired that the vehicle will try to centre itself within the track when it is excited by an external perturbation, e.g. track irregularities or wind gusts. Of course, the application of pure numerical integration can show the reaction of the vehicle following an external excitation. However, the result that the vehicle's motions die out for a certain excitation at a certain driving speed does not guarantee that this will happen in every case at this driving speed. If the considered driving speed actually lies between the non-linear and the linear critical speed, it is possible that the vehicle will perform permanent vibrations. Since the model of a railway vehicle usually has a high number of states, it is practically impossible to investigate all possible scenarios. Therefore, the knowledge of the attractors is very helpful for a safe design of the vehicle, because the pure existence of an attractor leading to permanent vibrations implicates that the system has the possibility to reach this attractor, even this may be seem highly improbable. In other words: An attractor leading to permanent vibrations can be overseen.

## 2.3 The method of the path-following

As mentioned before, it can be very difficult to find an attractor at a certain driving speed by a pure application of the time-integration. A further disadvantage of this method is that it can take a long integration time, until all transient processes have died out and the attractor itself is left. Therefore, the question is if there is a method which avoids these problems.

In many cases, an attractor is a limit cycle, i.e. a periodic motion of the system. If $\mathbf{z}(t)$ is the state vector of the system and $T$ is the duration of the period, then the periodicity can be expressed by:

$$\mathbf{z}(t) = \mathbf{z}(t+T) \tag{2.5}$$

Usually, the equations of motion are given in the following form:

$$\dot{\mathbf{z}}(t) = \mathbf{f}\left(\mathbf{z}(t), t\right)\ ,\ \mathbf{z}(t=t_0) = \mathbf{z}_0 \tag{2.6}$$

This is an initial value problem where $\mathbf{z}_0$ is the initial value. The current state of the system depends on the time and the initial state; therefore, it can be written:

$$\mathbf{z} = \mathbf{z}(\mathbf{z}_0, t) \tag{2.7}$$

By defining a residual vector $\mathbf{r}$, the problem for a periodic motion can be formulated as:

$$\mathbf{r} = \mathbf{z}(\mathbf{z}_0, t_0+T) - \mathbf{z}_0 = \mathbf{0} \tag{2.8}$$

For the solution of this problem, the initial state $\mathbf{z}_0$ and the duration of the period $T$ are obtained. The calculation of the state $\mathbf{z}(\mathbf{z}_0, t_0+T)$ is still done by a numerical time integration.

However, the determination of $\mathbf{z}_0$ and $T$, i.e. the "coordinationöf the calculation, is done by an additional program. This method is called "direct calculation".

If $p$ is a parameter, then the equation of motion can be expressed as:

$$\dot{\mathbf{z}}(t) = \mathbf{f}\left(\mathbf{z}(t), t, p\right)\ ,\ \mathbf{z}(t = t_0) = \mathbf{z}_0 \tag{2.9}$$

The path-following method is able to analyse the system's behaviour depending on the parameter. If an initial state $\mathbf{z}_{0,i} = \mathbf{z}_0(p_i)$ for a certain value $p_i$ of the parameter is known, the path-following method variates the parameter to $p_{i+1}$ and predicts a new initial state $\widetilde{\mathbf{z}}_0(p_{i+1})$. Starting with $\widetilde{\mathbf{z}}_0(p_{i+1})$, the direct calculation determines the solution $\mathbf{z}_0(p_{i+1})$ and the duration of the period $T(p_{i+1})$. The stepsize $\Delta p_{i+1} = p_{i+1} - p_i$ is adapted automatically to provide an appropriate prediction $\widetilde{\mathbf{z}}_0(p_{i+1})$, from which the algorithm for the direct calculation converges towards $\mathbf{z}_0(p_{i+1})$.

The path-following method has been used for a long time in the investigation of non-linear systems. However, in many cases, the systems investigated were of comparatively low order with respect to their number of states. It was the achievement of Schupp to apply the path-following method on more complex systems like railway vehicles, cp. [1] and [2]. In addition, Schupp also succeeded in treating systems containing constraints, i.e. systems which were described by differential-algebraic equations (DAEs). The multi-body simulation program used was SIMPACK, version 8.604, the path-following algorithm was the program PATH, developed by Kaas-Petersen at the Technical University of Denmark (DTU), cp. [3].

The practical use of the path-following method contains the following steps:

1. Calculate a periodic solution for a certain parameter $p = p_0$ , i.e. $\mathbf{z}_0(p = p_0)$ and $T = T(p = p_0)$. This has to be done "manually", i.e. the equation of motion $\dot{\mathbf{z}} = \mathbf{f}\left(\mathbf{z}(t), t, p\right)$ has to be integrated until a periodic behaviour is reached.

2. Starting from this initial input, the path-following algorithm is varying the parameter $p$ in the desired direction, e.g. decreasing the driving speed $v_0$, and tries to find the initial state $\mathbf{z}_0(p)$ and the duration of the period $T(p)$. The integration of the equations of motion over one period $T(p)$ starting with the initial state $\mathbf{z}_0(p)$ yields the evolution of the system's state $\mathbf{z}(t, p)$. From this, the maximum and the minimum values of each coordinate can be extracted.

It should be pointed out, that the "simple" coupling of the multi-body dynamics program and the path-following algorithm were not the only thing to cope with, but also internal modifications of the multi-body dynamics program were necessary. Usually, the longitudinal motion of the essential components (wheelsets, bogie frames, car body) is described by an absolute coordinate. If the entire vehicle is moving along the track, the values of these coordinates increase. However, for the application of the path-following method, all states of the system have to be periodic. It is possible to impose a longitudinal motion with constant velocity on these components, so that the longitudinal motion is no longer a degree of freedom. However, this elimination of degrees of freedom may distort the dynamical behaviour of the system. Another possibility which Schupp implemented in SIMPACK is the use of relative coordinates. The longitudinal motion $s(t)$ is now expressed as a superposition of a motion with a constant driving speed $v_0$ and a relative motion $\tilde{s}(t)$. By using an offset $s_0$, the position, the velocity and the acceleration of the longitudinal motion are expressed as:

$$s(t) = s_0 + v_0 t + \tilde{s}(t) \tag{2.10}$$

$$\dot{s}(t) = v_0 + \dot{\tilde{s}}(t) \tag{2.11}$$

$$\ddot{s}(t) = \ddot{\tilde{s}}(t) \tag{2.12}$$

In this formulation, the relative motion can be periodic. A analogous description is used for the overturning motion of the wheelsets.

The advantage of the path-following method is the automatic calculation with defined accuracy without external intervention. Once started, the calculation including the variation of the parameter is performed automatically. The direct calculation of the periodic motion ensures that within a predefined tolerance a periodic solution is calculated. However, since the direct calculation exploits the periodicity of the solution, its application is limited to periodic motions, i.e. it is not applicable for the calculation of quasi-periodic or chaotic motions.

# 3 Point of departure

## 3.1 The program

The program used for this investigation is a special version of the multibody simulation software SIMPACK. The path-following program PATH was integrated into the SIMPACK version 8.604 by Schupp. The program PATH for the path-following method had been developed by Kaas-Petersen at the Technical University of Denmark, cp. [3]. It should be pointed out that PATH is program mainly for research purposes, not a commercial tool. This was also mentioned by Schupp, see [1]. The integration algorithm is based on the DASSL code which has been adapted for the use in PATH.

Since the release of the version 8.604, many changes within SIMPACK were done which make an integration of PATH into a newer version of SIMPACK difficult. Therefore, this version compiled by Schupp had to be used. In the version 8.604, some newer features were not yet available. One of them is the modelling of the wheel-rail contact as an elastic multipoint contact. Thus, an elastic single point contact was always used in this investigation.

As mentioned before, some further modifications of the multibody formulation were necessary to apply the path-following method on a model of a railway vehicle. The joint of the type 07 which describes a motion of a body along a railway track was modified to use a relative formulation for the motion along the track and the overturning motion. An analogous modification was done for the revolute joint of type 02 which describes the motions of the axle-boxes. In both cases, the motion is described by superposing a reference motion with constant velocity or angular velocity and a relative motion which is the degree of freedom.

## 3.2 The model

The model which was analysed in this study was provided by Bombardier Transportation. It describes a double-deck passenger coach equipped with two bogies, each bogie having two wheelsets. To approximate the structural elasticity, the carbody and the bogie frames consist of two separate bodies which are linked by springs. In the case of the carbody, one half can perform a rotation around the longitudinal axis relative to the other half. In the case of the bogie frame, a relative rotation between the two halfs around the lateral axis is possible. Each wheelset is supported by a standard wheel-rail track element which can perform lateral, vertical and roll motions. The body of the track element is supported by linear springs and dampers. The default parameters provided by SIMPACK were used, as indicated in Fig.3.1:

- Mass of the sleeper body: 330 kg
- Moment of inertia of the sleeper: 10 kg $m^2$
- Stiffness of the vertical springs: $2 \cdot 10^7$ N/m
- Damping of the vertical dampers: 49000 Ns/m
- Stiffness of the lateral springs: $7.5 \cdot 10^7$ N/m
- Damping of the lateral dampers: 94000 Ns/m
- Lateral distance between the vertical springs: 1.5 m

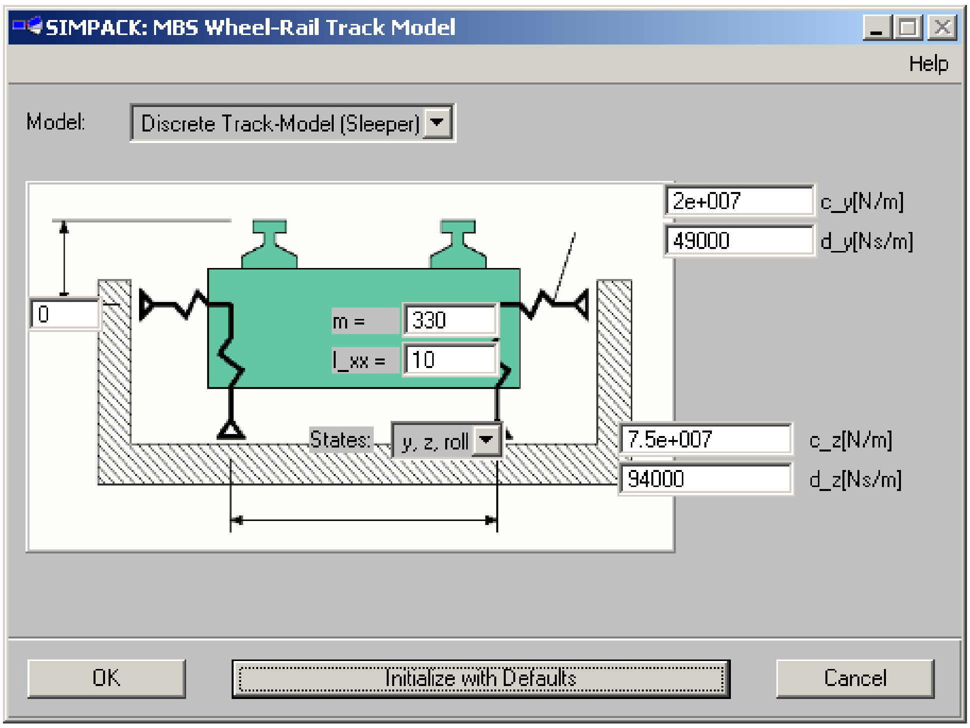


Abbildung 3.1: Parameters used in the track model

In total, the model has 88 states and 16 constraints, thereby it has 72 degrees of freedom. An overview on the model is given in Fig.3.2.

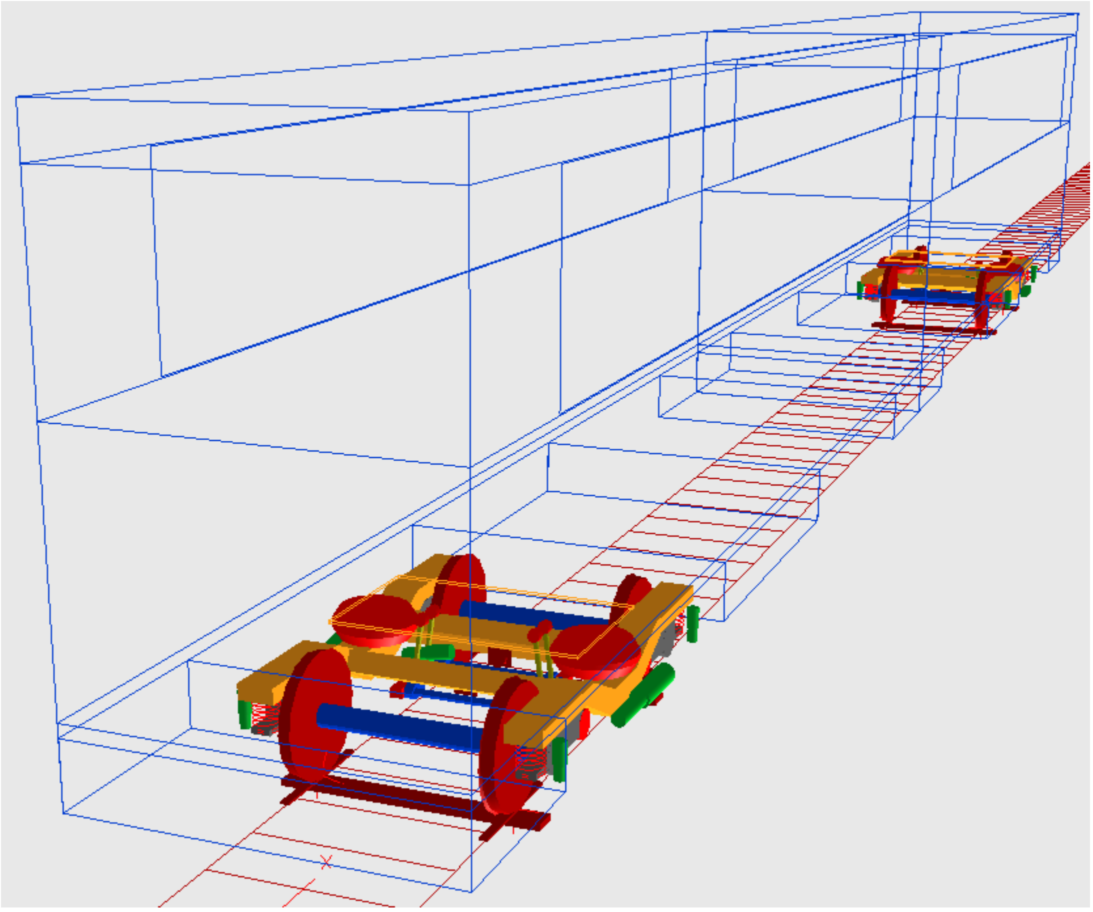

Abbildung 3.2: Overview on the multibody model of the passenger coach

Several configurations of the model were analysed. The following combinations of profiles for wheels and rails were used:

- Wheel: S1002, rail: WRS003, rail inclination: 0.0, gauge: 1435 mm
- Wheel: S1002, rail: EN52E1, rail inclination: 1:20, gauge: 1435 mm
- Wheel: STDW07, rail: UIC54, rail inclination: 1:20, gauge: 1435 mm
- Wheel: S1002, rail: UIC60, rail inclination: 1:20, gauge: 1450 mm

Furthermore, different force laws of the yaw dampers were used. The law of the damping force consists of a linear range around 0 m/s where a damping constant of 600 kNs/m is used. Beyond a certain limit which will be called "blow-off forceïn the following, the further increase of the damping force over the velocity is very small. Four configurations were used:

- Without yaw damper
- Blow-off force $F_{YD} = 6.0$ kN
- Blow-off force $F_{YD} = 12.0$ kN
- Blow-off force $F_{YD} = 18.0$ kN

In addition, different friction coefficients were applied to the wheel-rail contact in some cases. If now other value is explicitly indicated, a friction coefficient of $\mu = 0.4$ is used.

From the original model, a reduced model was derived in which the two wheelsets moved along the track with constant driving speed but performed no relative motions. The expression ”reduced model”will be used in the following when this model is meant. The reduced model is shown in Fig.3.3.

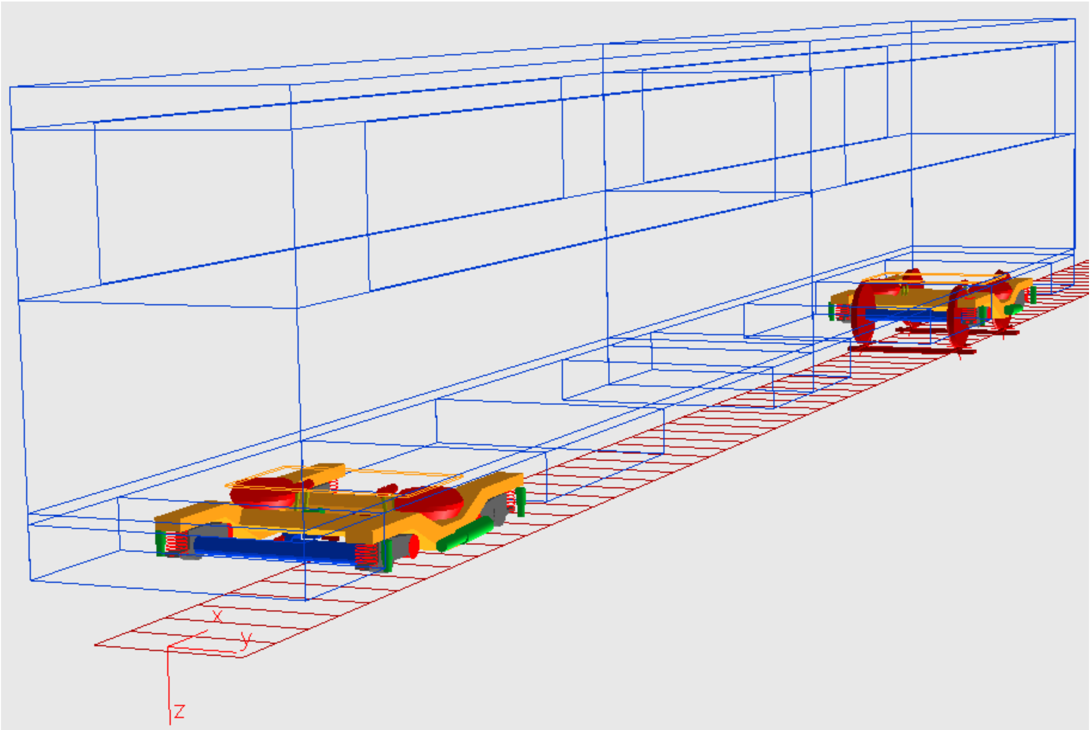


Abbildung 3.3: Reduced multibody model of the passenger coach

Since the hunting behaviour is usually important for the straight running of the vehicle, a straight track was always used. Because the path-following method tries to find the attractors of the system, the track was always undisturbed.

The essential results of PATH are written in files with the extension `*.path.soli`. This file has the following structure:

- Column 1: Number of the step
- Column 2: Value of the variated parameter, in this case $v_0$
- Column 3: Duration $T$ of the period
- Column $3i + 1$: Minimum value of the $i$-th state
- Column $3i + 2$: Maximum value of the $i$-th state
- Column $3i + 3$: Amplitude of the $i$-th state

In the analysed model, the wheelsets are the first four bodies. In the multibody model, each wheelset has 6 degrees of freedom, and the lateral motion is the second degree of freedom. However, since the angular position of the wheelset's overturning motion has no influence on the motion of the entire system due to the rotational symmetry of the wheelset, the overturning motion is not regarded as a degree of freedom by PATH. Therefore, the lateral motions have the numbers 2 and 7 for the wheelsets of the front bogie and the numbers 12 and 17 for those of the rear bogie. So the maximum lateral displacements of the wheelsets are contained in the columns 8 (bogie 1, wheelset 1), 23 (bogie 1, wheelset 2), 38 (bogie 2, wheelset 1) and 53 (bogie 2, wheelset 2).

# 4 Results

In each case, the maximum lateral displacement of the wheelsets will be regarded.

In every diagram diplaying the lateral displacement of the wheelsets, colours are used in the following way:

1. Bogie 1, wheelset 1: Red
2. Bogie 1, wheelset 2: Magenta
3. Bogie 2, wheelset 1: Blue
4. Bogie 2, wheelset 2: Cyan

## 4.1 Profile combination S1002 / WRS003

For the combination of the wheel profile S1002 and the rail profile WRS003, periodic behaviour occurs several cases. Furthermore, coexisting periodic solutions have been found, i.e. hunting of both bogies and hunting only of the front bogie. As an example, the two coexisting solutions are illustrated by the phase portraits of the wheelsets' lateral motions in Fig.4.1. It should be pointed out once more that the models were absolutely identical. The only difference leading to the different solutions were the initial conditions of the states.

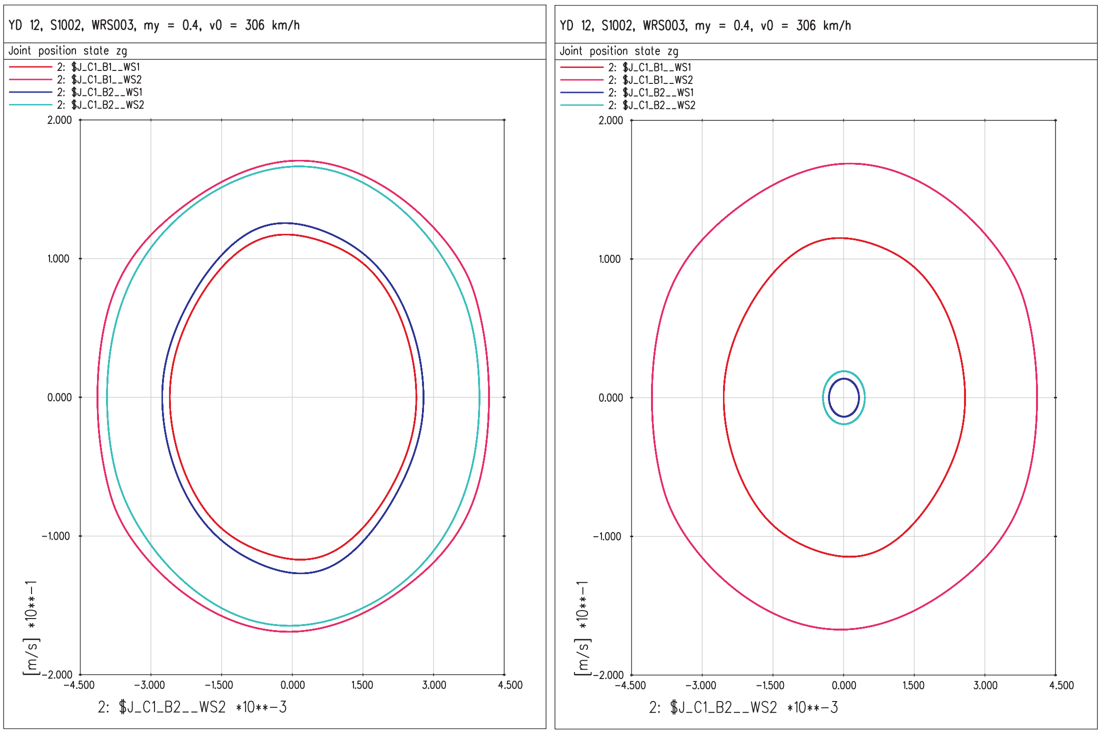


Abbildung 4.1: Coexisting solutions: Hunting of both bogies (left) and hunting only of the front bogie (right)

### 4.1.1 Hunting of both bogies

For the case, that both bogies perform ”real hunting”, i.e. a lateral displacement of several millimetres, periodic behaviour could only be observed, if the yaw dampers with the blow-off force $F_{YD} = 12$ kN or $F_{YD} = 18$ kN were used. Without yaw dampers or using the yaw damper with $F_{YD} = 6$ kN, only quasiperiodic motions occurred. The results of the path-following calculation are displayed in Fig4.2.

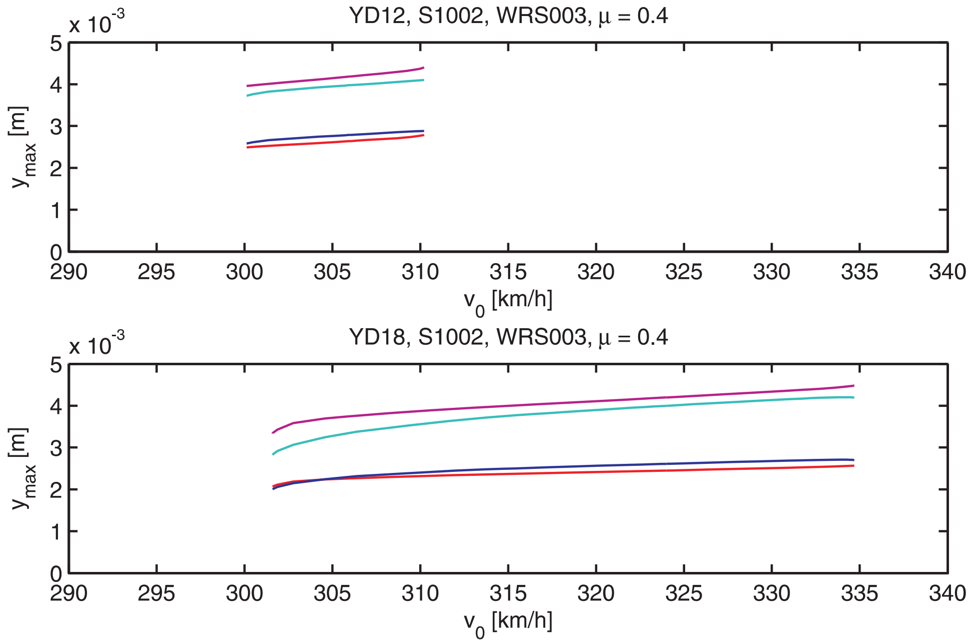


Abbildung 4.2: Hunting of both bogies, variation of the yaw dampers

The ranges of periodic behaviour found by the path-following method are:

- $F_{YD} = 12$ kN: $v_0 = 83.368$ m/s $\approx 300$ km/h up to $v_0 = 86.172$ m/s $\approx 310$ km/h
- $F_{YD} = 18$ kN: $v_0 = 83.774$ m/s $\approx 302$ km/h up to $v_0 = 92.971$ m/s $\approx 335$ km/h

Apparently, the range of periodic behaviour is enlargened by stronger yaw damping. The critical speed $v_{crit,nonlin}$ is shifted only slightly upwards by the stronger yaw damper, i.e. from $v_{crit,nonlin} \approx 300$ km/h up to $v_{crit,nonlin} \approx 302$ km/h. It can be assumed that for comparatively small periodic motions occurring at the critical speed the relative velocity of the damper’s ends hardly leaves the linear range for the blow-off force of $F_{YD} = 12$ kN. Thus, for the damper with $F_{YD} = 18$ kN, the same velocity stays within the linear range. This means that for both dampers the damper force at the critical speed is nearly the same, which explains the similar behaviour.

### 4.1.2 Hunting of the front bogie

In this case, only the front bogie performs a notable lateral displacement while the rear bogie stays nearly centred within the track. For this type of motion, periodic behaviour could be found for all four configurations of the yaw dampers. In Fig.4.3, the results of the path-following calculations for the front bogie hunting are displayed.

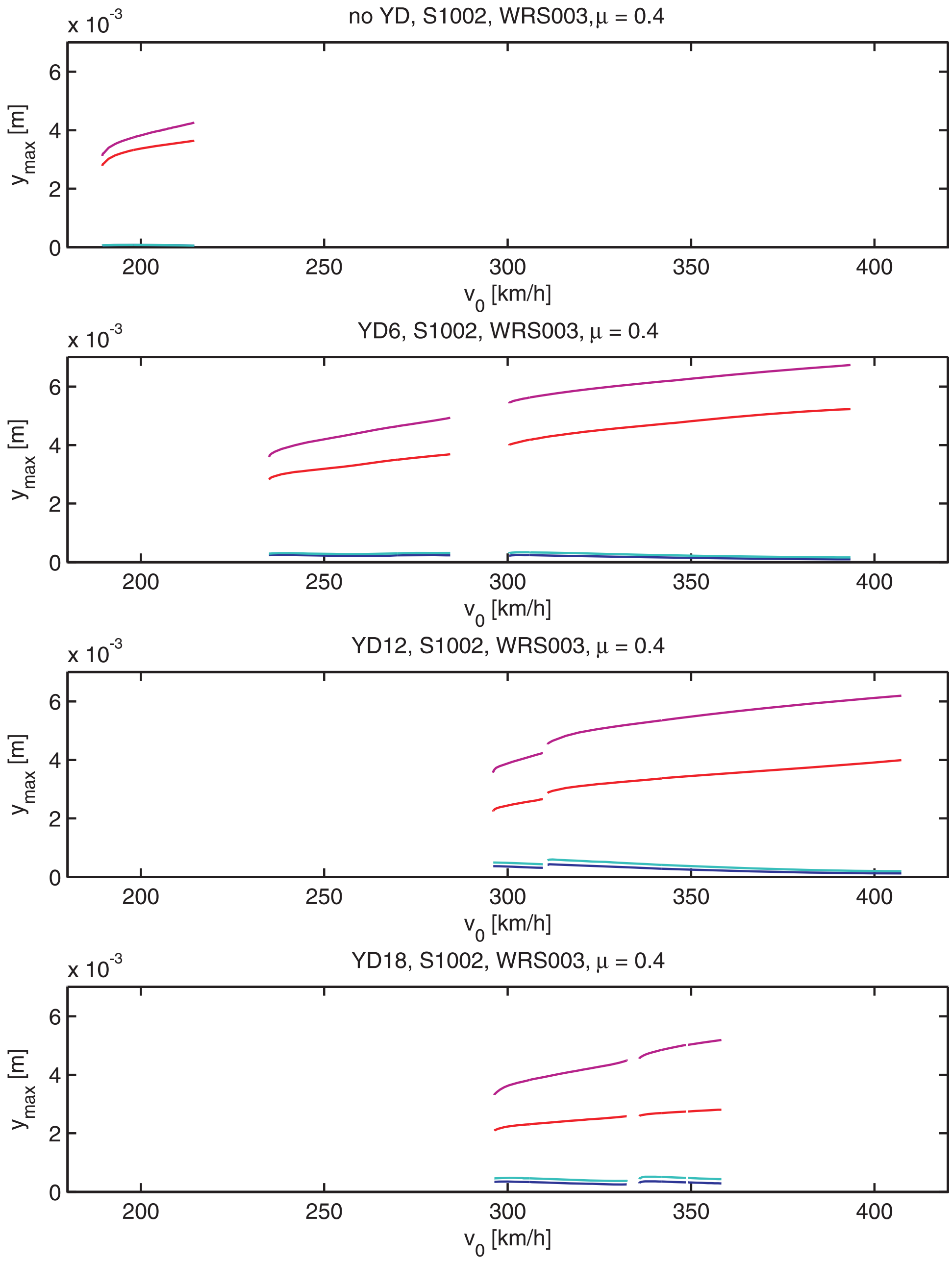


Abbildung 4.3: Hunting of the front bogie, variation of the yaw dampers

The values of the critical speed $v_{crit,nonlin}$, i.e. the lowest driving speed $v_0$ at which periodic motions occur, for the different configurations are:

- no yaw damper: $v_{crit,nonlin} = 54.821$ m/s $\approx 197$ km/h
- $F_{YD} = 6$ kN: $v_{crit,nonlin} = 65.259$ m/s $\approx 235$ km/h
- $F_{YD} = 12$ kN: $v_{crit,nonlin} = 82.241$ m/s $\approx 296$ km/h

- $F_{YD} = 18$ kN: $v_{crit,nonlin} = 82.324$ m/s $\approx 296$ km/h

As already observed for the hunting of both bogies, the critical speed is nearly the same for $F_{YD} = 12$ kN and $F_{YD} = 18$ kN. For the front bogie hunting, the difference is even smaller, probably because the velocity of compression and extension of the damper stays within the linear range. It is remarkable that the critical speed for the front bogie hunting is lower than for the hunting of both bogies.

It can be seen that the range of periodic motions is very small for the configuration without yaw dampers. For yaw dampers with $F_{YD} = 6$ kN, wide ranges of periodic behaviour appear, but an increase of the blow-off force diminishes the ranges. This will tried to be explained in the following.

If the front bogie is hunting, it excites the carbody and thereby also the rear bogie. In the case of the front bogie hunting, the amplitudes of the rear bogie's motions stay below the limit of self-excitation, i.e. regarding the diagram 2.1 below the broken line. If no yaw dampers are used, the bogie quickly reaches the self-excitation. Therefore, after the front bogie starts to hunt, its motions increase quickly with increasing driving speed and push the rear bogie over the low threshold of self-excitation. If yaw dampers with $F_{YD} = 6$ kN are used, the threshold to self-excitation is higher. Therefore, stronger motions of the front bogie are necessary to initiate the hunting of the rear bogie. If the blow-off force is higher and thereby the dampers are stronger, the transmission of forces from the front bogie to the rear bogie via the secondary suspensions and the carbody is stronger. One the one hand, the threshold to the self-excitation of the rear bogie is higher, which results in a shifting of the critical speed to a higher value. But on the other hand the "pushing" over the threshold is stronger. This leads to a shrinking of the range of the front bogie hunting.

Furthermore, the friction coefficient was varied using the values $\mu = 0.2$, $\mu = 0.4$ and $\mu = 0.6$. The configuration without yaw dampers was used. The results are displayed in Fig.4.4.

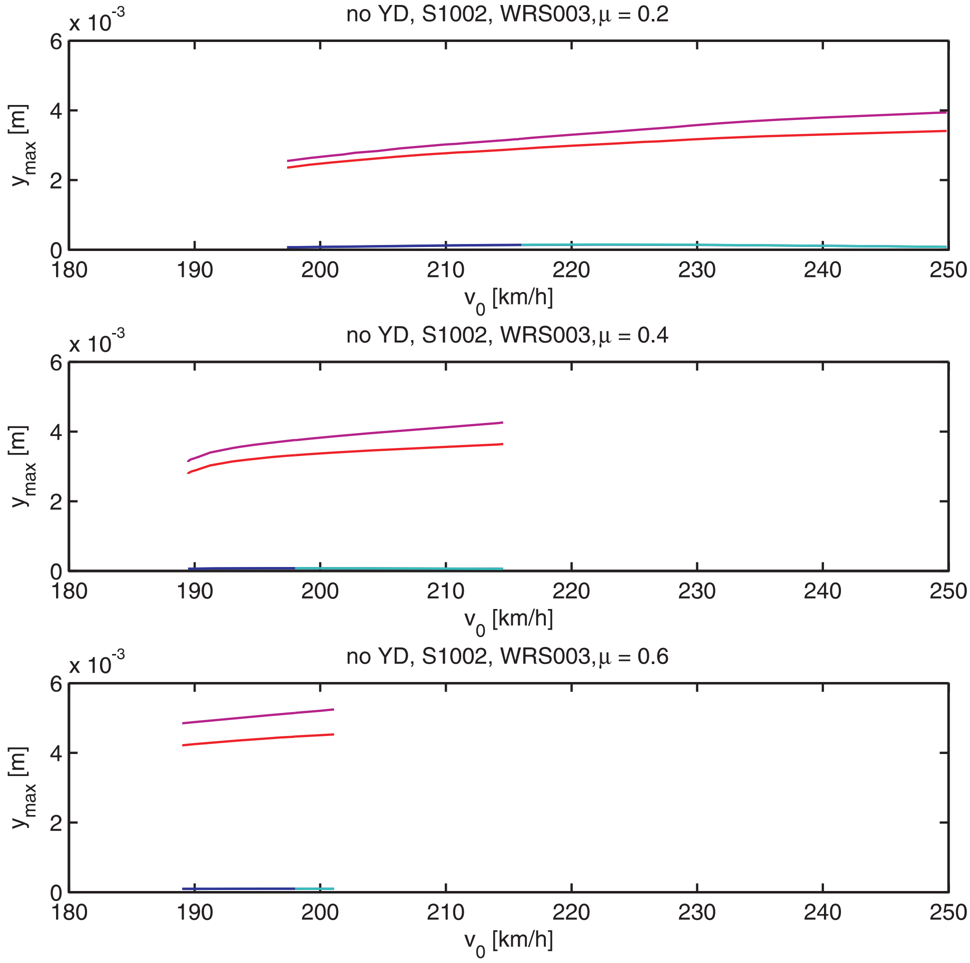


Abbildung 4.4: Influence of the friction coefficient

The nonlinear critical speed where the limit cycle behaviour starts is

- $v_{crit,nonlin} = 54.821\ \text{m/s} \approx 197\ \text{km/h}$ for $\mu = 0.2$
- $v_{crit,nonlin} = 52.634\ \text{m/s} \approx 189\ \text{km/h}$ for $\mu = 0.4$
- $v_{crit,nonlin} = 52.510\ \text{m/s} \approx 189\ \text{km/h}$ for $\mu = 0.6$

A higher friction shifts down the critical speed $v_{crit,nonlin}$ because it increases the self-excitation of the system. This also explains the higher amplitudes for higher friction coeffiecients. Furthermore, the range of periodic motions is wider for lower friction coefficients. If the frictional forces acting in the wheel-rail contact are lower, the tendency to self-excitation is weaker and the threshold to hunting is higher. Therefore, for a lower friction coefficient, stronger external forces transmitted by the secondary suspensions and the carbody are necessary to push the rear bogie over the threshold to hunting. Furthermore, the motions of the hunting front bogie are smaller, so that the forces applied by the bogie to the carbody are weaker. This explains why a reduction of the friction coefficient causes such a distinct widening of the range of the front bogie hunting.

## 4.2 Profile combination S1002 / EN52E1

For the combination of the wheel profile S1002 and the rail profile EN52E1 with an inclination of 1/20, no notable ranges of periodic behaviour could be found. Only quasi-periodic motions including hunting of both bogies were observed. No coexisting periodic or quasi-periodic motions with only one bogie hunting could be found.

Because the original model showed no periodic behaviour, an analysis using the reduced model was carried out. For this model, wider ranges of periodic behaviour were found that were analysed using the path-following method. As an example, the periodic behaviour for different configurations of the yaw dampers is shown in Fig.4.5. Because the reduced model is used, only the lateral motions of the front bogie's wheelsets are shown.

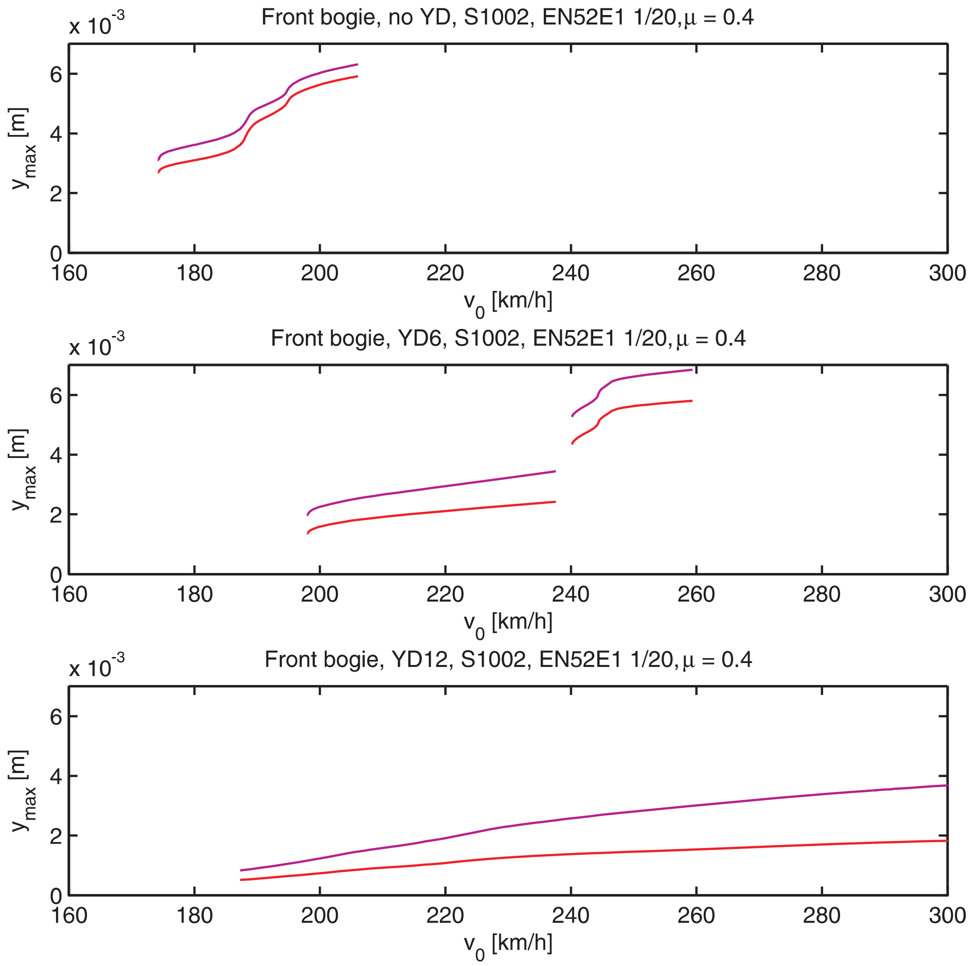


Abbildung 4.5: Reduced model, variation of the yaw dampers

The important information for the vehicle design is the driving speed at which the drastic increase of the lateral motions occur. It can clearly be seen that a stronger yaw damper shifts this speed to higher values.

## 4.3 Profile combination STDW07 / UIC54

For the combination of the wheel profile STDW07 and the rail profile UIC54 with an inclination of 1/20, the vehicle model without yaw dampers showed only quasi-periodic behaviour. However, if the yaw dampers with the blow-off force of $F_{YD} = 18$ kN were used, a wide range of periodic behaviour appeared. The results are displayed in Fig.4.6.

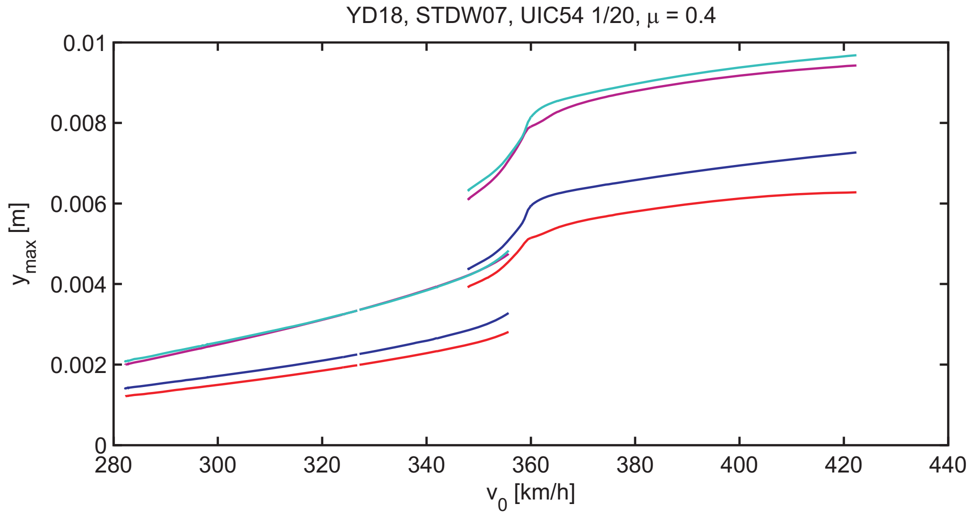


Abbildung 4.6: Periodic behaviour for strong yaw damping

It should be pointed out that the results had to be calculated in several runs due to numerical difficulties. – It can be seen that between $v_0 = 348$ km/h and $v_0 = 356$ km/h lies a range where two limit cycle attractors are coexisting.

## 4.4 Profile combination S1002 / UIC60, track gauge 1450 mm

For the combination of the wheel profile S1002 and the rail profile UIC60 with an inclination of 1/20 and a track gauge of 1450 mm, periodic motions could be observed over wide ranges. However, the application of the path-following turned out to be very difficult because of numerical problems of the integration method. The results of the calculations are displayed in Fig.4.7.

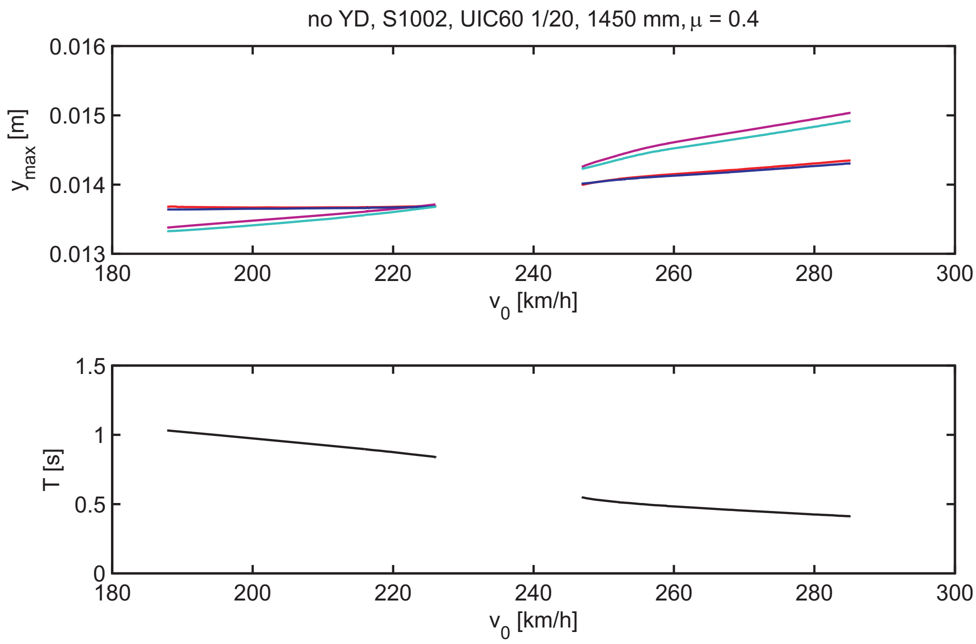


Abbildung 4.7: Maximum lateral displacements of the wheelsets (above) and duration of the period $T$ for no yaw dampers, S1002 / UIC60, inclination 1/20, track gauge 1450 mm

It is remarkable that the frequency of the motion is comparatively low, e.g. $f = 1.117$ Hz at $v_0 = 216$ km/h or $f = 1.944$ Hz at $v_0 = 252$ km/h. In contrast, the frequencies which occurred for the other profile combinations were considerably higher, e.g. $f = 5.565$ Hz for the front bogie hunting with the profile combination S1002 / WRS003 and $\mu = 0.2$. However, the evolution of some states over time can be quite complicated, as shown in Fig.4.8.

The time-integration over such a long interval requires a high accuracy of the integration method, especially if such complicated evolutions of some states occur. Apparently, some limits of the DASSL algorithm were reached.

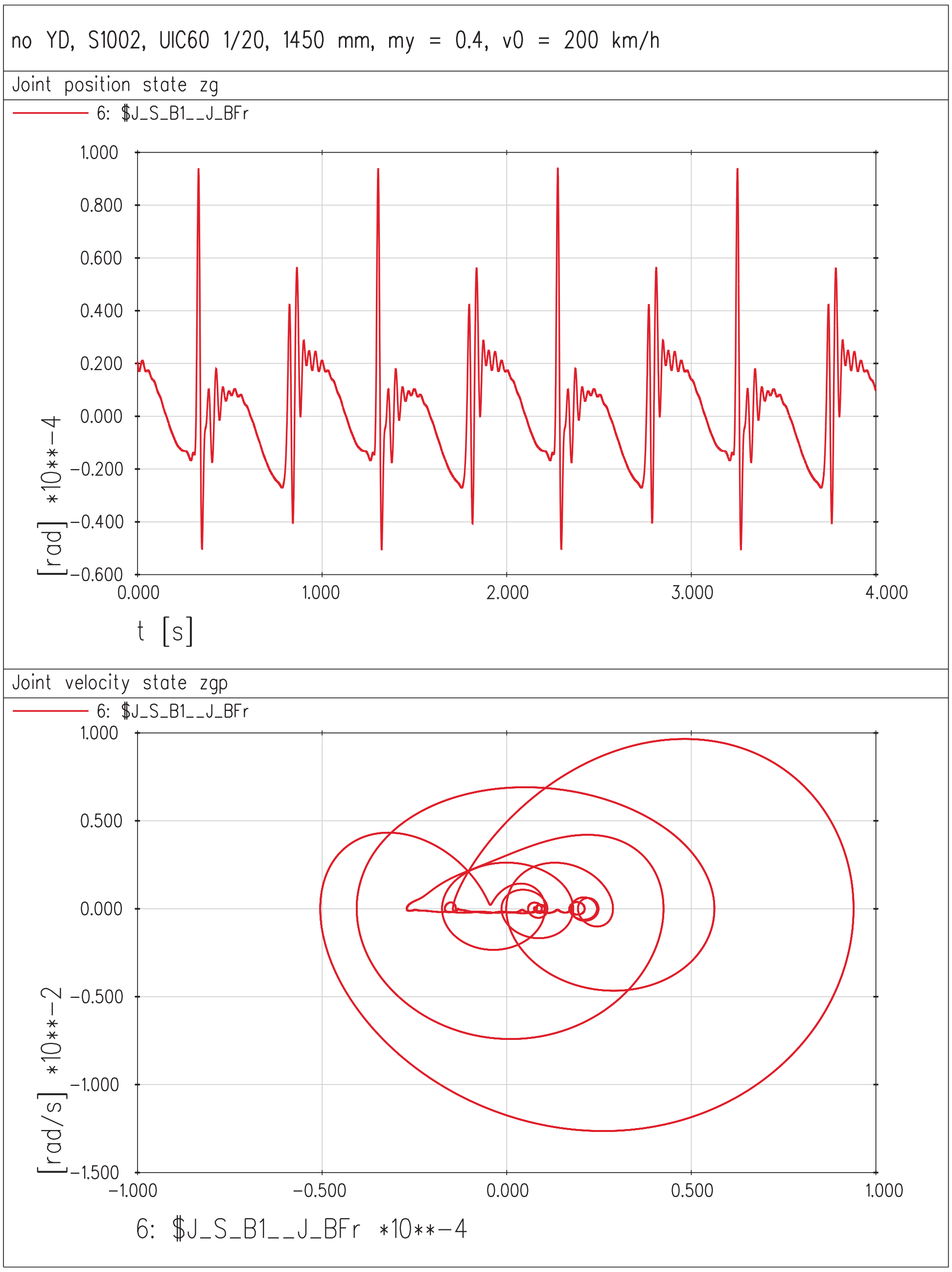


Abbildung 4.8: No yaw dampers, S1002 / UIC60, inclination 1/20, track gauge 1450 mm: Time plot (above) and phase portrait for the pitch motion of the frame of the front bogie

# 5 The quasi-periodic motion

The results obtained with the different configurations of the systems showed that in many cases quasi-periodic motions occurred which made the application of the path-following method impossible. The question arises what the reason for this behaviour is.

Usually, quasi-periodic motions occur, if a system is excited by two frequencies which slightly differ from each other. Let $\Omega$ and $\omega$ be angular velocities belonging to a large and a small frequency, so that $|\Omega| > |\omega|$. By considering the superposition of two vibrations with slightly different angular velocities, namely $\Omega - \omega$ and $\Omega + \omega$, and applying the theorems for the trigonometric functions, one obtains:

$$\begin{aligned} & \sin(\Omega t - \omega t) + \sin(\Omega t + \omega t) \\ = \; & \sin(\Omega t)\cos(\omega t) - \cos(\Omega t)\sin(\omega t) + \sin(\Omega t)\cos(\omega t) + \cos(\Omega t)\sin(\omega t) \\ = \; & 2\sin(\Omega t)\cos(\omega t) \end{aligned} \tag{5.1}$$

Here, the structure of the quasi-periodic motion is clearly visible: There is an "envelope" $\cos(\Omega t)$ with the low frequency. Inside the envelope, the system performs high-frequent vibrations $\sin(\omega t)$. In acoustics, this results in a periodic increasing and decreasing of the volume. This phenomenon is called "beat".

Regarding a railway passenger coach with two bogies, there are in fact two sources of excitation, namely the hunting motions of each bogie due to the self-excitation. The wheelset and thereby the bogie is a highly non-linear system, especially due to the non-linearity of the contact geometry. A typical characteristic of a non-linear system is the dependency between the amplitude of the system's motion and the duration of the period. Since the motion of both bogies can have slightly different amplitudes, the frequencies of their motions can also differ from each other. This might be the reason for the excitation of the carbody by two slightly different frequencies. It also explains why for the reduced model with only one "active" bogie, mostly periodic motions occurred, because in this case there is only one source of excitation.

Furthermore, if two systems, each of them having a self-excitation mechanism, are coupled, a synchronisation can occur. This means that both systems are influencing each other resulting in a motion with one common frequency. One of the best known examples for this effect is an orchestra: A string instrument like a violin generates its sound by self-excited vibrations. It is practically impossible to tune several instruments to exactly the same frequency, so there will always be a slight mistuning. However, if the mistuning is not too big, a pure tone without beats can be heard, because due to the dynamic interaction of the instruments via the vibrating air, a common frequency is generated.

It is obvious that a stronger coupling of the subsystems enforces their interaction and thereby the synchronisation. This explains why stronger yaw dampers lead to wider ranges of periodic behaviour. In the case of low-frequent hunting as for the profile combination S1002 / UIC60 with a track gauge of 1450 mm, large motions of the carbody are excited. This also leads to a stronger interaction of the two bogies, even without yaw dampers, and thereby to periodic behaviour.

# 6 Conclusion

The investigation has shown that the application of the path-following method which is integrated into SIMPACK can be applied even to complex models like the model of a double-decker passenger coach. However, there are some restrictions.

Due to the structure of the coach, namely a carbody with two bogies, quasi-periodic motions can occur, especially for a comparatively soft connection by the secondary suspension. Since the path-following method exploits the periodicity of the motions for the direct calculation, the case of quasi-periodic motions cannot be handled.

It has turned out that even for a vehicle with two bogies the synchronisation of both bogies' motions not always occur, which results in quasi-periodic behaviour. Therefore, it is very doubtful whether the application of the path-following method on models of vehicles which have even more bogies like articulated trains is sensible. In this case of an articulated train, more bogies mean more sources of excitation. Furthermore, the interaction of the bogies might be weaker. Considering articulated train having two segments and three bogies, the interaction of the front bogie and the rear bogie can only be done by transmissions of the vibrations over two segments which have a high inertia.

Furthermore, it should be mentioned that for the application of the path-following method a more stable integration method is desirable. In some cases, numerical problems occurred, i.e. the algorithm could not solve the equations of motion, especially for closer tolerances. For an industrial application with high reliability, a more stable algorithm than the used DASSL code is highly desirable.

On the other hand, the path-following method allows studies which would be very laborious if done by "trial and error", i.e. by changing a parameter, integrate the system, check its behaviour etc. The application to the reduced system, i.e. the car having only one bogie, has provided interesting results of the influence of certain parameters like the strength of the yaw dampers or the friction coefficient. Although the behaviour of only one bogie doesn't exactly represent the behaviour of the entire vehicle, it shows however trends that changes of parameters cause. This might be a helpful method to find an optimal mechanical design of the vehicle.

As a result, it can be said that the application of the path-following method on complex mechanical models of railway vehicles is possible. However, some cases cannot be handled by this method, and for an industrial use, some improvements of the numeric still have to be done.

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