Minimum Cost Flow phase unwrapping supported by multibaseline unwrapped gradient

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Abstract

The objective of the TanDEM-X Mission is the generation of a global high resolution Digital Elevation Model (DEM). To carry out this goal, two interferograms with different baselines will be acquired. We propose a novel two-stage multibaseline phase unwrapping method. Maximum Likelihood Estimation (MLE) is used to reduce the ambiguity and errors in gradient estimation on a pixel-by-pixel basis. Based on these estimates, Minimum Cost Flow (MCF) is used to unwrap the phase accounting for the overall conservative condition of the gradient. Hence the advantages of both techniques are efficiently integrated. Results on simlated data using TerraSAR-X parameters are reported.

1 Introduction

TanDEM-X Mission will start at the end of this year with the primary objective of generating a consistent global Digital Elevation Model (DEM) with an unprecedented accuracy. The whole land mass will be mapped twice with two different baselines in order to reduce the difficulty of phase unwrapping while achieving the required accuracy. Indeed, phase unwrapping is a crucial step to obtain this high quality DEM.

Classically DEMs have been generated from a single interferogram. Constantini [1] proposed a branch cut based phase unwrapping algorithm (Minimum Cost Flow, MCF). At DLR, a new MCF implementation optimized both in terms of memory and time consumption was developed [2]. Its efficiency has been proved during the SRTM mission. This algorithm follows a global approach incorporating the prior that the gradient of the unwrapped phase should be a conservative field. It is based on gradient estimates. Thus it tries to correct their ambiguities. However problems could arise in this process.

One of the novelties of the TanDEM-X Mission is its multibaseline approach. Several methods for multibaseline phase unwrapping have been suggested. Based on Maximum Likelihood Estimation (MLE) extensions have been developed [3]. Most of them work on a pixel-by-pixel basis. Hence no overall structural information is accounted for. Reconstruction quality could be increased using an *a priori* model [4]. Nonetheless, processing time is too long.

In our case, only two interferograms will be available during the first two mission years. The method we propose here combines both MCF and MLE. It is structured in two stages. Firstly, MLE is used to unwrap phase gradients on a pixel-by-pixel basis. The required search interval for gradient MLE is much smaller than the one for phase MLE. Thus computation is considerably faster. Secondly, phase unwrapping of the most accurate interferogram is performed with the MCF algorithm. Since gradient MLE has already solved or reduced the ambiguity error in gradient estimates, errors related to its estimation are reduced. Moreover, MCF introduces the overall conservative condition on the gradient, compensating the locality of the MLE stage. As a consequence, the advantages of both MCF and MLE are efficiently combined into a single robust framework.

2 Multibaseline gradient unwrapping to support MCF algorithm

The objective of phase unwrapping is to derive an estimate $\hat{\phi}(i,k)$ of the *true* phase $\phi(i,k)$ given its wrapped values $\psi(i,k) = \mathrm{W}\{\phi(i,k)\} \in [-\pi, \pi)$. Our idea is to calculate a *gradient estimate* $\hat{\nabla}\psi(i,k)$ of $\nabla\phi(i,k)$. This estimate is then integrated to determine $\hat{\phi}(i,k)$ with the help of the MCF algorithm.

2.1 Multibaseline Maximum Likelihood Estimation of the Gradient

The gradient is estimated by computing partial derivatives of $\psi(i,k)$ and wrapping them back if they exceed $\pm\pi$

$$\hat{\nabla\psi}(i,k) = \begin{pmatrix} \mathrm{W}\{\Delta_i\psi(i,k)\} \\ \mathrm{W}\{\Delta_k\psi(i,k)\} \end{pmatrix} = \begin{pmatrix} \mathrm{W}\{\psi(i+1,k)-\psi(i,k)\} \\ \mathrm{W}\{\psi(i,k+1)-\psi(i,k)\} \end{pmatrix}. \quad (1)$$

Given the probability density function (*pdf*) of an interferogram sample [5]

$$pdf(\phi;\gamma,L) = \frac{\Gamma(L+\frac{1}{2})(1-\gamma^2)^L \mid \gamma \mid \cos(\phi-\phi_0)}{2\sqrt{\pi}\Gamma(L)(1-\gamma^2\cos^2(\phi-\phi_0))^{L+\frac{1}{2}}} + \frac{(1-\gamma^2)^L}{2\pi} {}_2\mathrm{F}_1(L,1;\frac{1}{2};\phi^2\cos^2(\phi-\phi_0)), \quad (2)$$

the *pdf* of the phase difference between two neighbouring statistically independent samples is derived as the convolution of *pdf* of those samples. Hence the gradient distribution $pdf(\nabla\phi)[i,k]$ is [6]

$$pdf(\phi;\gamma,L)[i+1,k] * pdf(-\phi;\gamma,L)[i,k]. \quad (3)$$

Due to the wrapping operator W this *pdf* is 2π-periodic and centered at $\Delta\phi \pm 2n\pi$. Nevertheless n varies only maximum between $[-2,2]$ since it is very unlikely to have a very high gradient.

Given a set of interferograms $\{\psi_l\}_{l\in\{1,...,L\}}$ and once one has been selected as reference $\big(l=1\big)$, the distributions of the gradients of the others may be converted accounting for the ratio of baselines a_l defined as

$$a_l = \frac{B_1}{B_l}, \quad (4)$$

where B_l is the baseline of the interferogram l. As a consequence, the pdf of the reference remains 2π-periodic, while that of interferogram l is $2\pi a_l$-periodic.

These independent gradient estimations can be combined through MLE. The *joint pdf* of the gradient is the product of each *pdf* of the interferograms converted to the reference geometry

$$pdf(\nabla\phi_1,\nabla\phi_2,...,\nabla\phi_N)[i,k] = \prod_l^N pdf(\phi_l;\gamma_l,L)[i+1,k] * pdf(-\phi_l;\gamma_l,L)[i,k]. \quad (5)$$

Given the density of rational numbers in the real ones and the finite precision of the estimated baselines, the ratios $\{a_l\}_{l\in\{1,...,L\}}$ can be approximated by rational numbers. Then this joint pdf is also periodic, with an extended period P which can be given by

$$P = m.c.m.\big\{2\pi q_l\big\}_{l\in\{1,...,L\}}, \quad (6)$$

where $m.c.m.$ stands for the minimum common multiple. Thus the ambiguity on the gradient value is solved or reduced. The maximum of the *joint pdf* in the interval $[-P/2, P/2)$ constitutes our MLE estimate.

We obtain a multibaseline gradient estimate which does not correspond to any of the ambiguities of the original interferograms. It follows that we round it to the nearest ambiguities of the interferogram of highest baseline, since its precision is the highest. This process is a requirement for the second stage of the algorithm.

2.2 Adaptation of Minimum Cost Flow algorithm to use unwrapped gradients

The MCF approach solves the following global minimization problem

$$\min_{d_i d_k}\left\{\sum_{i,j} c_i(i,k)|d_i(i,k)| + \sum_{i,j} c_k(i,k)|d_k(i,k)|\right\}. \quad (7)$$

In usual MCF approach, $d_i(i,k)$ and $d_k(i,k)$ are the residue fields and have values equal to $0,-2\pi,2\pi$. They are used in order to correct the gradient estimate, making it conservative.

In our approach, new residue fields are calculated with the help of the unwrapped gradients from section 2.1. Thus, values can be now integer multiples of 2π.

The cost functions are $c_i(i,k)$ and $c_k(i,k)$. They can depend on coherence, phase gradient variance, etc. In our case they have to be used as a connection between MLE and MCF and are derived from a quality estimator of the gradient estimate. It is the spread of the resulting distribution given by

$$q[i,k] = \sqrt{\frac{\sum\Big(pdf(\nabla\phi_l)[i,k](\nabla\phi - argmax(pdf(\nabla\phi_l)[i,k]))^2\Big)}{max(pdf(\nabla\phi_l)[i,k])}}. \quad (8)$$

These costs show a dependency on the coherence and is linked to the number of peaks.

3 Results

Following [7], multibaseline interferograms have been simulated using a DEM obtained from a repeat-pass TerrarSAR-X (TSX) interferogram and real TSX geometrical parameters. Hence our data is realistic regarding geometrical aspect but without any atmospheric artefacts for example. Moreover, we can control the level of noise of these simulated phases.

We simulated two interferograms with two different baselines. The first interferogram has a height of ambiguity of 52.4 m/cycle and the second one of 39.1 m/cycle, analogous to TanDEM-X operational configuration. Search interval for gradient MLE is three cycles of the *reference* interferogram.

Multibaseline gradient estimation has been performed to remove the gradient ambiguity for each interferogram. Figure 1 shows two cases of *joint pdf*, both in terms of single and joint gradient pdf. In the first plot, the estimation is correct whereas the gradient has been estimated badly in the second one.

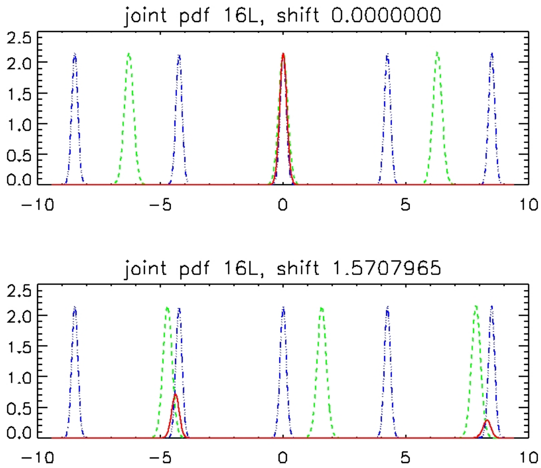

Figure 1: Example of *pdf* (dashed) and *joint pdf* (solid) for two different configurations of the gradients *pdf* for $\gamma = 0.8$

Unwrapped gradients in range and azimuth are shown in figure 2. We can notice that the amount of wrong estimations increases in areas of low coherence.

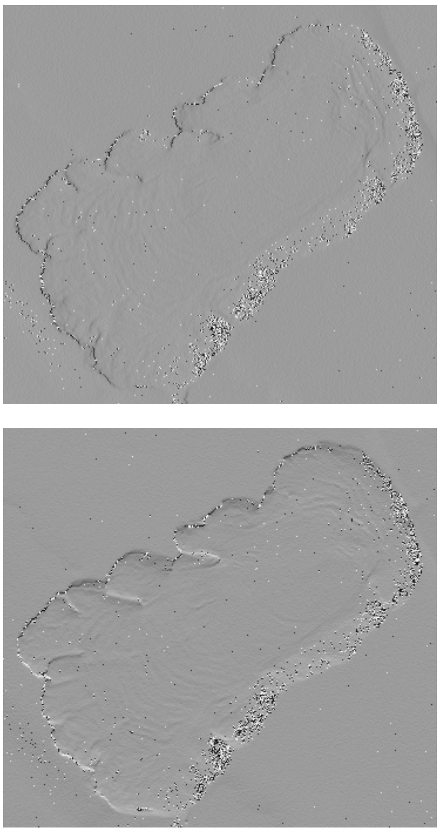

Figure 2: Unwrapped gradients in range (top) and in azimuth (bottom) using MLE.

Then these unwrapped gradients in range and azimuth are used in MCF. New residue fields are obtained (Fig. 3). More residues are obtained but most of the branch-cuts are successfully removed, so that the resulting unwrapped phase present less errors. Figure 4 illustrates both gradient fileds after optimization with MCF. Edges of the footprint have been retrieved correctly.

3).

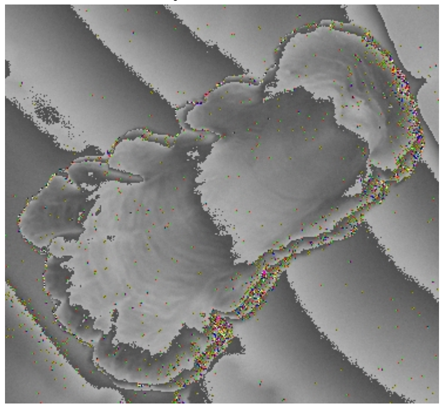

Figure 3: Wrapped phase, residues (red and green, bigger than 2π: orangem smaller than -2π: yellow) and branch-cuts (blue, multiple cuts: pink) found by adapted Minimum Cost Flow algorithm.

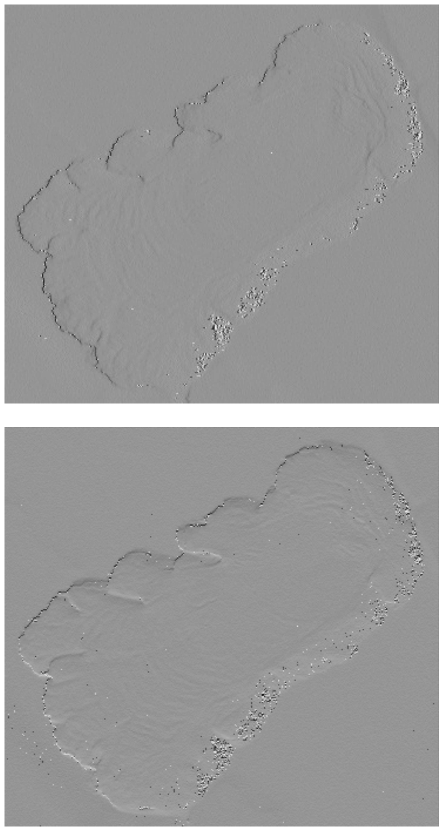

Figure 4: Gradients in range (top) and in azimuth (bottom) after optimization with MCF

Figure 5 is a comparison of phase unwrapping results using usual MCF (on the left-hand side) and the MCF supported by the multibaseline unwrapped gradients (on the right-hand side). Both unwrapped phases are compared to the simulated noiseless phase. In the usual approach, the footprint has been completely bad unwrapped while our approach solved correctly the ambiguities but introduced some additional noise in the low coherent areas.

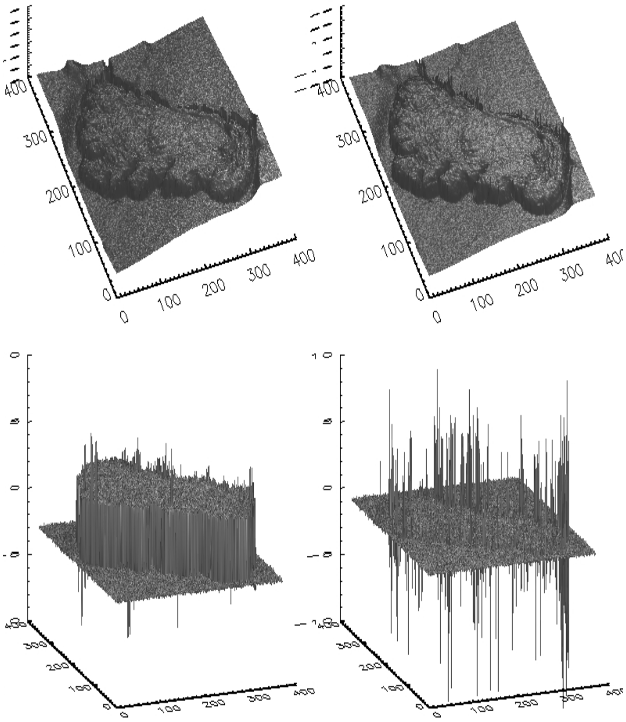

Figure 5: Comparison of the unwrapped phases. Top: on the left-hand side, using usual MCF; on the right-hand side, using the MCF supported by the multibaseline unwrapped gradients. Bottom: difference with the simulated noiseless phase.

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