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A Wave Model for Sound Generation in Circular Jets

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Summary

A wave model is used for the investigation of sound generation in circular jets. The source term of the Lighthill equation is expressed by a Fourier series in the azimuthal angle, is Fourier-transformed with respect to time, and each component is assumed to be of wave-type in jet direction. A far-field solution for the sound pressure is derived in this way for single azimuth-frequency components. It is found that the sound field depends strongly on a convection parameter and on a jet thickness parameter. The influence of axisymmetric and azimuthal source components is discussed. For a special source term with arbitrarily chosen amplitude distributions the convection factor and the jet thickness factor of sound intensity are calculated and discussed. The influence of the amplitude distribution of the source components is found to be of minor importance.

Ein Wellenmodell für die Schallerzeugung in runden Freistrahlen

Übersicht

Ein Wellenmodell wird für die Untersuchung der Schallerzeugung in runden Freistrahlen benutzt. Der Quellterm der Lighthillgleichung wird durch eine Fourierreihe in bezug auf den Umfangswinkel dargestellt und fouriertransformiert in bezug auf die Zeit. Ferner wird jede Komponente als wellenförmig in Freistrahlrichtung angenommen. Auf diese Weise wird eine Schalldruck-Fernfeldlösung für einzelne Azimutal-Frequenzkomponenten hergeleitet. Es zeigt sich, daß das Schallfeld von einem Konvektionsparameter und einem Freistrahldickenparameter stark beeinflußt wird. Der Einfluß von axialsymmetrischen und azimutalen Quellkomponenten wird diskutiert. Für eine spezielle Form des Quellterms mit freigewählter Amplitudenverteilung wird der Konvektions- und der Freistrahldickenfaktor der Schallintensität berechnet und diskutiert. Es zeigt sich, daß die Amplitudenverteilung der Quellkomponenten nur einen geringen Einfluß hat.

Contents

1. Introduction

Recent investigations of sound generation in jets were mostly based on an acoustic model introduced by LIGHTHILL[1]. He showed that with respect to the radiated sound, the source term of the inhomogeneous wave equation - which is often called Lighthill equation - can be interpreted as a distribution of acoustic quadrupoles in a medium otherwise at rest. An alternative approach based on simple sources was proposed by RIBNER [2]. The results of these methods of replacing the fluctuating flow by virtual acoustic sources helped to explain many features of the aerodynamic sound generation. Taking the convection speed of the turbulent eddies into account, the directivity of the noise pattern could be deduced by means of the correlation function of the turbulent stress (see, for example, FFOWCS-WILLIAMS [3]). By this method it is, however, difficult to take the specific structure of the turbulence into account, which may be quite different in different jets. Therefore MOLLO-CHRISTENSEN [4] stated "that the theory of aerodynamic noise could never be checked experimentally in detail".

With respect to noise emission MOLLO-CHRISTENSEN, KOLPIN & MARTUCCELLI [5] found experimentally that spectral components of low and high frequency radiate sound quite differently. Hence it may be supposed that a method only based on the correlation function can hardly reveal all the properties of the sound field radiated by the flow, since there is a strong loss of information by using overall-correlations only. With this in mind, it seems to be worthwhile to look for other models for sound generation in jets.

MOLLO-CHRISTENSEN [4] suggested "that turbulence may be more regular than we think it is. The experimental data are telling us that turbulence comes in packages containing components of all frequencies, and that different frequency components preserve their phase relationships over a few jet diameters". The wave-like character of turbulence seems to be reasonable, since wave-like disturbances are known to exist at least in the laminar-turbulent transition region. Therefore MOLLO-CHRISTENSEN & NARASIMHA [6] emphasized the intimate connexion between jet instability and noise generation. Wave-like phenomena of jet turbulence were also found experimentally by LAU, FUCHS & FISHER [7]. Furthermore, a waveguide model for turbulent shear flow has successfully been used by LANDAHL [8].

With respect to the source term of the Lighthill equation previous theories of jet noise generation have mostly used cartesian coordinates. For a circular jet, however, a cylindrical coordinate system is surely adequate. Thus one can easily take into account that for fixed time the jet turbulence has to be periodic with respect to the azimuthal angle, and so ~~has to be~~ does the source term. This periodicity in the azimuthal angle is trivial and self-evident, since the instantaneous turbulent fluctuations must have unique values for fixed time at each point of the jet flow independent ~~on~~ of any multiple of 2π.

Considering these facts, an alternative approach is proposed here to study the sound generation in circular jets. Unlike previous theories, we shall use a solution of the Lighthill equation in which the source term is given in cylindrical coordinates. We take into account that the source term, although random in time, has to be periodic with respect to the azimuth φ and can, therefore, be represented by a Fourier series in φ with the period 2π. The second point is that we deal with frequency components by means of a Fourier transform with respect to time. In specifying the jet turbulence to be wave-like, we finally assume that each Fourier component of the source term has wave character in the jet flow direction. It is hoped that this "wave model" will explain some features of the mechanism of sound generation which are difficult to obtain otherwise.

2. A solution of the Lighthill equation for wave-type jet turbulence

The basic equation for the aerodynamic sound generation due to LIGHTHILL [1] can be written as an inhomogeneous wave equation

$$(2.1) \qquad \frac{1}{a_o^2}\frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x_i^2} = q$$

The inhomogeneous part q stands for the acoustic source properties of the fluctuating flow and is given by

$$(2.2) \qquad q \equiv \frac{\partial^2}{\partial x_i\,\partial x_j}\left[\rho c_i c_j\right] + \frac{1}{a_o^2}\frac{\partial^2}{\partial t^2}\left[p - a_o^2\,\rho\right]$$

Here p is the pressure, c_i the velocity vector, ρ the density and a_o the sound speed in the medium at rest surrounding the flow. The Lighthill equation (2.1) with the source term (2.2) can be derived from the inviscid equation of motion and the continuity equation. By means of the appropriate Green's function the solution of (2.1) can be obtained in integral form, if the source term q retarded in time is assumed to be known.

Since we are dealing with circular jets, we use cylindrical coordinates (x,r,φ) where the x-direction coincides with the jet axis. Then with no solid walls bounding the flow the solution of (2.1) becomes except for an additive constant

$$(2.3)\quad p(x,r,\varphi,t) = \frac{1}{4\pi}\iiint\limits_V q(x',r',\varphi',t - r_o/a_o)\,\frac{r'}{r_o}\,dx'dr'd\varphi'$$

where

$$(2.4)\quad r_o = \sqrt{(x'-x)^2 + r^2 + r'^2 - 2rr'\cos(\varphi' - \varphi)}$$

is the distance between the source point (x',r',φ') and the measuring point (x,r,φ). In general the source term q vanishes or is negligabl small outside the flow. Hence the integration can be restricted to a cylindrical volume V outside of which the source term is zero. Then we have $0 \leq \varphi' \leq 2\pi$. For values (x,r,φ) outside the flow, the solution (2.3) gives the pressure fluctuations - in the uniform medium at rest with the sound speed a_o - produced by the fluctuating flow. The integral solution takes care of all effects of jet noise generation including those of convection and refraction and the different mechanism of self- and shear-noise. They do, however, not appear explicitly (cf. LIGHTHILL [1]).

2.1 Fourier representation of the Lighthill integral

We already stated that for fixed time t the source term q has to be periodic with respect to the azimuth φ. Hence, in specifying the solution (2.3), the source term q can be expressed by the following Fourier series

$$(2.5)\quad q(x,r,\varphi,t) = \sum_{m=o}^{\infty}\left[q_m(x,r,t)\,e^{im\varphi} + \bar{q}_m(x,r,t)\,e^{-im\varphi}\right]$$

where the superscript $^-$ denotes a conjugated complex value. The real value of m is the azimuthal wave-number. The functions q_m are complex and random in time t. Thus the rms-value of the Fourier series (2.5) is independent of φ. Introducing (2.5) into (2.3) and noting that the distance r_o in the integrand depends only on $\chi = \varphi' - \varphi$, it follows that the pressure p can also be expressed by a Fourier series

$$(2.6) \quad p(x,r,\varphi,t) = \sum_{m=o}^{\infty} \left[p_m(x,r,t)\, e^{im\varphi} + \bar{p}_m(x,r,t)\, e^{-im\varphi} \right]$$

Then the m-th component of the pressure is given by

$$(2.7) \quad p_m(x,r,t) = \frac{1}{4\pi} \iiint\limits_V q_m(x',r',t - r_o/a_o)\, \frac{r'}{r_o}\, e^{im\chi}\, dx'dr'd\chi$$

which again is random in time.

The second step is now to consider only single frequency components. By means of the Fourier transform

$$(2.8) \quad \begin{aligned} P_{m\omega}(x,r) &= \int_{-\infty}^{\infty} p_m(x,r,t)\, e^{i\omega t}\, dt \\ Q_{m\omega}(x,r) &= \int_{-\infty}^{\infty} q_m(x,r,t)\, e^{i\omega t}\, dt \end{aligned}$$

one easily obtains from (2.7)

$$(2.9) \quad P_{m\omega}(x,r) = \frac{1}{4\pi} \iint\limits_{A_Q} dx'dr' Q_{m\omega}(x',r')r' \int_o^{2\pi} d\chi\, \frac{1}{r_o}\, \exp\left[i(kr_o + m\chi)\right]$$

where

$$(2.10) \quad k = \omega/a_o$$

is the wave-number of sound. The area A_Q can be restricted to that region where $|Q_{m\omega}| > 0$. This condition defines a limiting radius R of the source region. In x-direction the Fourier component $Q_{m\omega}$ of the source term will also exist only in a finite region. Hence a length L can be defined with a non-zero integrand in $0 \leqq x' \leqq L$ only. R and L can generally depend on both m and ω and the flow parameters as well. The values of R and L can, however, only be estimated by experiments.

The last step introduces the wave-model concept by assuming that each source component $Q_{m\omega}$ is wave-like with respect to the x-direction. Hence the amplitude distribution of each wave component is determined by

$$(2.11) \quad \tilde{Q}_{m\omega}(x,r) = \int_{-\infty}^{\infty} q_m(x,r,t)\ e^{-i(\alpha x - \omega t)} dt = e^{-i\alpha x}\ Q_{m\omega}(x,r)$$

where the phase of the complex $\tilde{Q}_{m\omega}(x,r)$ has to be independent of x for wave character. $\tilde{Q}_{m\omega}$ and the wave-number α will generally depend on m and ω as well as on the flow parameters, e.g. the Mach number.

It is convenient to introduce polar coordinates for the measuring point in replacing (x,r) by

$$(2.12) \quad x = \tilde{r}\cos\Theta \quad ; \quad r = \tilde{r}\sin\Theta$$

Then (2.4) becomes

$$(2.13) \quad r_o = \sqrt{\tilde{r}^2 - 2x'\tilde{r}\cos\Theta - 2r'\tilde{r}\sin\Theta\cos\chi + x'^2 + r'^2}$$

and equ. (2.9) yields with (2.11)

$$(2.14) \quad P_{m\omega}(\tilde{r},\Theta) = \frac{1}{4\pi}\int_o^L\!\!\int_o^R dx'dr'\ \tilde{Q}_{m\omega}(x',r')\ r'e^{i\alpha x'}\int_o^{2\pi} d\chi\ \frac{1}{r_o}\ \exp i(kr_o + m\chi)$$

If we denote the Fourier transform of the total pressure p in (2.6) by P_ω , the spectral density of p is related to the Fourier components $P_{m\omega}$ by

$$(2.15) \quad |P_\omega|^2 = \sum_{m=o}^{\infty} |P_{m\omega}|^2$$

if the different functions p_m in the Fourier series (2.6) are assumed to be uncorrelated. (2.15) means that the directivity of the spectral density is a superposition of those of the various m-components. $|P_\omega|$ is a quantity which could be compared with measurements and which would approximately agree with the rms-value of a narrow-band-width pressure signal.

2.2 The sound field far from the source region

In the following we shall restrict our attention to a pressure component $P_{m\omega}$ far from the source region. Then we have $\tilde{r} \gg R$, $\tilde{r} \gg L$ and the distance r_o defined by (2.13) can be expanded as follows

$$(2.16) \quad \tilde{r}_o = \tilde{r}\left[1 - (x'/\tilde{r})\cos\Theta - (r'/\tilde{r})\sin\Theta\cos\chi + O((L/\tilde{r})^2, (R/\tilde{r})^2)\right]$$

Introducing (2.16) into the last integral of (2.14) and retaining the lower order terms only, we readily find

$$(2.17) \quad \int_0^{2\pi} d\chi \frac{1}{r_o} \exp\left[i(kr_o + m\chi)\right] = 2\pi i^{-m}\, \tilde{r}^{-1} \exp\left[ik(\tilde{r} - x'\cos\Theta)\right] \; J_m(kr'\sin\Theta)$$

where J_m is the Bessel function of order m. With (2.17) and from (2.14) the sound pressure component far from the source region becomes

$$(2.18) \quad P_{m\omega}(\tilde{r},\Theta) = i^{-m}\, \tilde{r}^{-1} e^{ik\tilde{r}}\, I_{m\omega}(\Theta)$$

Here

$$(2.19) \quad I_{m\omega} = \frac{1}{2}\int_0^L\int_0^R dx'\,dr'\,\tilde{Q}_{m\omega}(x',r')\,r'\,J_m(kr'\sin\Theta)\; e^{i\alpha(1-M_c\cos\Theta)x'}$$

and the convection Mach number

$$(2.20) \quad M_c = k/\alpha = (\omega/\alpha)/a_o = c_{ph}/a_o$$

is the ratio of phase velocity c_{ph} and sound speed a_o of the Fourier component. The spectral density of this component is

$$(2.21) \quad |P_{m\omega}|^2 = |I_{m\omega}|^2 / \tilde{r}^2$$

The double integral $I_{m\omega}$ depends on the angle Θ. This leads to a directivity of the radiated sound field which will be different for each component.

It should be noted here that the source term q, which due to (2.2) is a nonlinear function of the velocity components in the jet, can also be expressed by the pressure fluctuations p' in the jet. Since the flow quantities have to satisfy the nonlinear compressible flow equations, we can write the source term

$$(2.22)\quad q \equiv \frac{1}{a_o^2}\,\frac{\partial^2 p'}{\partial t^2} - \left[\frac{\partial^2 p'}{\partial r^2} + \frac{1}{r}\frac{\partial p'}{\partial r} + \frac{1}{r^2}\frac{\partial^2 p'}{\partial \varphi^2} + \frac{\partial^2 p'}{\partial x^2}\right]$$

We see that the source term q is a linear operator of the pressure fluctuations p'. Hence it follows that the various sound pressure components $P_{m\omega}$ far from the source region are merely determined by the corresponding pressure components in the jet. Therefore, by measuring the relative magnitude of the various components of the pressure fluctuations p' in the jet, one could obtain an estimate of the relative magnitude of $\tilde{Q}_{m\omega}$ which appears in the integrand of (2.19).

2.3 Discussion of the sound pressure component $P_{m\omega}$

Some essential conclusions may be drawn by means of the solution (2.18), even if the amplitude distribution $\tilde{Q}_{m\omega}$ is not known in detail. First of all, we can say that $P_{m\omega}$ will depend on $\tilde{Q}_{m\omega}(x',r')$, i.e. on the flow properties. ~~This term essentially determines the Mach number dependence of the sound radiation~~. By using a characteristic velocity U_1, density ρ_o and length L_o, it follows from (2.22) with $p' \sim \rho_o U_1^2$ that $q \sim \rho_o U_1^2\, L_o^{-2}\, [O(M^2) + O(M^o)]$. ~~The term of $O(M^o)$ do not vanish necessarily, as it is mostly assumed~~. MICHALKE [9] found ~~at least~~ that for a disturbed free shear flow, which is a solution of the linearized compressible equations, the term of $O(M^o)$ exists inside the shear layer, i.e. for gradients of the mean velocity and density normal to the flow direction. This source term was explained to be a consequence of the oscillations of the free shear layer, whereas the source term of $O(M^2)$ is due to the density fluctuations. It may be noted, however, that the Mach number dependence of $\tilde{Q}_{m\omega}$ is not basically a phenomenon of the sound generation itself.

It is evident from (2.19) that the directivity of a single sound pressure component is essentially influenced by two acoustic parameters which correspond to two different mechanisms. The first mechanism is due to the radial interference of sound sources in the jet. It is expressed by the term $kR\sin\Theta$ which occurs in the argument of the Bessel function of (2.19). This term is a parameter for the acoustic jet thickness, since kR is essentially the ratio of the source region to the sound wavelength. This effect of non-zero acoustic jet thickness is mostly ignored in previous theories. The second mechanism is due to the axial interference and convection of sound sources in the jet. It is expressed by the term $\alpha L(1 - M_c\cos\Theta)$ in the exponential term of (2.19).

If we introduce the maximum jet velocity U_1 and the radius R of the source region as characteristic quantities, the terms kR and M_c can be expressed by the Mach number $M = U_1/a_o$. By using the Strouhal number $S = 2Rf/U_1$ with the frequency $f = \omega/2\pi$, we find from (2.10)

$$kR = \pi SM \tag{2.23}$$

and from (2.20)

$$M_c = (c_{ph}/U_1)\, M \tag{2.24}$$

Here the normalized phase velocity c_{ph}/U_1 of the Fourier component like the normalized wave-number $\alpha R(L/R)$ may depend on m and S as well as on the flow parameters, e.g. the Mach number M.

Let us first discuss the influence of the jet thickness parameter $kR\sin\Theta$ which accounts for the radial interference of sources within the jet. The influence of the jet thickness parameter is symmetric around $\Theta = \pi/2$. The behaviour of the Bessel function infers that for a fixed amplitude distribution $\tilde{Q}_{m\omega}(x',r')$ and $kR\sin\Theta \ll 1$, the value of the double integral $I_{m\omega}$ is much smaller for $m \geqq 1$ than for $m = 0$. This means that for fixed $\tilde{Q}_{m\omega}$ an axisymmetric component of the source term radiates much more sound than a non-axisymmetric component with $m \geqq 1$. Hence in this case the directivity of the spectral density due to (2.15) is approximately given by $|P_\omega|^2 \approx |P_{o\omega}|^2$. On the other hand, for fixed $\tilde{Q}_{m\omega}$ and $m = 0$ it follows that with increasing $kR\sin\Theta$ the integral $I_{m\omega}$ becomes smaller, since the Bessel function J_o decreases and can change the sign. This will occur, when the radius R and the sound wavelength will be comparable, i.e. for $kR = \pi SM > 2$ approximately. Then the sound intensity of the axisymmetric component which is proportional to $|P_{o\omega}|^2$ has a maximum in jet direction ($\Theta = 0^o$) and is very small normal to the jet ($\Theta = \pi/2$). Contrary to this, azimuthal source components $\tilde{Q}_{m\omega}$ with $m \geqq 1$ mainly radiate normal to the jet direction, with no radiation in the direction of the jet axis ($\Theta = 0$, $\Theta = \pi$). Hence it follows from (2.15) that the spectral density is $|P_\omega|^2 = |P_{o\omega}|^2$ for $\Theta = 0$ or $\Theta = \pi$, whereas for $0 < \Theta < \pi$ at least $|P_{o\omega}|^2$ and $|P_{1\omega}|^2$ can be of the same order of magnitude.

The convection parameter $\alpha L(1 - M_c\cos\Theta)$ in the exponential term of

(2.19) accounts for the axial interference of the sources in the jet. It leads to an asymmetry of sound radiation with respect to Θ. For a convection Mach number with $0 < M_c < 1$ the sound radiation of a component will be enhanced in the direction of the jet ($\Theta = 0$), but reduced in upstream direction ($\Theta = \pi$). Furthermore, for $M_c > 1$, the sound pressure component (2.18) can reach a maximum value for $\cos\Theta = M_c^{-1}$, since the reducing influence of the exponential term then drops out. In this case the sources at different x'-positions will not interfere, and a peak in the directivity can appear at this angle Θ. This effect is called "Mach wave radiation" in the literature.

The present theoretical results (2.18),(2.21) and (2.15) are difficult to compare with those previously obtained by correlation function technique (c.f. FFOWCS-WILLIAMS [3]). The overall-intensity derived in this way can be obtained from the present results only by integrating the spectral density (2.15) over all frequencies ω. But for this purpose, additional assumptions had to be made which would lead to a greater arbitrariness. On the other hand, from narrow-band-width measurements, some directivities of jet noise components were reported by MOLLO-CHRISTENSEN, KOLPIN & MARTUCCELLI [5] and by KRISHNAPPA & CSANADY [10]. It is felt that a comparison with these results could be more reasonable. Before this will be done in section 3.4, and in order to discuss the results in more detail, the general solution (2.18) will be specialized in the following section 3.

It should, however, be noted that the solution (2.18) of this section 2.3 obtained for the wave model can be generalized, if the assumption (2.11) of the wave character of the source components would be suppressed. The integral solution (2.9) would then lead to a more general integral $I_{m\omega}$ instead of (2.19) valid far from the source region

$$(2.25) \qquad I_{m\omega} = \frac{1}{2}\int_0^L\int_0^R dx'dr' \; Q_{m\omega}(x',r')r'J_m(kr'\sin\Theta) \; e^{-ikx'\cos\Theta}$$

which depends on the acoustic parameters $kR\sin\Theta$ and $kL\cos\Theta$. The first one is again the jet thickness parameter which accounts for the radial interferences of sources in the jet. Its influence on the integral $I_{m\omega}$, as discussed in section 2.3, remains unchanged even in the generalized case (2.25). The second parameter $kL\cos\Theta$ of the generalized $I_{m\omega}$ accounts for the axial interference of sources in the jet. It is obvious from

(2.25) and (2.19) that the special assumption (2.11) of the wave character only introduces the effect of convection and additional interference of the wavy source distribution. If in contrast to (2.11) the phase of the complex $Q_{m\omega}$ would depend nonlinearily on x, a more complicated type of non-uniform convection would be found.

3. The directivity due to a special wave-like source term

The spectral density $|P_{m\omega}|^2$ of an azimuth-frequency component is determined by the double integral (2.19). It depends essentially, as noted in section 2.3, on the flow properties via the source amplitude distribution $\tilde{Q}_{m\omega}(x',r')$ and on the acoustic properties via the jet thickness- and the convection parameter. In order to discuss these influences on the directivity of sound intensity in more detail, we shall now specialize our assumptions. For this reason, we assume for simplicity that the amplitude distribution $\tilde{Q}_{m\omega}$ is independent of m and ω. Then the directivities of different (m,ω)-components can well be compared. The additional assumption, besides of the wave-like character assumed in (2.11) , is made that the amplitude distribution $\tilde{Q}_{m\omega}$ of each source term component is similar in x-direction. Both conditions will likely not be satisfied in a turbulent jet, but they may be used in order to examine the main properties of the solution (2.18). Then we have

$$(3.1)\quad \tilde{Q}_{m\omega}(x,r) = \tilde{Q}(r)\ g(x)$$

where $\tilde{Q}(r)$ is generally complex, while g(x) is real. Then the integrations over x' and r' in the double integral $I_{m\omega}$ (equ. (2.19)) can be treated separately.

We introduce (3.1) into the integral (2.19) valid far from the source region and require additionally $k\tilde{r} \gg 1$. According to (2.21) the sound intensity of a Fourier component can then be written

$$(3.2)\quad \frac{|P_{m\omega}|^2}{\rho_o a_o} = \frac{M}{\rho_o U_1}\,\frac{|A|^2}{\tilde{r}^2}\,|\hat{I}_r|^2\,|\hat{I}_x|^2$$

Here

$$(3.3)\quad A = \frac{i^{-m}}{2}\int_0^R\int_0^L \tilde{Q}(r')g(x')r'dr'dx'$$

is the maximum source strength and

$$(3.4)\quad \hat{I}_r = \int_0^1 d\hat{r}\ \hat{Q}(\hat{r})\hat{r}\ J_m(kR\sin\Theta\,\hat{r}) \Big/ \int_0^1 d\hat{r}\ \hat{Q}(\hat{r})\hat{r}$$

and

$$(3.5)\quad \hat{I}_x = \int_0^1 d\hat{x}\ \hat{g}(\hat{x})\ e^{i\alpha L(1-M_c\cos\Theta)\hat{x}} \Big/ \int_0^1 d\hat{x}\ \hat{g}(\hat{x})$$

The superscript ^ denotes quantities normalized by the maximum values:

$$(3.6)\quad \hat{Q} = \tilde{Q}/\tilde{Q}_{max}\ ;\ \hat{g} = g/g_{max}\ ;\ \hat{r} = r'/R\ ;\ \hat{x} = x'/L$$

Here $|\hat{I}_r|^2 \leqq 1$ is a jet thickness factor which accounts for the radial interference of the sources in the jet for finite radius R and which is a function of the parameters $kR\sin\Theta$ and m. $|\hat{I}_x|^2 \leqq 1$ corresponds to a convection factor and depends on the parameter $\alpha L(1 - M_c\cos\Theta)$. Both factors together determine the directivity of sound radiation.

In the following the influence of the radial and axial amplitude distribution of the source term on the directivity will be estimated and discussed separately. For this purpose the jet thickness factor and the convection factor are calculated for arbitrarily chosen distributions $\hat{Q}(\hat{r})$ and $\hat{g}(\hat{x})$.

3.1 Evaluation of the jet thickness factor for various $\hat{Q}(\hat{r})$

The influence of the jet thickness factor $|\hat{I}_r|^2$ will be examined by assuming two different radial amplitude distributions $\hat{Q}(\hat{r})$ both functions being real for simplicity. The first one chosen is

$$(3.7)\quad \hat{Q}(\hat{r}) = \frac{27}{4}\hat{r}^2(1 - \hat{r})$$

which has a maximum at $\hat{r} = 2/3$ in the outer part of the source region and will be denoted by (a). The second one is

$$(3.8)\quad \hat{Q}(\hat{r}) = 1 - \hat{r}^2$$

which has a maximum at the axis at $\hat{r} = 0$ and will be denoted by (b). Both radial distributions vanish at $\hat{r} = 1$, and it is assumed that $|\hat{Q}| \equiv 0$ for $\hat{r} \geqq 1$. With (3.7) and (3.8) the jet thickness factor $|\hat{I}_r|^2$ was evaluated numerically from (3.4). The results are shown in Figure 1 as function of

$kR\sin\Theta$ for an axisymmetric source term component with $m = 0$ and for the first azimuthal component with $m = 1$. We see that the curves (a) and (b) for the different $\hat{Q}(\hat{r})$-distributions chosen quite differently show the same tendency. This implies that the special radial amplitude distribution of the source term components is of minor importance. For $m = 0$ the jet thickness factor decreases from unity with increasing $kR\sin\Theta$. It is only for real $\hat{Q}$ that there are zeroes of $|I_r|^2$ for large values of $kR\sin\Theta$ which depend on the special $\hat{Q}$. Then, for sufficiently large values kR, there are directions Θ where no sound will be radiated. As mentioned in section 2.3, for $kR \rightarrow 0$ we find $|\hat{I}_r|^2 \rightarrow 1$ for $m = 0$ and $|\hat{I}_r|^2 \rightarrow 0$ for $m = 1$.

For $m = 0$ the jet thickness factor yields a reduction of sound radiation normal to the jet. Contrary to this, a non-axisymmetric source term component with $m \geqq 1$ does not radiate in the direction of the jet axis ($\Theta = 0$ or $\Theta = \pi$), and for $kR < 2$ its contribution may be negligible compared with the axisymmetric component as can be seen in Figure 1 for $m = 1$. On the other hand, in the region $2 < kR < 5$, i.e. for higher Strouhal numbers, the azimuthal components with $m \geqq 1$ can dominate.

The Θ-variation of the jet thickness factor $|\hat{I}_r|^2$ is shown in Figure 2 for different values kR and m. We see that the curves are symmetric around $\Theta = 90^\circ$ as mentioned in section 2.3. The influence of the special distribution $\hat{Q}(\hat{r})$ seems to be not very significant. For $kR = 1$ the variation of the jet thickness factor with Θ is small. For $kR = 3$, however, the influence of $|\hat{I}_r|^2$ is very important. Its value for the axisymmetric and the first azimuthal component can then be of the same order of magnitude for $40^\circ \leqq \Theta \leqq 140^\circ$ as stated in section 2.3.

3.2 Evaluation of the convection factor for various $\hat{g}(\hat{x})$

The convection factor $|\hat{I}_x|^2$ was evaluated for four different axial distribution functions $\hat{g}(\hat{x})$. The simplest one is

(3.9) $\hat{g}(\hat{x}) = 1 \quad \text{for } 0 < \hat{x} < 1$

denoted by (I). The second one denoted by (II) is linear i.e.

(3.10) $\hat{g}(\hat{x}) = \hat{x} \quad \text{for } 0 \leq \hat{x} < 1.$

The third distribution denoted by (III) is parabolic

(3.11) $\hat{g}(\hat{x}) = 4\,\hat{x}(1 - \hat{x}),$

while (IV) is given by

$$(3.12)\quad \hat{g}(\hat{x}) = \frac{27}{4}\,\hat{x}^2(1 - \hat{x})$$

It is again assumed that $\hat{g}(\hat{x}) \equiv 0$ for $\hat{x} < 0$ and $\hat{x} > 1$. The functions (I) and (III) are symmetric with respect to the maximum value, while (II) and (IV) are asymmetric. The integrals in (3.5) can be obtained analytically for the chosen $\hat{g}(\hat{x})$. For instance, by using (I) and the abbreviation $C = \alpha L(1 - M_c \cos\Theta)$ one simply obtains

$$(3.13)\quad |\hat{I}_x|^2 = 2(1 - \cos C)/C^2$$

and for (II)

$$(3.14)\quad |\hat{I}_x|^2 = 4\left[(1 - \cos C)^2 + (\sin C - C)^2\right]/C^4$$

The asymptotic behaviour of the convection factor $|\hat{I}_x|^2$ for $|C| \to \infty$ is with (I) and (II) of $O(C^{-2})$, with (III) and (IV) of $O(C^{-4})$. The curves of the convection factor as function of $C = \alpha L(1 - M_c \cos\Theta)$ are shown in Figure 3 for the various $\hat{g}(\hat{x})$. We see that the curves are similar for all distributions $\hat{g}(\hat{x})$ except that their asymptotic behaviour is different and except that a symmetric $\hat{g}$ like (I) and (III) leads to zeroes of $|I_x|^2$ for certain values of $\alpha L(1 - M_c \cos\Theta)$. Hence it follows that the axial amplitude distribution too seems to be of minor importance.

For fixed αL and M_c the convection factor $|I_x|^2$ varies as function of Θ from the value at $\alpha L(1 - M_c)$ in jet flow direction ($\Theta = 0$) to the value at αL normal to the jet ($\Theta = \pi/2$) and to the value at $\alpha L(1 + M_c)$ in upstream direction ($\Theta = \pi$). Therefore, if $M_c \ll 1$, the convection factor is nearly independent of Θ and determined only by the value of αL. For values $0 < M_c < 1$ the convection factor yields a sound radiation with a maximum value for $\Theta = 0$ and smaller values for $\Theta > 0$. This is the typical convection effect. On the other hand, if $M_c > 1$, the convection factor becomes unity at $\Theta = \arccos M_c^{-1}$ as mentioned above and is very small at $\Theta = \pi$. This leads to a maximum of the convection factor for $\Theta > 0$ which is due to Mach wave radiation.

The typical Θ-variation of the convection factor for $0 < M_c < 1$ is shown in Figure 4 and for $M_c > 1$ in Figure 5 for the various distributions $\hat{g}(\hat{x})$ mentioned above. It is assumed that $\alpha L = 6$ in both Figures, and $M_c = 0.5$ in Figure 4, while $M_c = 1.5$ in Figure 5. In the latter case the peak due to the Mach wave radiation is roughly at $\Theta = 48^o$ for this convection Mach number M_c. It is obvious that, regarding the general tendency, the influence of the distributions $\hat{g}(\hat{x})$, chosen quite differently, turns out to be not very significant.

3.3 Discussion of the directivity

The directivity of sound intensity of a single azimuth-frequency component is after (3.2) given by the product of $|\hat{I}_r|^2$ and $|\hat{I}_x|^2$. From section 3.1 and 3.2 it follows that for small kR the directivity is determined mainly by the convection factor of the axisymmetric component. For larger values of kR, however, the sound intensity of axisymmetric and first azimuthal components can be of the same order of magnitude, although their directivities differ considerably.

For $M_c > 1$ and $m = 0$ the peak of the convection factor for $\Theta > 0$ as shown in Figure 5 can possibly be compensated by the valley of the jet thickness factor for $\Theta > 0$ as shown in Figure 2 for $kR = 3$. Then the total directivity can again reach a maximum only for $\Theta = 0$. Contrary to this, for $M_c > 1$ and $m = 1$, the peak of the convection factor for $\Theta > 0$ will be pronouced by the peak of the jet thickness factor for $\Theta > 0$. Then the total directivity will be zero at $\Theta = 0$ and $\Theta = \pi$ and have a strong peak for $0 < \Theta < \pi/2$.

For a subsonic flow with $M_c < M < 1$ the convection factor is of the type shown in Figure 4 with a maximum for $\Theta = 0$ only. For large kR and $m = 0$ the total directivity will again show a maximum only at $\Theta = 0$, but very small sound radiation normal to the jet because of the interference of the sources across the jet. Contrary to this, for $m = 1$ the total directivity will be zero at $\Theta = 0$ and $\Theta = \pi$ and have a peak for $\Theta > 0$ even in the subsonic case. This peak, however, is not a consequence of Mach wave radiation, which can occur only for $M_c > 1$, or of Refraction, but only a consequence of the coherence of the first azimuthal source components distributed across the jet.

In order to illustrate these results, some directivities of single rms-sound pressure components $\sim |\hat{I}_r| \cdot |\hat{I}_x|$ have been calculated for a subsonic flow. The convection Mach number is assumed to be $M_c = 0.5$. For both $m = 0$ and $m = 1$ the amplitude distributions are chosen as in (3.7) and (3.14). For $\alpha L = 3$ and $\alpha L = 6$ the results are shown in Figure 6 for $kR = 1$ and in Figure 7 for $kR = 4$. We see quite clearly the different types of the directivity produced by axisymmetric and first azimuthal components. For $kR = 1$ in Figure 6 the sound pressure level of the axisymmetric component ($m = 0$) exceeds that of the first azimuthal component ($m = 1$), whereas for $kR = 4$ (Figure 7) and for $40^\circ \leqq \Theta \leqq 140^\circ$ we find the opposite to be true.

3.4 Comparison with previous results

In previous theoretical investigations the directivity pattern has been discussed mainly with respect to the convection effect, while the jet thickness effect has been ignored. The directivities obtained by correlation function technique were overall-values. Therefore a quantitative comparison with the present results, obtained for azimuth-frequency components only, is difficult. Furthermore, the directivities derived among others by LIGHTHILL [1], FFOWCS-WILLIAMS [3], RIBNER [2] and JONES [11] differ considerably. Nevertheless the main tendency of the results is in agreement with that of our convection factor as shown, for example, in Figures 4 and 5. The remarkable reduction in intensity at small angles of Θ, which was sometimes found in experiments by MOLLO-CHRISTENSEN, KOLPIN & MARTUCCELLI [5], by KRISHNAPPA & CSANADY [10] and others, could not be described by the convection factors previously derived. RIBNER [12], JONES [11] and KRISHNAPPA & CSANADY [10] explained this phenomenon by means of the "refraction effect".

Some narrow-band-width measurements of the directivity were reported by MOLLO-CHRISTENSEN, KOLPIN & MARTUCCELLI [5]. Their far-field measurements were concerned with subsonic jets. In Figure 8 their rms-sound pressure directivity is shown for frequency components with Strouhal numbers $S < 0.5$. As mentioned above, these curves should approximately agree with the directivity of $|P_\omega|$ defined by (2.15). It is found that there apparently is a maximum for $\Theta = 0$. All curves for various Mach numbers $M \leqq 0.9$ show the same tendency as our directivity (Figure 6) for $m = 0$ and $kR = 1$ calculated for the special source type (3.1). In fact, assuming that the jet diameter D is roughly twice the radius R of the source region, it follows that according to $S < 0.5$ and $M < 0.9$ we have $kR < 1.4$. Hence we can conclude from our theoretical results that the directivity is determined only by the convection factor, and sound is mainly radiated from axisymmetric components.

In Figure 9 the directivity is shown for frequency components with Strouhal numbers $S > 2$. It is found that the directivity has a remarkable peak at roughly $\Theta = 40^{o}$ and apparently tends to zero for $\Theta = 0^{o}$. This peak of the directivity cannot be a consequence of Mach wave radiation, since the flow is subsonic and hence the convection Mach number M_c cannot exceed 1. In the view of our results it can be supposed that this type of directivity can only be a consequence of a dominating sound emission from non-axisymmetric source components with $m \geqq 1$ as shown in Figure 7 for $m = 1$ and $kR = 4$. In fact, for $S > 2$ and $M > 0.6$ we find $kR > 3.6$ i.e. the condition for a dominating sound emission of the first and higher azimuthal source components is satisfied. The shape of the directivity for $\Theta \rightarrow 0$ suggests that due to (2.15) the contribution $|P_{o\omega}|^2$ of the axisymmetric source component $Q_{o\omega}$ is small compared with that of azimuthal ones at these Strouhal numbers $S > 2$.

Similar results were found by KRISHNAPPA & CSANADY [10], although here a peak at $\Theta \rightarrow 0$ appears even at smaller values of kR. The authors explain this phenomenon as being "produced by the sum of an x-x and x-r quadrupole, if the intensity of the latter is slightly greater than twice the intensity of the former". They concluded that for higher frequencies the x-r quadrupole dominates and, additionally, the refraction effect becomes important, if the product of Strouhal and Mach number exceeds unity. Both effects would shift the position of the intensity peak to larger Θ. The importance of the parameter SM, which after (2.23) is proportional to kR, is in agreement with our results. But an alternative explanation in the light of our results may be that in the jet, investigated in [10] the azimuthal source components with $m \geqq 1$ dominated the axisymmetric one even at smaller Strouhal numbers. After (2.15) this would lead to a strong dominance of the directivities of azimuthal sound components even at small kR.

It follows that the different shape of the directivities, measured for the same frequency bands in [5] and [10], may be a consequence of a different relative strength of the axisymmetric and azimuthal source components in both jets. Such a different structure of jet turbulence will surely depend on whether a laminar-turbulent transition takes place with a dominating strength of axisymmetric components (ring vortices), or whether the jet is already fully turbulent at the nozzle exit with a dominating strength of three-dimensional components. The laminar instability process in a jet with large jet core will mainly produce axisymmetric pressure fluctuations with Strouhal numbers in the order of $S < 0.02 \sqrt{Re}$ approximately. Here the Strouhal number S and the Reynolds number Re are based on the jet diameter and Re = $10^4 ...... 10^5$.

The different structure of jet turbulence can certainly be checked by measuring the azimuthal pressure correlation in the near-field of the jet like that made by MOLLO-CHRISTENSEN [4]. This method should be applied to narrow-band-width pressure fluctuations. It seems to be a promising way to study the structure of jet turbulence in this way with respect to axisymmetric and azimuthal components. Their influence upon the far-field noise can then be predicted by the present analysis. Similarily, axial correlation measurements could be used for checking the wave-concept and for determining the wave-number α and the length L of each wave component.

Finally a remark may be given on the Mach number dependence of sound radiation. By means of this wave model for aerodynamic sound generation the Mach number dependence of sound radiation cannot be derived, since it is implicitly given in the source term q. The derivation of the Mach number dependence of q, however, is rather a question of Gasdynamics and Turbulence Theory. If we assume that the source term q consists only of a term of $O(M^2)$ as LIGHTHILL [1] did, then (3.2) would lead to the well-known result that the sound intensity is of $O(M^5)$. If the source term, however, would consist of terms of $O(M^2)$ and $O(M^0)$ as mentioned in section 2.3, then the sound intensity could consist of terms of $O(M^5)$, $O(M^3)$ and $O(M^1)$. Terms of this type have, in fact, been found by FFOWCS-WILLIAMS & GORDON [13] in experimental investigations of circular jets and by WEBSTER [14] for plane jets.

In summarizing we can say that the wave model and the method used here for discussing the sound generation mechanism in circular jets, led to a far-field solution of the Lighthill equation which is relatively simple and can easily be interpreted. Some results obtained by the wave model apparently demonstrate new aspects of aerodynamic sound generation. The main result is the remarkable different sound radiation by axisymmetric and azimuthal source components. It suggests that a considerable reduction of jet noise could be achieved by suppressing axisymmetric pressure components in the jet. Separate discussion of the influence of the jet thickness parameter on one hand and of the convection parameter on the other hand were found to be helpful. The results for different, arbitrarily chosen amplitude distributions of the source term components imply that their influence on the directivity is only of minor importance. As a result, some experimental directivities can be explained by the present theory.

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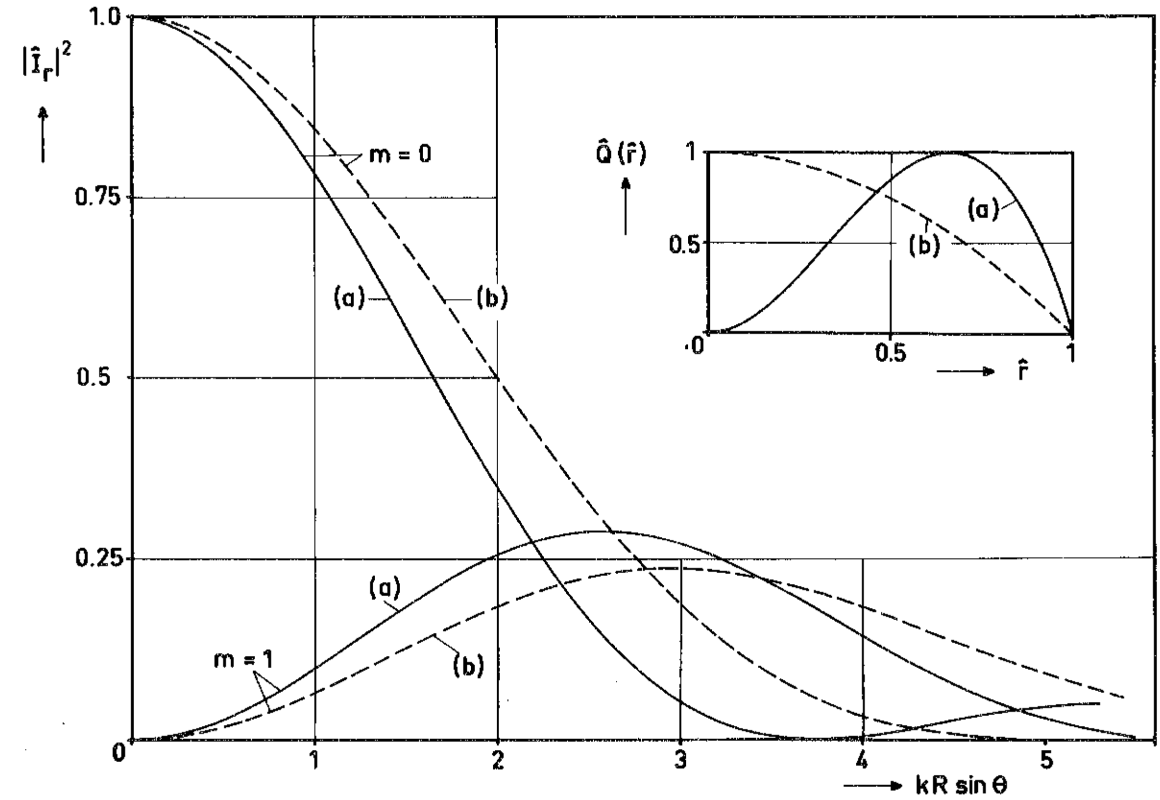

Figure 1 The jet thickness factor of axisymmetric and first azimuthal components as function of the jet thickness parameter for various radial amplitude distributions of the source term.

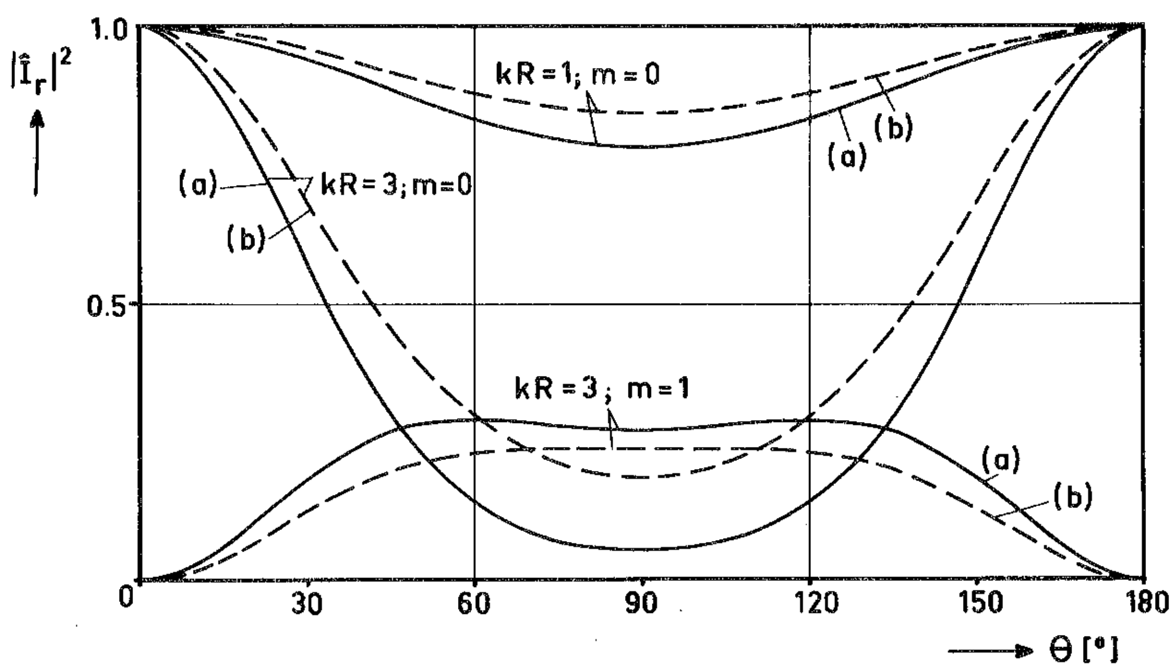

Figure 2 The jet thickness factor of axisymmetric and first azimuthal components as function of the jet angle for various values of kR and various radial amplitude distributions of the source term.

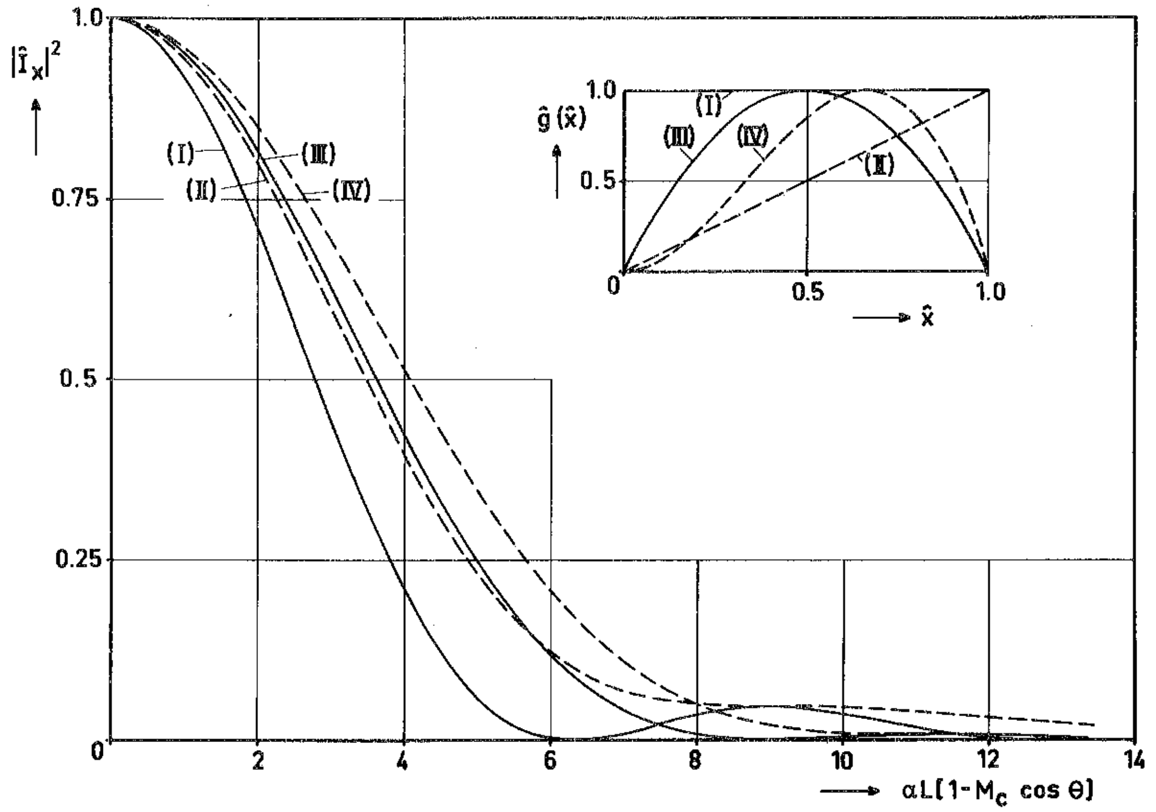

Figure 3 The convection factor as function of the convection parameter for various axial amplitude distributions of the source term.

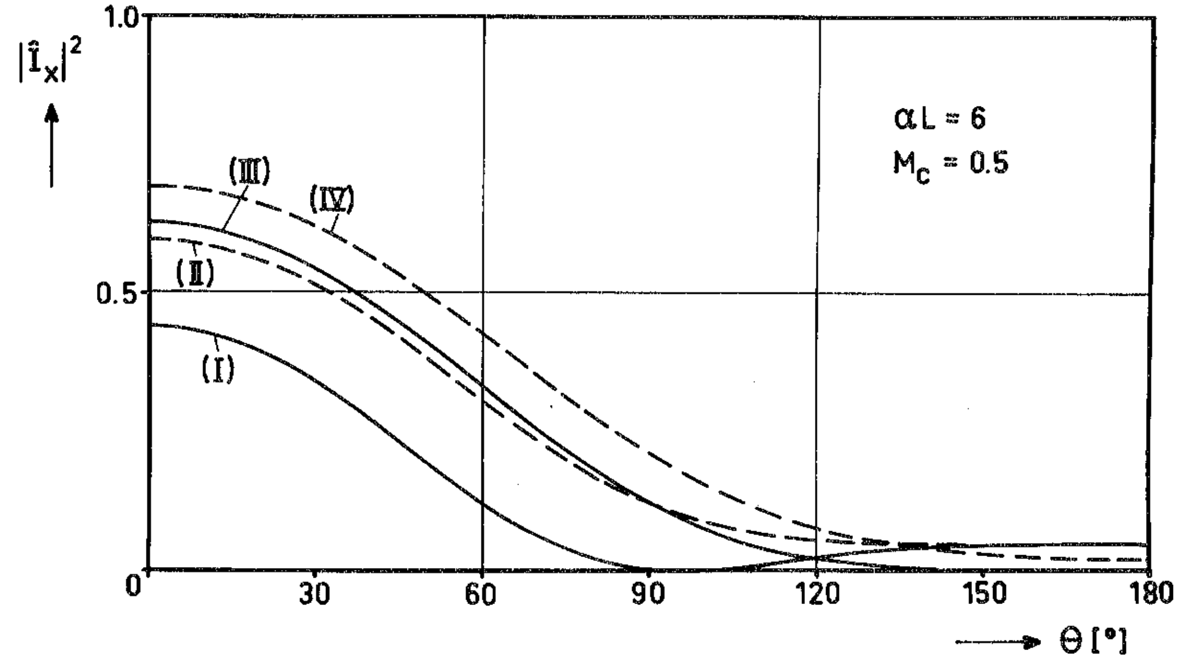

Figure 4 The convection factor of a subsonic flow as function of the jet angle for various axial amplitude distributions of the source term.

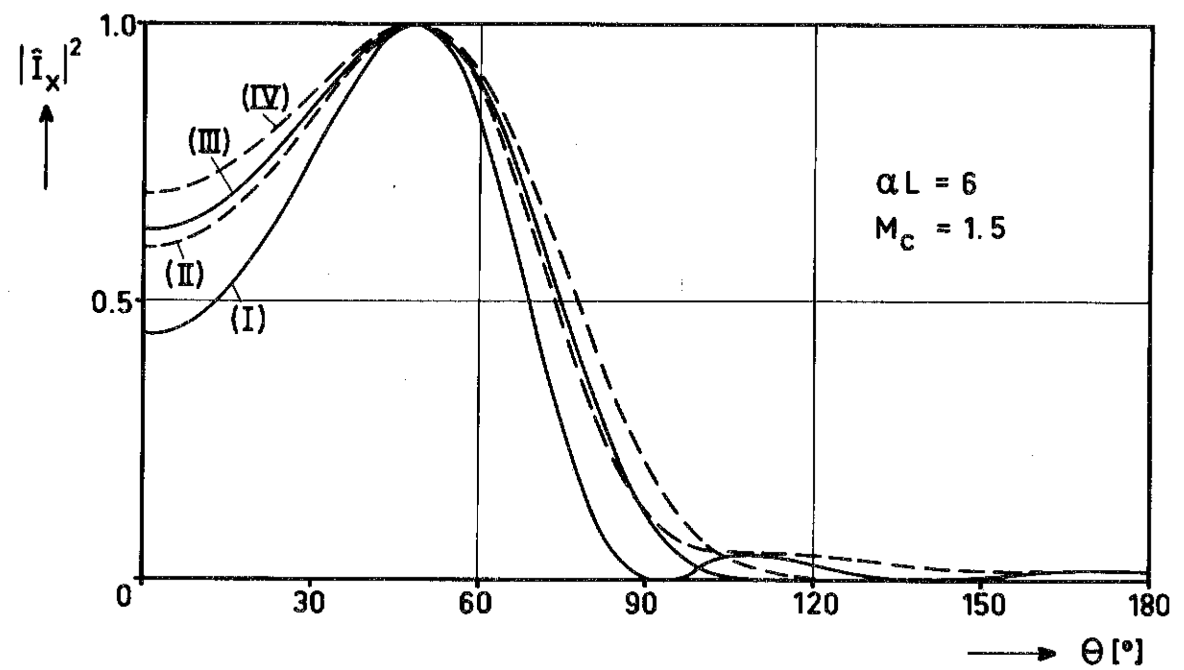

Figure 5 The convection factor of a supersonic flow as function of the jet angle for various axial amplitude distributions of the source term.

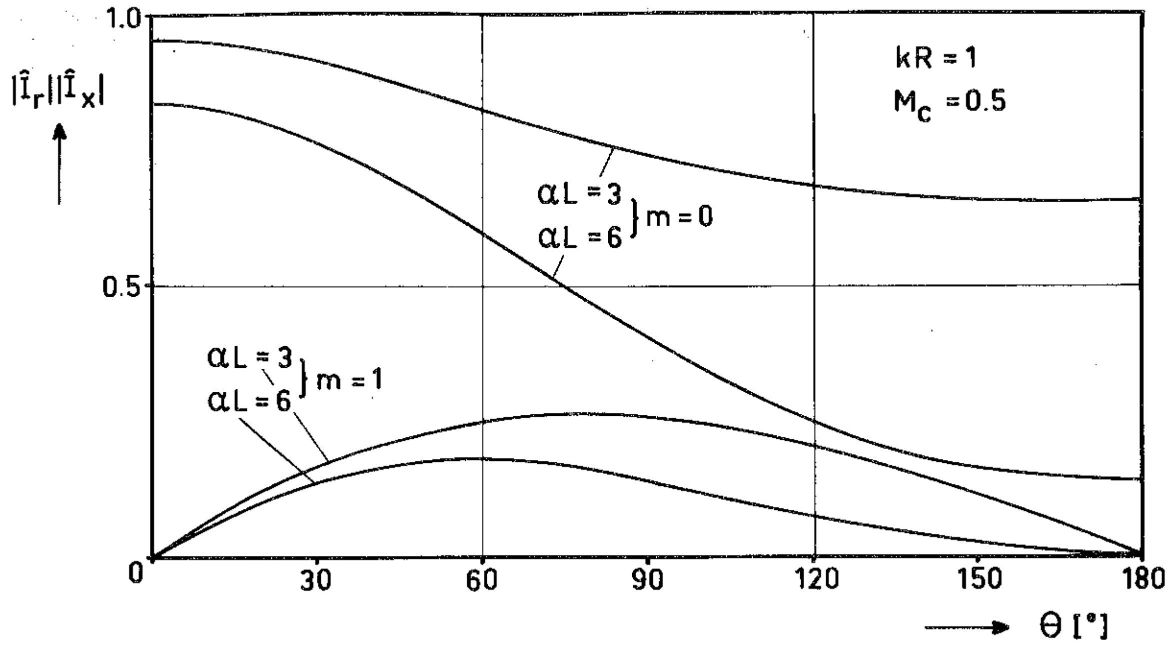

Figure 6 The directivity of rms-sound pressure components for kR = 1 and M_c = 0.5. The amplitude distribution of the axisymmetric and first azimuthal source components are due to (3.7), (3.14)

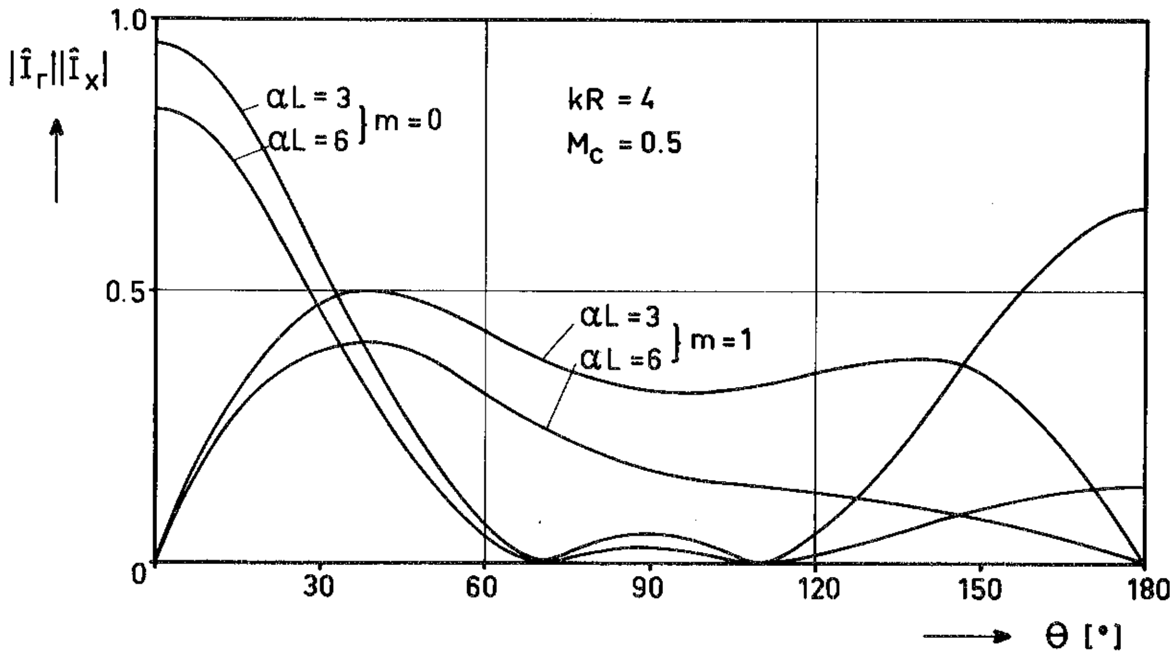

Figure 7 The directivity of rms-sound pressure components for kR = 4 and M_c = 0.5. The amplitude distribution of the axisymmetric and first azimuthal source components are due to (3.7), (3.14)

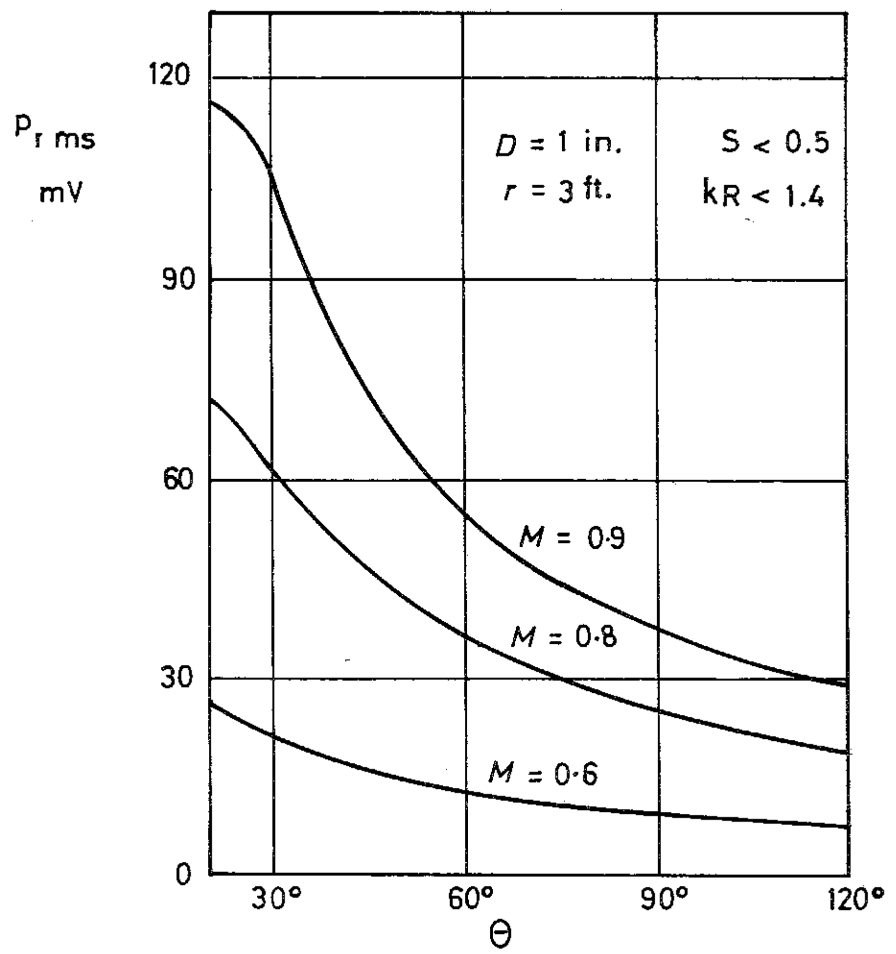

Figure 8 RMS-sound pressure directivities for Strouhal numbers S < 0.5 and various Mach numbers M measured by MOLLO-CHRISTENSEN, KOLPIN & MARTUCCELLI [5]

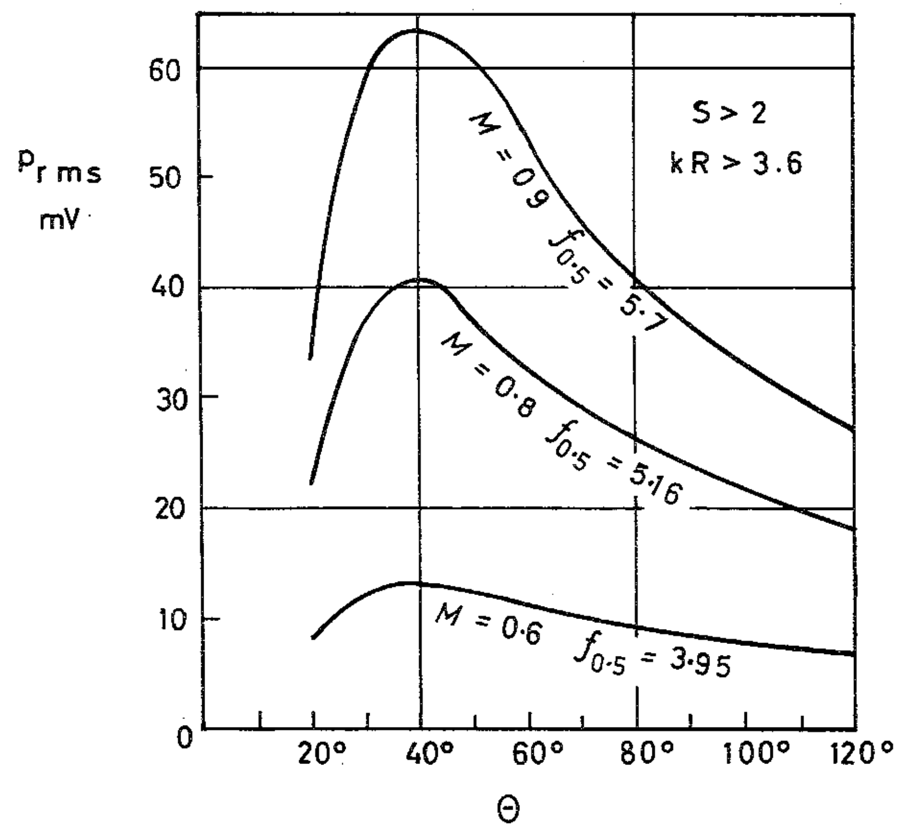

Figure 9 RMS-sound pressure directivities for Strouhal numbers S > 2 and various Mach numbers M measured by MOLLO-CHRISTENSEN, KOLPIN & MARTUCCELLI [5]