Effect of Multipath on Code-Tracking Error Jitter of a Delay Locked Loop

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BIOGRAPHY

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ABSTRACT

It is very well known that multipath propagation is a major source of error for the measurements performed by a GNSS receiver, and in particular, that it is the cause of a bias in pseudorange estimation. Nevertheless, there is another side effect that seems to have received scant attention: the tracking error jitter alteration. Additive channel noise induce fluctuations in the value of tracking error and multipath components alter the relationship between input noise and these fluctuations. In this contribution the impact of multipath on the tracking error variance is investigated, adding a useful tool in the characterization of multipath effects and in the evaluation of performance of GNSS receiver.

INTRODUCTION

The impact of multipath on code tracking accuracy is often represented as a multipath-induced tracking error envelope representing the maximum error resulting from one single multipath in-phase and out of phase with the LOS, as a function of the relative delay of the reflected ray. This so called "*multipath envelope*", represents a bias in the tracking error due to multipath propagation. However, from a closer investigation it has come out that this is not the only effect of multipath; not only does multipath changes the average value of the tracking error, but also its variance. In other words, the multipath alters the way the input noise affects the tracking process.
The full description of the role played by channel noise in a synchronous control system as the Delay Locked Loop (DLL) poses a very complex mathematical problem, owing to the specific nonlinearity of the system. Nevertheless, if some assumptions are done, the problem can be still solved satisfactorily with linear theory, and with this approach it has been possible to determine the variance of the tracking error as a function of noise, signal and receiver parameters. The idea underlying this work is to extend this approach to the case in which the impinging signal is corrupted by multipath. The ultimate aim is to provide a new "*metric*" in the characterization of the mul-

tipath error; in contrast to the well estabilished two-paths *multipath envelope*, here it is proposed a more complete *metric*: for a given realization of a multipath channel with an arbitrary number of paths, the bias and the variance of the tracking error are calculated; and these two quantities are then merged into the Root Mean Square Error (RMSE). In this way, in only one parameter will be summarized the joint effect of noise and multipath propagation. With this new *metric* defined, several DLL structures will be compared and the Double Delta Correlator, ”popular” multipath mitigation technique that has been thought with the aim of reducing the multipath bias, will be explored from this new perspective.

SIGNAL MODEL

In order to characterize the influence of multipath, we model the GNSS signal impinging on the receiver, and after the Doppler removal and the base-band conversion, as:

$$\begin{aligned} r(t) =& \sqrt{P}\, c(t-\tau) + \\ & \sqrt{P} \sum_{n=1}^{N} \alpha_n c(t-\tau-\tau_n) e^{j(2\pi D_n t + \vartheta_n)} + n(t) \ , \end{aligned} \tag{1}$$

in which:

$c(t)$: Pseudo-noise (PN) code of unitary power.

P : Signal carrier power.

τ : Code phase of the LOS component.

f_0 : Carrier frequency of the LOS (Line-Of-Sight) component. It is the sum of the carrier frequency of the GNSS signal plus the LOS Doppler.

N : Number of the multipath rays.

α_n : Multipath relative amplitude of the n-th ray.

τ_n : Multipath excess Delay of the n-th ray. From now on, simply Multipath delay.

D_n : Relative Doppler shift or residual Doppler. Said f_n the carrier frequency on the n-th ray, then $D_n = f_n - f_0$.

ϑ_n : Carrier phase for the n-th path.

$n(t)$: White Gaussian noise.

The PN code is a DS-CDMA signal spreaded with a BPSK signal:

$$c(t) = \sum_{k=-\infty}^{\infty} a_j p(t - kT_c) \tag{2}$$

with

T_c : Chip interval.

a_i : Binary pseudo-noise signature sequence. Its elements are random, independent,aperiodic equally likely. $a_i \in \{0;1\}$.

$p(t)$: Impulse response of the pulse shaping filter.

For long sequences the power spectrum of the PN code is asymptotically equal to the spectrum of the pulse. Holding this, the cross-correlation between the incoming PN code and the locally generated code can be expressed as a function of the pulse spectra:

$$\begin{aligned} R_{c\hat{c}}(\varepsilon) =& \frac{1}{T_p} \int_{T_p} c(t-\tau)\hat{c}(t-\tau)\mathrm{dt} = \\ =& \int_{-B}^{B} C(f)\hat{C}^*(f) e^{j2\pi f \varepsilon} \mathrm{df} \ , \end{aligned} \tag{3}$$

where:

$C(f)$: Pulse spectrum.

$\hat{c}(t)$: Locally generated PN code.

$\hat{C}(f)$: Spectrum of the pulse of the locally generated PN code.

$\hat{\tau}$: Estimate of the code phase τ.

T_p : Integration time.

B : Pulse bandwidth.

$\varepsilon = \tau - \hat{\tau}$: Tracking error.

By defining the cross-correlation in the frequency domain as in (3), it is possible to carry on a general formulation which is independent of the pulse used. In this model the multipath is fully described by time invariant deterministic parameters, that can be arranged in the vectors $\boldsymbol{\alpha}$, $\mathbf{D}$, $\boldsymbol{\tau}$, $\boldsymbol{\vartheta}$, each of which containing N entries, one for every multipath ray. Modeling the multipath in this fashion for a DLL requires the channel, and hence the multipath parameters, to be static in a time span equal to the inverse of the loop bandwidth [1].
The objective of the DLL is to estimate the code phase (time-delay) τ and to track this quantity as the users and the satellite move. Describing mathematically the tracking performance of a DLL is equivalent to providing a statistical characterization of the tracking error. The most important statistical characterization of an estimation error involves the determination of its bias and variance. In other words, the tracking error is to be characterized in terms of moments of the first and second order. In doing this we will make two assumptions:

Steady-state tracking : the DLL is already *”tracking”* or *”in-lock”* and the joint effect of multipath and noise is such not to cause a loss-of-lock.

Small tracking jitter : The tracking point oscillates around the lock point in a restricted set of values, for which the composite discriminator is approximately linear.

DISCRIMINATOR FUNCTIONS IN MULTIPATH

When as input to the DLL there is the composite incoming signal (1), rather than only the desired Line-Of-Sight (LOS), the cross-correlation with the reference PN code gets distorted. This happens because the multipath is a sort of disturbance that is highly correlated with the useful signal. As a consequence of that also the discriminator function is distorted, and with it also the overall tracking performance of the DLL is altered. We call the distorted discriminator *”composite discriminator”*, and we denote it by $S_c(\varepsilon)$. If the multipath propagation is modeled as in (1), it is possible to express the composite discriminators as function of the multipath parameters, the pulse spectra of the transmitted PN code and of the locally generated PN code and of the correlator spacing. The expression depends of course on the kind of DLL structured used.

In Table 1 the several composite discriminators are reported. The we have used the notation $S_{c,DLL\,type}(\varepsilon)$, where the field ”DLL type” stands for the DLL structure:

Coh : Coherent DLL.

Nc : Non-Coherent DLL.

Dp : Dot-product DLL.

$\Delta\Delta$: Double delta DLL (Coherent).

The term $\vartheta_c(\epsilon)$ appearing in Table 1 is the so called ”*composite phase*”. This is the steady-state error bias that due to the inability of the PLL to discern the carrier phase of the LOS from those of the other multipath components. The composite phase amounts to:

$$\vartheta_c(\epsilon) = \arctan\left\{ \frac{\sum_{k=1}^{M} \alpha_k \mathrm{sinc}(T_p D_k) R_{c\hat{c}}(\varepsilon + \tau_k)\sin(\vartheta_k)}{R_{c\hat{c}}(\varepsilon) + \sum_{k=1}^{M} \alpha_k \mathrm{sinc}(T_p D_k) R_{c\hat{c}}(\varepsilon + \tau_k)\cos(\vartheta_k)} \right\} \quad (4)$$

The term ”Double Delta DLL” is a general expression for a DLL which employs two correlator pairs with different correlator spacings. The circulating expressions ”High Resolution Correlator” (HRC), ”Strobe Correlator”, ”Pulse Aperture Correlator” (PAC) are all equivalent from the conceptual point of view. In the sources (see [2]) nothing is said about whether the Double Delta is based on a coherent down-converted baseband signal or on a non-coherent one.. In this work we will consider only the *Coherent Double Delta*, but also a *Non-coherent Double Delta* or eventually a *Dotproduct Double Delta* are possible. In writing the discriminator of the Double Delta DLL, we will indicate by $2\Delta_1$ the smaller correlator spacing and by $2\Delta_2$ the larger one.

The zero crossing of the composite discriminators, to which the maximum of the cross-correlation correspond, will no more necessarily be at the point $\varepsilon = 0$, as discussed in [5] and in other sources. Nevertheless in this analysis we will focus specifically at the variance of the tracking error.

$S_{c,Coh}(\varepsilon) =$	$2\sum_{k=0}^{N}\alpha_k \mathrm{sinc}(T_p D_k)\cos(\theta_c(\varepsilon)-\theta_k)\cdot$ $\cdot\left\{\int_{-B}^{B} C(f)\hat{C}^*(f)\sin(2\pi f(\varepsilon+\tau_k))\sin(2\pi f\Delta)\mathrm{df}\right\}$
$S_{c,Nc}(\varepsilon) =$	$\sum_{n=0}^{N}\sum_{m=0}^{N}\alpha_n \mathrm{sinc}(T_p D_n)\alpha_m \mathrm{sinc}(T_p D_m)\cos(\vartheta_n-\vartheta_m)\cdot$ $\cdot\left\{\int_{-B}^{B} C(f)\hat{C}^*(f)e^{j2\pi f(\varepsilon-\Delta+\tau_n)}\mathrm{df}\cdot\right.$ $\int_{-B}^{B} C(f)\hat{C}^*(f)e^{j2\pi f(\varepsilon-\Delta+\tau_m)}\mathrm{df}+$ $-\int_{-B}^{B} C(f)\hat{C}^*(f)e^{j2\pi f(\varepsilon+\Delta+\tau_n)}\mathrm{df}\cdot$ $\left.\cdot\int_{-B}^{B} C(f)\hat{C}^*(f)e^{j2\pi f(\varepsilon+\Delta+\tau_m)}\mathrm{df}\right\}$
$S_{c,Dp}(\varepsilon) =$	$\sum_{n=0}^{N}\sum_{m=0}^{N}\alpha_n \mathrm{sinc}(T_p D_n)\alpha_m \mathrm{sinc}(T_p D_m)\cos(\vartheta_n-\vartheta_m)$ $\left\{\int_{-B}^{B}\lvert C(f)\rvert^2\cos(2\pi f(\varepsilon+\tau_n))\mathrm{df}\cdot\right.$ $\left.2\int_{-B}^{B}\lvert C(f)\rvert^2\sin(2\pi f(\varepsilon+\tau_m))\sin(2\pi f\Delta)\mathrm{df}\right\}$
$S_{c,\Delta\Delta}(\varepsilon) =$	$2\sum_{k=0}^{N}\alpha_k \mathrm{sinc}(T_p D_k)\cos(\vartheta_c(\varepsilon)-\vartheta_k)\cdot$ $\left\{\int_{-B}^{B} C(f)\hat{C}^*(f)\sin(2\pi f(\varepsilon+\tau_k))\cdot\right.$ $\left.\cdot\left[\sin(2\pi f\Delta_1)-\frac{1}{2}\sin(2\pi f\Delta_2)\right]\mathrm{df}\right\}$

Table 1 Composite discriminators

TRACKING ERROR VARIANCE IN MULTIPATH

A full non linear description of the effect of noise on a Phase Lock Loop (PLL) is to be found in [6]. Most of the principle apply also to a Delay Locked Loop. Nevertheless in this paragraph we will derive the expression of the tracking error variancein presence of multipath by linearizing the model. Indeed, as we will show, the the linear model can be modified and multipath can be included in it. The error signal of a DLL that is functioning in presence of multipath propagation is:

$$e[k] = \begin{cases} \sqrt{P}\, S_c(\varepsilon;k) + n_T[k], & \text{Coh and } \Delta\Delta \\ P\, S_c(\varepsilon;k) + n_T[k], & \text{Nc and Dp} \end{cases} \quad (5)$$

where $S_c(\varepsilon;k)$ is the composite S-curve at the k-th epoch, and $n_T[k]$ is the noise term. Both the composite S-curve and the noise term of the error signal depend on the DLL structure. The multipath alters only the discriminator, but not the noise. So the statistical characterisation of the noise term $n_T[k]$ is exactly the same as in the single path case.

A discriminator unaffected by multipath shows a linear behavior around the point $\varepsilon = 0$. This means that for values of the tracking error around 0, a linearization of the S-curve is reasonable. Multipath propagation distorts the S-curve by adding to the LOS S-curve other shifted and attenuated S-curves. The main result of this, is that the zero crossing of the composite S-curve does not take place any more at $\varepsilon = 0$, but at another point $\varepsilon = b_\varepsilon$. The shape of the S-curve is also distorted, but it is always possible to find a small region around the lock point $\varepsilon = b_\varepsilon$, for which the S-curve can be linearized. The extension of the linear region depends on the discriminator type, as well as on the multipath conditions. However, the linearization around $\varepsilon = b_\varepsilon$ is always licit for *”small”* elongations from the lock point, that is to say for:

$$|\varepsilon - b_\varepsilon| \simeq 0 \tag{6}$$

This is the equivalent condition of the *"small tracking"* error used to calculate the variance in the case of single path: simply here the elongations around the stable point must be small and not their absolute values. In the single path case the two things were coinciding. If (6) holds, then (5) can be linearized around the point $\varepsilon = b_\varepsilon$:

$$e[k] = \begin{cases} \sqrt{P}\, S_c'(b_\varepsilon; k)(\varepsilon - b_\varepsilon) + n_T[k], & \text{Coh and } \Delta\Delta \\ \\ P\, S_c'(b_\varepsilon; k)(\varepsilon - b_\varepsilon) + n_T[k], & \text{Nc and Dp} \end{cases} \tag{7}$$

where $S_c'(b_\varepsilon; k)$ is the derivative of the composite S-curve calculated at the point $\varepsilon = b_\varepsilon$, at the k-th epoch. The value of this quantity for a Dot-product DLL is given in Appendix A.

If we examine (7) without considering the noise term, it is evident the error signal is zero when $\varepsilon = b_\varepsilon$. When this happens there is no feed-back, and the loop is stable . We are interested in the oscillations of the tracking error around this lock point, more specifically in the centered moment of the second order of the random variable ε:

$$E\left\{ \left(\varepsilon - E\{\varepsilon\} \right)^2 \right\} = \sigma^2_{\varepsilon,mpath} \tag{8}$$

If the assumptions of steady state tracking are applicable, we can state that:

$$E\{\varepsilon\} = b_\varepsilon \tag{9}$$

Let us now define the tracking error around the stable point, or in other words biased tracking error as:

$$\varepsilon_b \triangleq \varepsilon - b_\varepsilon \tag{10}$$

in this way (7) can be rewritten as:

$$e[k] = \begin{cases} \sqrt{P}\, S_c'(b_\varepsilon; k)\varepsilon_b + n_T[k], & \text{Coh and } \Delta\Delta \\ \\ P\, S_c'(b_\varepsilon; k)\varepsilon_b + n_T[k], & \text{Nc and Dp} \end{cases} \tag{11}$$

In steady state tracking, the mean value of ε_b is zero and its means-square value is equivalent to (8):

$$E\{\varepsilon_b\} = 0 \tag{12}$$

$$E\{\varepsilon_b^2\} = \sigma^2_{\varepsilon,mapath} \tag{13}$$

The variance of the tracking error in the single path case has been found in [3] as a function of the noise power spectrum, by calculating the mean square value of the random variable ε: since ε is zero mean, the mean square value and the variance of ε were the same. If we now look at (11), the error signal is the same as in [3], with the only difference being:

- $S_c'(b_\varepsilon; k)$ in the place of $S'(0; k)$
- ε_b in the place of ε

This means that the effect of multipath on the error signal is twofold: changing the slope of the S-curve around the stable point, and shifting the stable point away from zero. While the latter effect was important for the bias, it plays no role in the variance of the tracking error: the variance of a random variable is an indication of how much the values of the random variable are spread around the mean, whatever the mean value is. Said that, we can calculate the mean-square error of ε_b, that is equivalent to the variance of ε (12), by following the same passages done in [3] and using (11) as the expression of the error signal.

$$\boxed{E\{\varepsilon_b^2\} = \sigma^2_{\varepsilon,mpath} = \begin{cases} \frac{2B_L\, N_T(0)}{P\,[S_c'(b_\varepsilon)]^2}, & \text{Coh and } \Delta\Delta \\ \\ \frac{2B_L\, N_T(0)}{P^2\,[S_c'(b_\varepsilon)]^2}, & \text{Nc and Dp} \end{cases} \quad (14)}$$

in which $N_T(f)$ is the power spectral density of the noise, that depends on the DLL type. The autocorrelation of the noise term $n_T[k]$ is not altered, and so its power spectrum; the quantity $N_T(0)$ is the same to be found in a multipath free analysis. For completeness, we report in Table 2 its value for all DLL types. By $R_{\hat{c}}$ it is indicated the autocorrelation of the local reference:

$$R_{\hat{c}}(\xi) = \int_{-B}^{B} |\hat{C}(f)|^2 e^{j2\pi f\xi} \mathrm{df}\ , \tag{15}$$

By Combining (14) with the quantities given in Table 1 and Table 2, it is possible to give an explicit formulation of the tracking error variances for all DLL types:

$$\sigma^2_{\varepsilon,mpath,Coh} = \frac{2B_L}{P[S'_{c,Coh}(b_\varepsilon)]^2} N_0[1 - R_{\hat{c}}(2\Delta)] \tag{16}$$

$$\sigma^2_{\varepsilon,mpath,Nc} = 2\frac{N_0 B_L}{P[S'_{c,Nc}(b_\varepsilon)]^2} \Big\{ 4R^2_{\hat{c}}(\Delta)[1 - R(2\Delta)] + \frac{N_0}{PT_p}\left[1 - R^2_{\hat{c}}(2\Delta)\right] \Big\} \tag{17}$$

$N_T(0)_{Coh} =$	$N_0[1-R_{\hat{c}}(2\Delta)]$
$N_T(0)_{Nc} =$	$4N_0PR^2_{\hat{c}}(\Delta)[1-R(2\Delta)]+\frac{N_0^2}{T_p}[1-R^2_{\hat{c}}]$
$N_T(0)_{Dp} =$	$PN_0[1-R_{\hat{c}}(2\Delta)]+\frac{N_0^2}{2T_p}[1-R_{\hat{c}}(2\Delta)]$
$N_T(0)_{\Delta\Delta} =$	$N_0\Big\{[1-R_{\hat{c}}(2\Delta_1)]+$ $\frac{1}{4}[1-R_{\hat{c}}(2\Delta_2)]-[R_{\hat{c}}(\Delta_2-\Delta_1)-R_{\hat{c}}(\Delta_2+\Delta_1)]\Big\}$

Table 2 DC component of the noise term $n_T[k]$ of the error signal.

$$\sigma^2_{\varepsilon,mpath,Dp} = \frac{2B_L N_0}{P} \frac{[1 - R\hat{c}(2\Delta)]}{[S'_{c,Dp}(b_\varepsilon)]^2} \left\{ 1 + \frac{N_0}{2T_p P} \right\} \quad (18)$$

$$\sigma^2_{\varepsilon,mpath,\Delta\Delta} = \frac{2B_L}{P[S'_{c,\Delta\Delta}(b_\varepsilon)]^2} N_0 \Big\{ [1 - R_{\hat{c}}(2\Delta_1)] + \frac{1}{4}[1 - R_{\hat{c}}(2\Delta_2)] - [R_{\hat{c}}(\Delta_2 - \Delta_1) - R_{\hat{c}}(\Delta_2 + \Delta_1)] \Big\} \quad (19)$$

In (16), (17), (18), (19) appears the derivative of the composite discriminator calculated at the point $\varepsilon = b_\varepsilon$. This quantity can be easily found by deriving the respective expression in Table 1.

TWO-PATH SCENARIO

Exemplarly we particularize these results for the well known two-paths propagation scenario. In the present figures the standard deviation (which is the root of the variance) is depicted, for the two cases in which the second path is either in phase ($\vartheta_1 = 0$) or in opposition of phase($\vartheta_1 = \pi$). In every figure the mulipath standard deviation for a particular DLL structure is shown. The parameters used to obtain these figures are represented in Table 3. In the all the four figures the multipath standard deviation is compared with the standard deviation of the tracking error in absence of multipath.

It is not difficult to notice that, for small multipath delays, the values of the tracking error variance differs highly from the value calculated for only the LOS. In particular in the case in which the phase of the second path is π, the multipath variance is higher than the LOS variance, and the case having the second path in-phase presents a multipath variance which is lower. The reason for this

Parameter	**Value**
Correlator spacing(2Δ)	$0.1T_c$
Loop filter bandwidth	1 Hz
Integration time (T_p)	1 ms
C/N_0	30 dB
Pulse type	rectangular pulse
Pulse bandwidth	10.23 MHz
multipath relative amplitutde(α_1)	0.2

Table 3 Parameters used for the calculation of the multipath standard deviation of the trackign error represented in the figures.

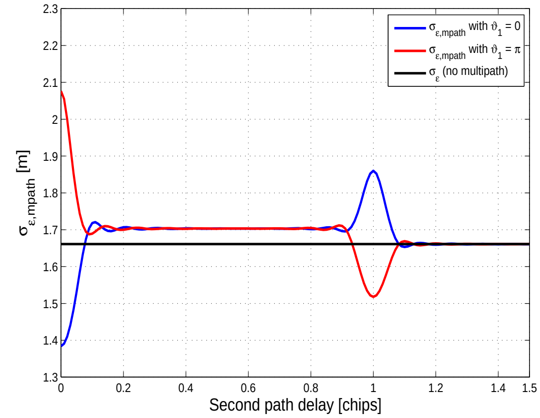

Fig. 1 Two-paths standard deviation of the tracking error for a **Coherent DLL**. The second path has a normalized amplitude $\alpha_1 = 0.2$. The other parameters are specified in Table 3.

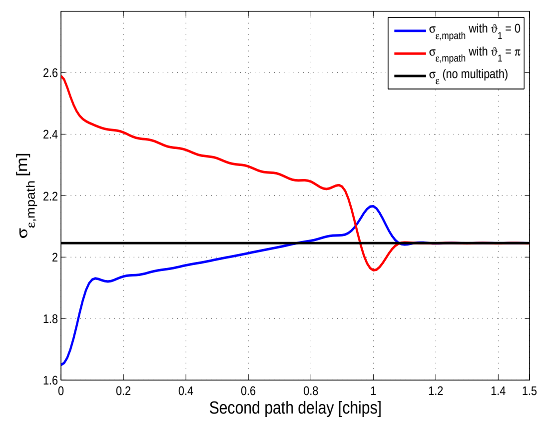

Fig. 2 Two-paths standard deviation of the tracking error for a **Non-coherent DLL**. The second path has a normalized amplitude $\alpha_1 = 0.2$. The other parameters are specified in Table 3.

can be explained in the following way. The composite S-curve is made of the LOS S-curve plus other terms, whose sum we shall call *"equivalent multipath S-curve"*. When the second path has a phase $\vartheta_1 = 0$, the slope of the equivalent multipath S-curve has the same sign of the slope of LOS S-curve. Thus , the slope of the linear region of the composite S-curve is increased, and as a result of that the variance is decreased (see (14) and (18)). When instead the multipath is in opposition of phase, the sign of the slope of the equivalent multipath S-curve is inverted: in this case the linear region of the composite S-curve is decreased and so the variance becomes larger.

Although these figures depict a restrictive propagation case, some general considerations can be drawn out of that. First of all, short multipath delays are the ones for

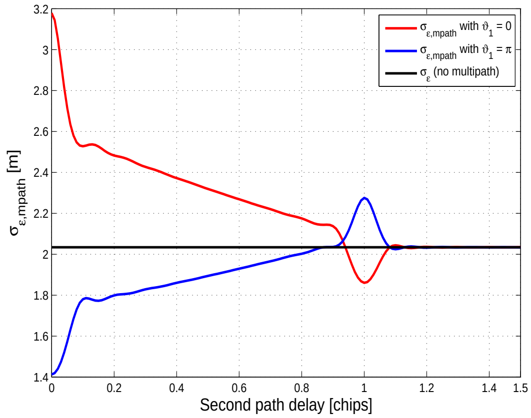

Fig. 3 Two-paths standard deviation of the tracking error for a **Dot-product DLL**. The second path has a normalized amplitude $\alpha_1 = 0.2$. The other parameters are specified in Table 3.

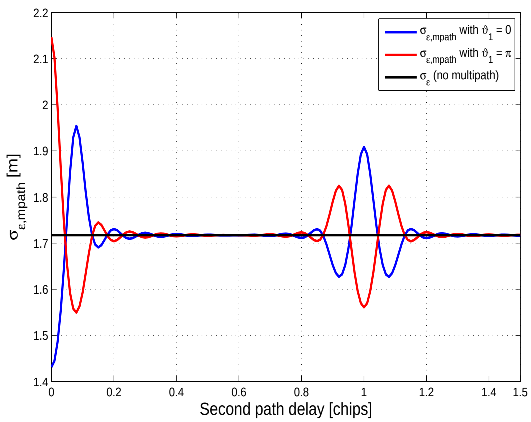

Fig. 4 Two-paths standard deviation of the tracking error for a **Double Delta DLL**. The second path has a normalized amplitude $\alpha_1 = 0.2$. The other parameters are specified in Table 3.

which the influce of multipath over the tracking jitter is most heavily felt. Even though in a realistic scenarios the multipath phases are usually to be averaged, one has to keep in mind that there can be unlucky cases in which the tracking jitter can be particularly high. Eventually these ”unlucky” cases can cause a *loss of lock*. As a last point, it is interesting to see, that the Double Delta DLL has an intersting behavior also from the point of view of the tracking jitter, Especially for short multipath delays.

CONCLUSION

In this work we have shed some light on the possibility of assessing the influence of multipath on the tracking jitter of a DLL by means of mathematical description. When the channel is known and static, or at most slowly varying , it is possible to calculate the bias and the variance of the tracking error. Knowing the two means knowing also the MSE, a very well known error metric. This allows to evaluate the tracking perforamance of a DLL by means of a mathematical formula avoiding time-consuming simualtions.
Future works on this line include a validation of the presented results by means of a hardware GNSS receiver, and also a further mathematical analysis on the relationship between multipath and the probability of losing lock.

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