Computation of Airframe Noise with the Discontinuous Galerkin Method

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Introduction

Airframe noise is generated through turbulence passing by the edges of (rigid) bodies. Considering the low noise engines of modern airliners, it is an important noise source during the approach phase.

It may be computed efficiently using a synthetic, unsteady, turbulent velocity field, which is generated stochastically based on time-averaged flow information from a RANS[1] computation. This way, broadband slat noise was computed very successfully [1]. In that work, the two-dimensional (2D) Acoustic Perturbation Equations (APE) were spatially discretized with the well-known Dispersion-Relation-Preserving (DRP) Finite Difference scheme [2] requiring a block-structured grid (which was made from as many as 25 blocks in [1]).

In general, constructing a block-structured grid around a complex body is cumbersome and time-consuming, though, and one may rather resort to a flexible, unstructured, triangular grid for the 2D spatial discretization.

The Discontinuous Galerkin (DG) method provides a robust, efficient, high-order accurate discretization even with such a grid-type. It has repeatedly and successfully been used in computational aeroacoustics already [3, 4]. But, first, no standard approach can yet be identified, how to apply this method, if the mean flow field, which underlies a system of partial differential equations like the APE, is space-dependent. Second its combination with a source term, which depends on a synthetic velocity field, is untried, too.

In the present work, the DG method will be applied to discretize the APE in case of a space-dependent mean flow field and a source term from a synthetic turbulent velocity field, respectively. The particular goal is the computation of airframe noise of a complex 2D airfoil configuration using a triangular grid, cp. the sketch of figure 1.

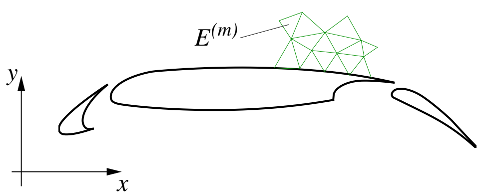

Figure 1: Complex two-dimensional airfoil configuration, for which airframe noise will be computed.

[1] Reynolds Averaged Navier Stokes

Method

The current DG discretization principally follows that proposed in [4]. One difference is, that another system of partial differential perturbation equations, namely the aforementioned APE, is discretized here. Thus, the unknowns are the temporal and spatial fluctuations p' of pressure, and u', v' of the x, $y-$directed velocity.

For constant sound speed the APE read in 2D matrix-vector notation:

$$\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{F}^x}{\partial x} + \frac{\partial \boldsymbol{F}^y}{\partial y} - \boldsymbol{S} = \boldsymbol{0}, \tag{1}$$

where $\boldsymbol{U} = [p', u', v']^T$ is the vector of the aforementioned unknowns. The flux vectors $\boldsymbol{F}^x$ and $\boldsymbol{F}^y$ obey $\boldsymbol{F}^x = \boldsymbol{AU}$ and $\boldsymbol{F}^y = \boldsymbol{BU}$, respectively, where the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ contain the time-averaged, i.e. steady, but generally space-dependent, mean flow quantities. $\boldsymbol{S}$ is a given source vector.

In one element $E^{(m)}$, *all* quantities appearing in (1) are approximated in the fashion of the following expansion, shown for one component of the vector $\boldsymbol{U}$ here:

$$U_r^{(m)} \approx \tilde{U}_r^{(m)} = \sum_{l=1}^{N} \hat{U}_{rl}^{(m)}(t) \cdot \Phi_l^{(m)}(x, y). \tag{2}$$

In (2), $\Phi_l^{(m)}$ are given shape-functions, which are calculated by coordinate transformation from reference shape-functions Φ_l^R defined in a reference triangle E^R. The overall approximate solution $\tilde{\boldsymbol{U}}$ may be *discontinuous* along the inter-element boundaries. The elementwise time-dependent coefficients $\hat{U}_{rl}^{(m)}(t)$ are the actual unknowns of the DG discretization and N depends on the polynomial degree p associated to the expansion.

As mentioned in the introduction, one issue of a DG discretization is a space-dependent mean-flow field. Following [4], both the mean flow quantities (entries of $\boldsymbol{A}$ and $\boldsymbol{B}$), as well as the unknowns (components of $\boldsymbol{U}$) are represented by an expansion of type (2) with polynomial degree p. (The mean-flow coefficients $\hat{A}_{rs,l}^{(m)}$ and $\hat{B}_{rs,l}^{(m)}$, are, of course, time independent.) Thus, the flux quantities, e.g. $\boldsymbol{F}^{x(m)}$, are actually polynomials of degree $2p$. In order to reduce computational efforts, they are also truncated to polynomials of degree p, though, and they are also represented according to (2). The calculation of the associated flux coefficients, e.g. $\hat{F}_{rl}^{x(m)}(t)$, is very easy, when use is made of the so-called "nodes", which are associated to shape-functions given by the well-known Lagrange polynomials.

This approach was implemented in a 2D Fortran DG-APE code, using p=3 Lagrange polynomials. As time integration is performed by a standard explicit fourth order Runge-Kutta scheme, the overall order of accuracy of the code is four, which was verified by test computations.

Results

To verify the DG-APE code with regard to sound transmission through a sheared mean flow, the case of a monofrequent, harmonic monopole sound source in an isothermal boundary layer with constant thickness was considered [4, 5]. The free-stream Mach-number was $Ma_\infty = 0.3$. The monopole model [4] was situated at the origin of the coordinate system, cp. figure 2. All coordinates and lengths were normalized with the boundary layer thickness, thus the boundary layer occupied the region $0 < y < 1$. The grid consisted of 18.136 triangles and was refined in the vicinity of the source. One wavelength was resolved by about 3 triangles everywhere, i.e., the grid was also refined upstream of the source and coarsened downstream, respectively. The solution obtained on this grid can be regarded grid-independent.

Figure 2 shows a snapshot of the pressure perturbation field from the DG computation. It looks very similar to the respective DNS[2] result [5] and exhibits all the physical phenomena related to the refraction of sound waves in the boundary layer.

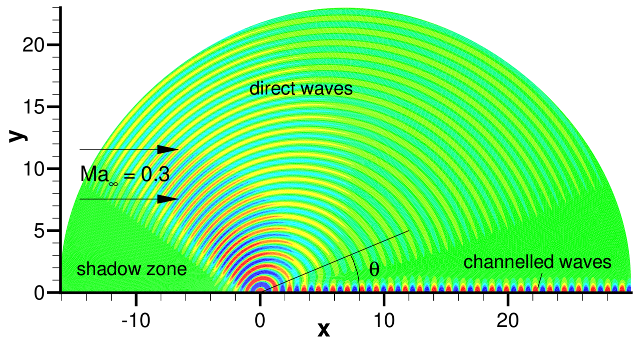

Figure 2: Snapshot of computed pressure perturbation field of a monopole sound source in a boundary layer.

Figure 3 qualitatively compares the DG-APE sound pressure amplitude directivity (left) to that obtained from the DNS (right, circles) [5]. The underlying pressure signals were recorded on a semi-circle with radius $R = 15$ around the coordinate origin. For clarity, the contribution of the channelled waves is removed in the plots. Both solutions are very similar and the maximum clearly occurs close to the theoretically predicted "critical angle" of $\theta = 126.3°$ [5].

The wavelength of the channelled waves in the DG-APE simulation was found to be $\lambda^{CH} = 1.166$. The theoretically predicted wavelength is $\lambda^{CH}_{th} = 1.121$ [5]. The small discrepancy may mainly be attributed to the fact, that the theoretical solution is based on the wave-operator

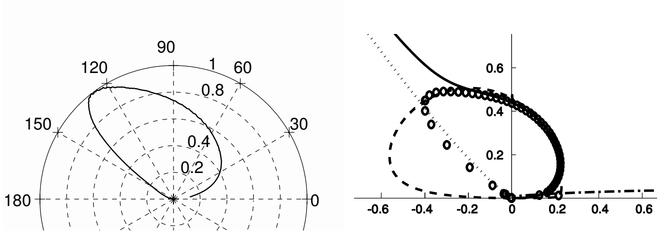

Figure 3: Qualitative comparison of sound pressure amplitude directivity from computation (left) with reference DNS solution (right, circles) from [5].

from linearized Lilley's equation, which is exact for a parallel shear-flow, whereas the wave operator encoded in the APE is actually exactly valid for a potential mean-flow field.

Conclusion & Outlook

The current DG discretization of the APE works very well even in presence of a non-uniform mean-flow field, and may principally be applied to compute airframe noise of the complex airfoil configuration, figure 1. But, before doing so, the mean-flow coefficients $\hat{A}^{(m)}_{rs,l}$ and $\hat{B}^{(m)}_{rs,l}$, respectively, as well as the unsteady source coefficients $\hat{S}^{(m)}_{rl}$ have to be provided. Both implies interpolation of data (generated on another grid by an extrinsic code) onto the nodes of the triangles in the DG grid.

Acknowledgement

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References

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[2]Direct Numerical Simulation