

# ENSEMBLE TIME IN GNSS - PERFORMANCE REQUIREMENTS AND ALGORITHM TESTS

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## Abstract

*Any Global Navigation Satellite System (GNSS) relies on a highly stable and reliable System Time that has to meet high-performance requirements to enable GNSS services suitable for navigation and timing communities. The challenge is to guarantee this high performance continuously. The Kalman filter algorithm implemented in GPS, called the GPS Composite Clock, is a mature method to generate such a highly robust System Time. The algorithm estimates the time offsets of every individual clock to the so-called implicit mean, which is a common component in all clock estimates. The common component offers the functionality of System Time and is understandable as a weighted average out of all ensemble clock readings. GPS Composite Clock performance is analyzed by simulations of a “light” GNSS configuration with 10 rubidium satellite clocks, including deterministic drift, six ground cesium clocks, and two ground active hydrogen masers. Besides evaluating the stability of an error-free clock constellation to define the regular performances, the behavior of the algorithm is investigated considering different operational scenarios: exclusion of clocks from the GNSS ensemble and occurrence of clock feared events (frequency steps in rubidium satellite clocks and ground H-masers).*

## INTRODUCTION

Precise timing is the backbone of all Global Navigation Satellite Systems (GNSS) like GPS or the future Galileo. Consequently, the aim is to establish a highly stable and reliable System Time which fulfills high performance requirements to enable GNSS services suitable for navigation and timing communities.

## PERFORMANCE REQUIREMENTS ON GNSS SYSTEM TIME

The main purpose of System Time is to provide a stable reference for precise prediction of satellite clocks. System Time should only marginally affect the prediction error of the time offsets compared to the general prediction error of the satellite clocks themselves. To assess the required stability of the System Time, satellite clock prediction errors were simulated for six different scenarios. The scenarios assume a typical stability of the Galileo satellite rubidium and varying System Time stability. Obviously,

the minimal prediction error is observed for a perfectly stable System Time. Here, the satellite clock prediction error is not affected by the System Time component at all and is equal to 1.03 ns ( $1\sigma$ ) (Figure 1, bar marked with “0”). However, comparing this result to scenarios where the System Time was 10, 5, or 3 times more stable than the satellite clocks, it can be noted that prediction error only marginally increases to 1.08 ns ( $1\sigma$ ) for the 3 times case (see the bar marked with “3”). Relating to the simulation results, it seems to be reasonable to require the System Time to be at least 3 times more stable than the satellite clocks. Here, the prediction error is only marginally disturbed by instability of the System Time and also the performance is not distinctly improved by using a more stable System Time.

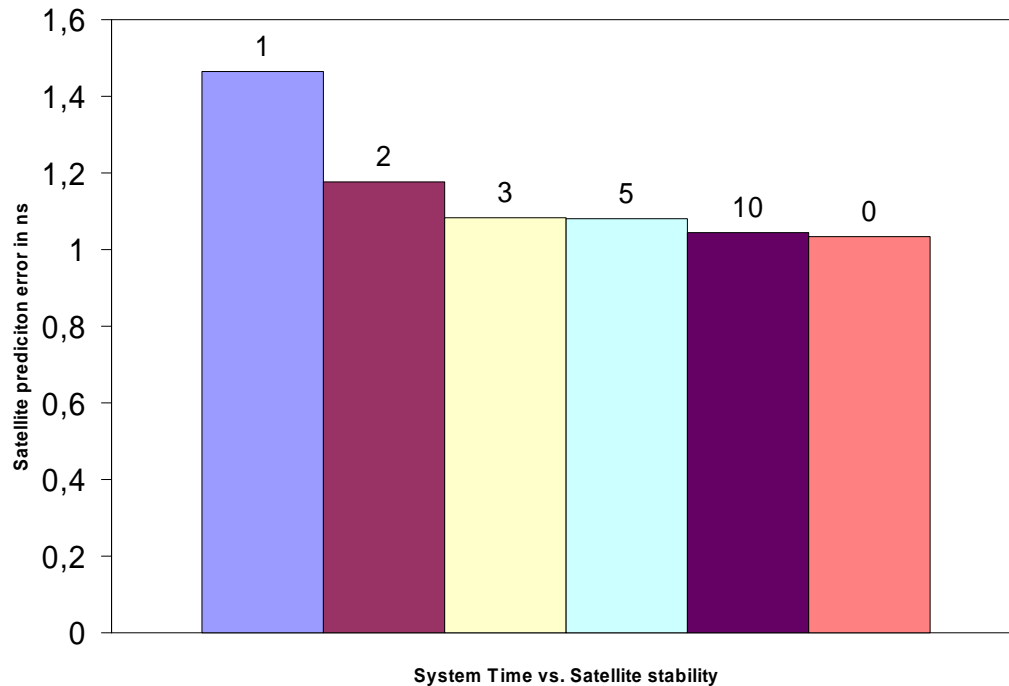


Figure 1. Prediction error of different System Time stabilities.

The second function of GNSS System Time is the support of timing services. The System Time has to be closely steered to UTC and the long-term stability has to be in levels of  $10^{-14}$ . However, in this paper we concentrate on the navigation-related (prediction of satellite clocks) performances.

## ESTABLISHING SYSTEM TIME USING COMPOSITE CLOCK ALGORITHM

In general, there are two approaches to establish System Time that meet the strict requirements of navigation and time services. The first approach, called the Master Clock, is to establish the System Time using a high-quality atomic clock, e.g. an active hydrogen maser (AHM). Its physical output defines System Time and all other GNSS clocks are referenced to it. This strategy, in which System Time had a physical representation, was used by GPS till 1991 [3]. However, implementing System Time by a hardware clock creates a single point of failure, since any Master Clock error like frequency or phase jumps, affects the System Time.

The second approach is using an Ensemble Time algorithm to establish System Time. Such algorithms estimate time offsets of every GNSS clock with respect to a “paper” System Time produced by the algorithm. In this approach, there is no physical representation and System Time is equal to a weighted average of all GNSS clocks. Since 1991, GPS makes use of a Kalman Filter that is called GPS Composite Clock (CC) [1]. Although the GPS Composite Clock does not depend on a single clock, every GNSS clock contributes with a different weight to the System Time. Hence, any clock error still affects System Time and, thus, disturbs all other time offsets. These effects will be investigated further in this paper.

## SIMULATED GNSS CLOCK ENSEMBLE

The robustness of GPS Composite Clock using 3-state clock models is investigated by simulating a “light” GNSS clock ensemble of 10 satellite rubidium clocks and two ground stations, each equipped with three cesium clocks and one active hydrogen maser (AHM). Figure 2 shows the Allan deviation of clock simulations of every clock type.

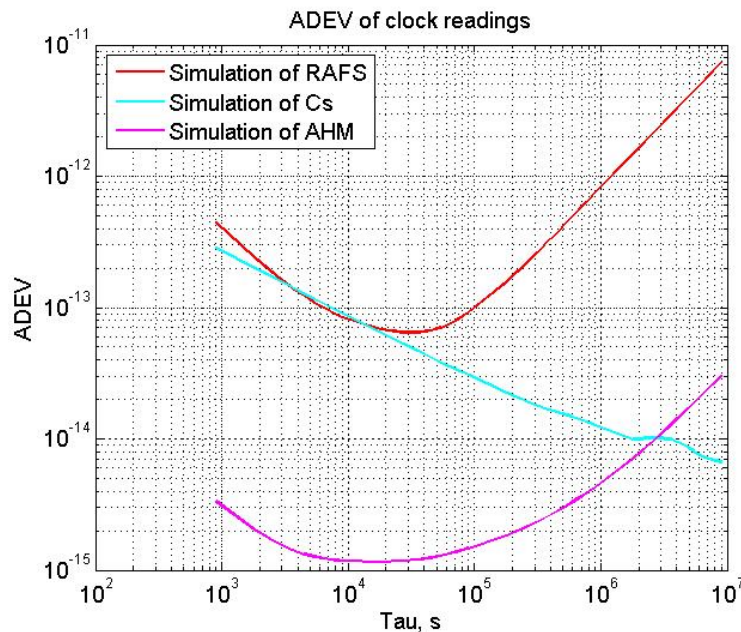


Figure 2. Stability of simulated clock readings.

Further, we simulated clock measurements by referencing all ensemble clocks to an AHM with a sample interval of 15 min and adding White Gaussian measurement noise of  $\sigma = 0.42$  ns ( $1\sigma$ ). Obviously, all time offsets are affected by the measurement noise in the short term, and in the long term the stability of Cs clocks is disturbed by the instability of the AHMs (Figure 3).

## STABILITY OF TIME OFFSETS WITH RESPECT TO THE COMPOSITE CLOCK

In order to operate the GPS Composite Clock, the individual clock process parameters  $q_1$ ,  $q_2$  and  $q_3$  are derived from the Allan deviation of every clock [2]. These parameters  $q_1$ ,  $q_2$  and  $q_3$  also define the clock weights in the Composite Clock algorithm [1]. Since the AHMs are almost 100 times more stable at the selected measurement interval (15min) than the other clocks, the filter implicitly assigns almost all weight in the ensemble to both AHMs (see Figure 4).

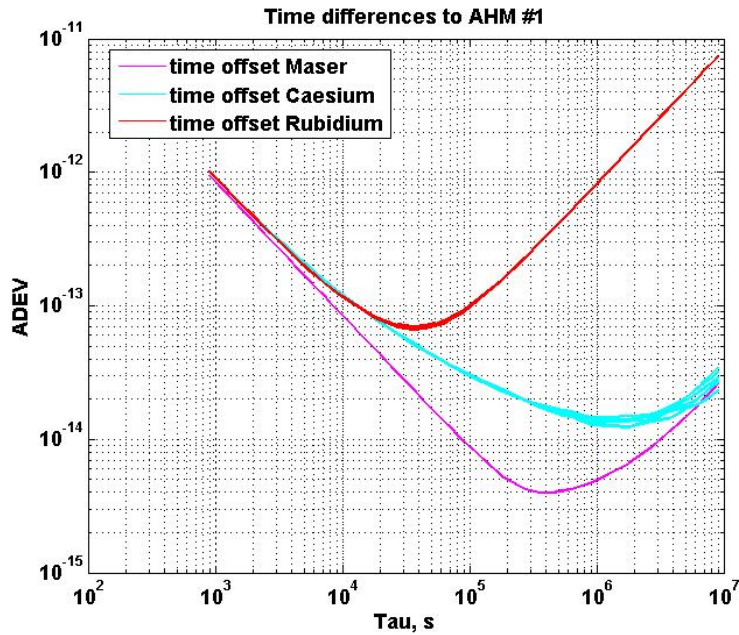


Figure 3. Stability of time offsets w.r.t. AHM.

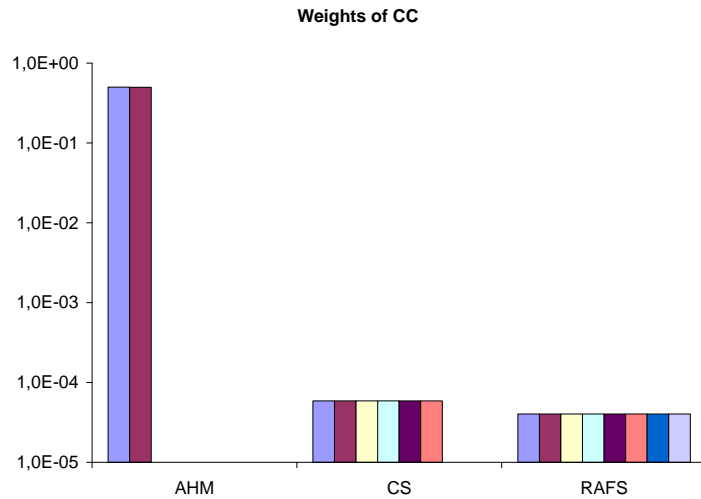


Figure 4. Weights implicitly assigned by the Composite Clock algorithm.

The CC algorithm iteratively processes the simulated time offsets, which are referenced to the selected AHM, and computes time offsets with respect to the implicit mean (Composite Clock). The stability of the computed time offsets is shown in Figure 5. All estimates benefit from application of the Kalman filter in the short- and medium-term, since measurement noise is mitigated. Further, the stability of rubidium and Cs clocks is unaffected by a Composite Clock in the short and medium terms, remaining (almost) equal to the stability of these clocks in a free-running mode. However, the stability of the Cs

clocks offset to the CC is disturbed by the CC's instability in the long term. Finally, the stability of both AHM is also disturbed by CC's short-term instability.

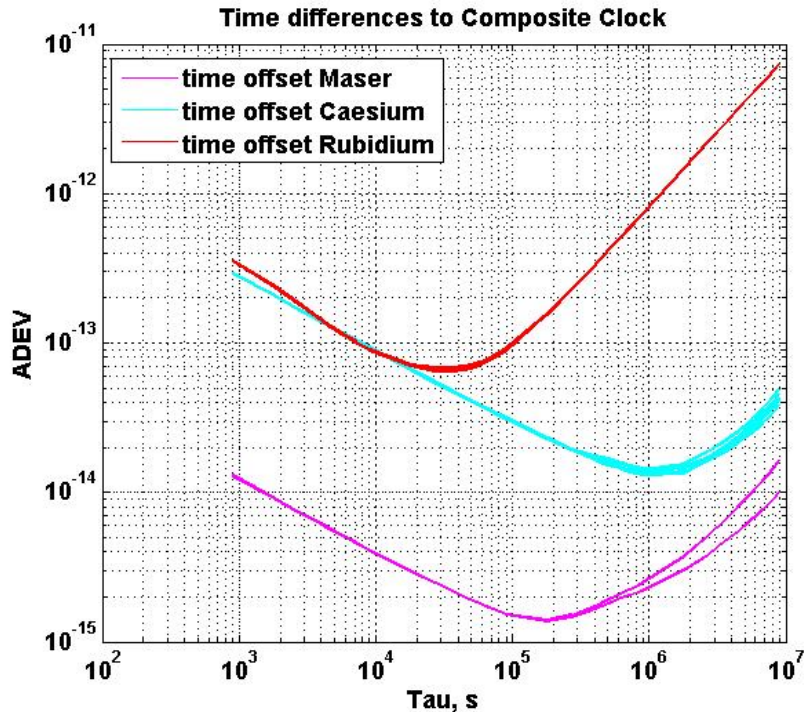


Figure 5. Stability of estimated time offsets w.r.t. Composite Clock.

### STABILITY OF “CORRECTED” CLOCKS

The theory of GPS Composite Clock outlines that the difference between simulated clock readings and Kalman filter estimates is equal to the common component Composite Clock or System Time plus a (steady-state) representation error. It is important to mention that these differences are usually called “corrected” clocks and are only computable by simulations, since the true clock readings are unknown in the real world. Every “corrected” clock represents Composite Clock or System Time with an individual and steady-state representation error.

Figure 6 compares stabilities of “corrected” and “uncorrected” cesium, rubidium, and AHM clocks. “Corrected” cesium and rubidium clocks behave similarly and are dominated by White Phase noise in the short and medium term. Only “corrected” AHM clocks are not characterized by White Phase noise and exhibit White Frequency noise in the short and medium term. However, the stability of “corrected” AHM clocks is worse than the stability of “uncorrected” AHM clocks.

### IMPACT OF CLOCK FAILURES ON THE COMPOSITE CLOCK

An important advantage of Ensemble Time algorithms is the generation of System Time which is independent of one individual (Master) clock. However, all clocks contribute to the System Time and, therefore, individual clock failures affect System Time. Several failure cases are investigated below.

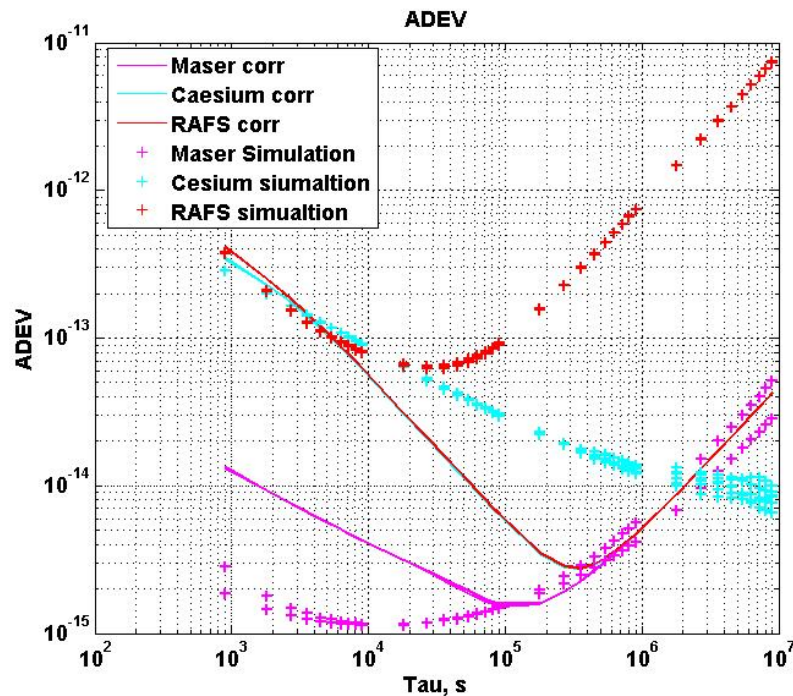


Figure 6. Stability of corrected vs. uncorrected clocks.

### LOSS OF ONE SATELLITE CLOCK

The first test case investigates the impact of a loss of one satellite clock on the Composite Clock. There are different methods to implement such a clock removal. We have chosen to reduce the filter dimension and to remove all rows and columns of the affected clock from the filter state and covariance matrix. The estimate and covariance matrix of the last step before the clock removal are used to reinitialize the reshaped filter.

In order to assess the impact of satellite clock removal, the filter estimate including all clocks is subtracted from the filter estimate with one satellite clock removed. At the time of removal, this difference averages to about 0.15 ps for all clocks (Figure 7).

### LOSS OF ONE GROUND STATION

The second test case analyzes the impact of losing one monitor station (1 AHM plus three Cs clocks) on the computed time offsets. Here, the difference in time offsets averages to about 5.4 ps at the time of removal (Figure 8). The impact is about 30 times higher than the one of the scenario when losing one satellite clock. But it is still at the level of picoseconds.

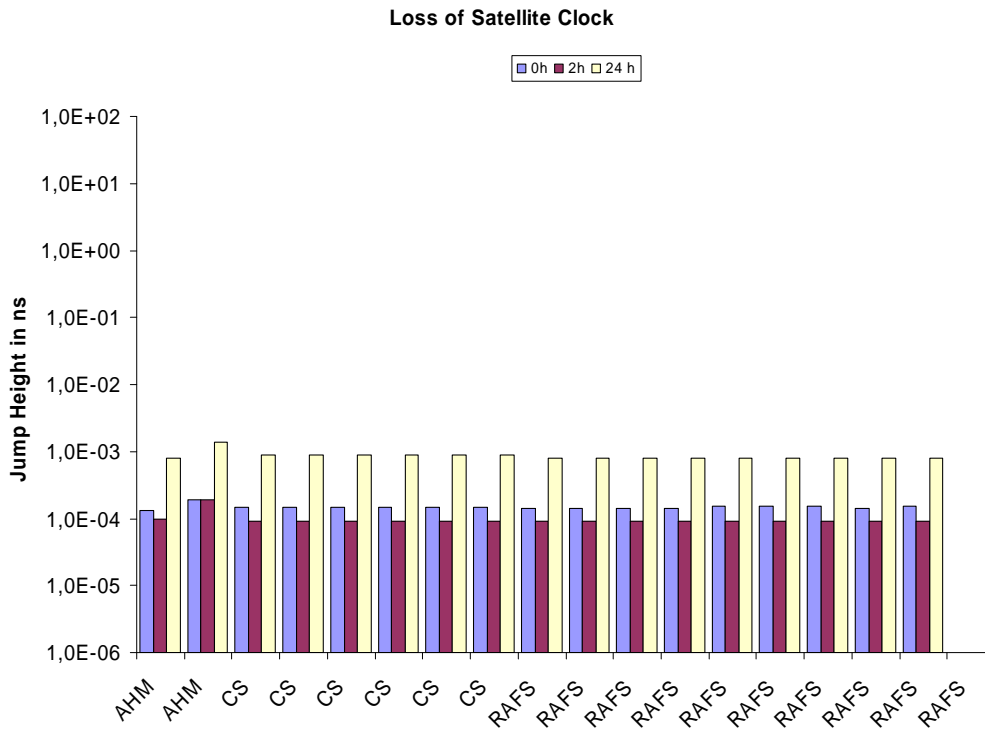


Figure 7. Differences in time offsets after loss of one satellite clock.

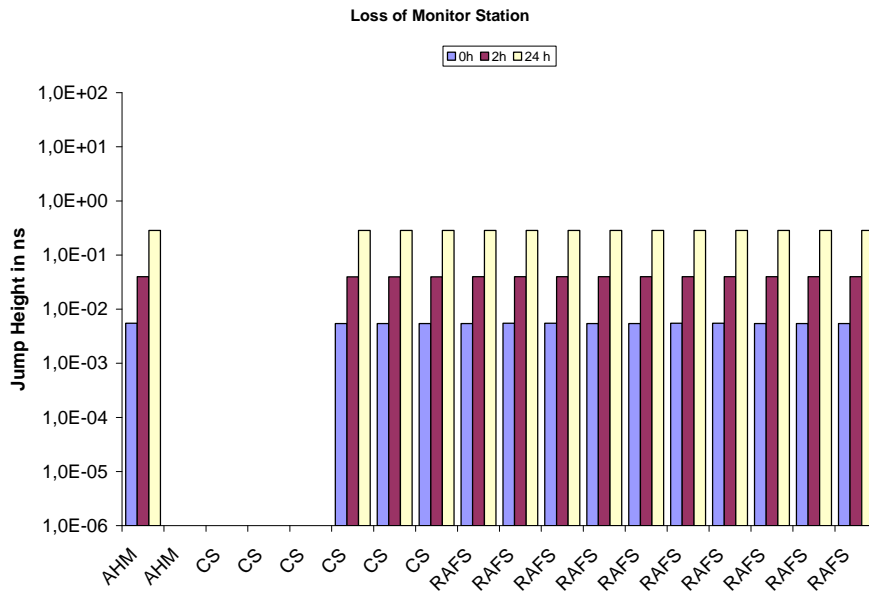


Figure 8. Differences in time offsets after loss of one ground station.

## FREQUENCY STEP OF A SATELLITE CLOCK

The next test cases focus on the impact of clock feared events (in particular, frequency steps) on the Kalman filter estimates. No algorithm modifications for this case were implemented. The measurements disturbed by a clock feared event were processed with the nominal Kalman filter configuration.

First, we simulated a frequency step of  $10^{-11}$  in one of the satellite clocks. Figure 9 compares the filter frequency estimates of the affected rubidium to an unaffected rubidium. Obviously, the filter responds to the frequency step and, after 48 h, its estimates totally reflect the jump size of  $10^{-11}$ . Remarkably, the other clock estimates were not significantly affected.

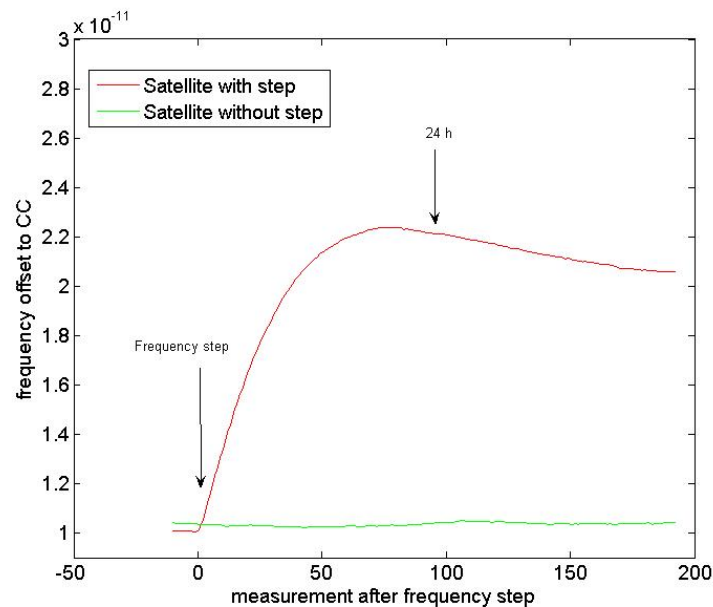


Figure 9. Frequency estimates of two RAFS: with a frequency step and without.

Again, the impact is assessed by differencing the filter estimates without the frequency step to the estimates with the step. The impact at the clocks without the step averages about 1 ps at one measurement step after the event time (see Figure 10).

## FREQUENCY STEP OF AN ACTIVE HYDROGEN MASER

The situation is different if a frequency step happens to a AHM with high weight in the CC algorithm. We analyzed the case when one AHM is disturbed by a simulated step of size  $5 \times 10^{-13}$ .

Both AHMs respond to the clock event and, after a time interval of 24 h, each of the estimates is affected about half of the step size (Figure 11).

However, the filter smooths the event and its output does not respond immediately to the event. The impact of the AHM frequency steps on the filter estimates of all clocks averages to about 10 ps (Figure 12).



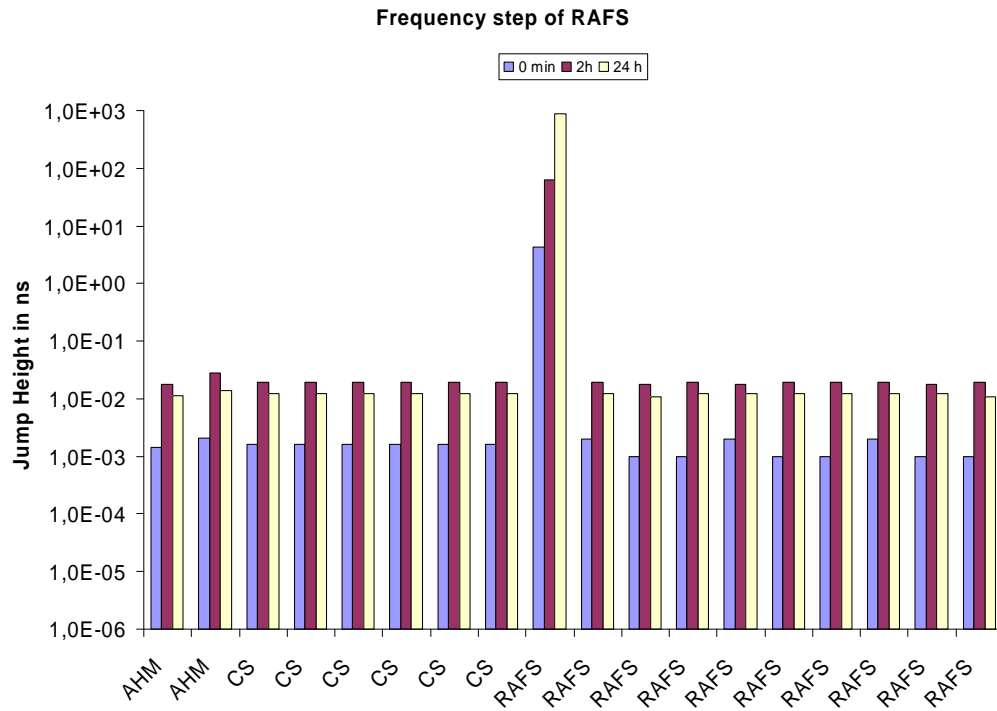


Figure 10. Differences in time offsets after a frequency step applied to a rubidium clock.

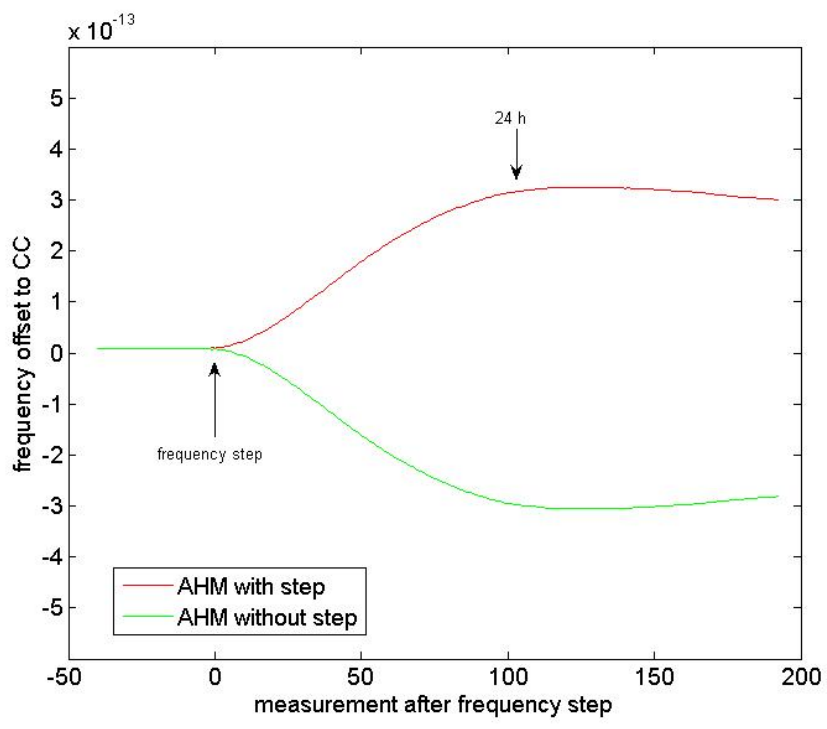


Figure 11. Frequency estimates of both an affected and an unaffected AHM.

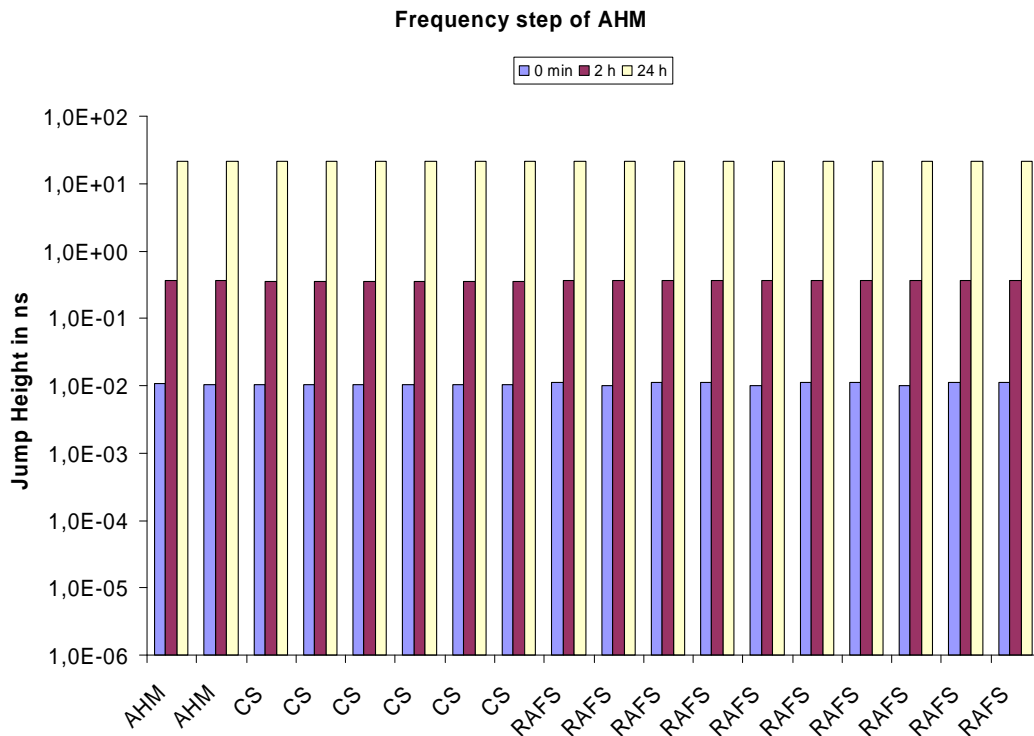


Figure 12. Differences in time offsets after a frequency step of an AHM.

## CONCLUSION

The Composite Clock Algorithm has been analyzed with a simulated GNSS clock constellation of ten Rb-clocks, two AHMs, and six Cs clocks. The choice of clock parameters  $q_1$ ,  $q_2$  and  $q_3$ , in accordance with the clock stability, has put the weights of both AHMs to almost 100%. Consequently, in the selected simulation scenario, two clocks (AHMs) of the ensemble establish System Time and the other clocks only marginally contribute to it.

The evaluation of the filter output points out that the short- and medium-term stability of rubidium and cesium clock offsets with respect to the Composite Clock (implicit mean produced by the Kalman filter) is very close to the stability of these clocks in a free-running mode. On the other hand, the instability of the AHMs-CC offsets is considerably higher compared to a free-running AHM due to instabilities of the CC itself. Furthermore, the instability of the cesium-clocks-CC offset is disturbed in the long term. Hence, the Composite Clock with the used q-parameter configuration is not able to benefit from the long-term stability of the cesium clocks.

Besides applying the algorithm to an error-free clock constellation, the impact of different clock failure scenarios was investigated. First, we tested the scenario when one satellite clock was excluded from the ensemble. The change in time offsets averages about 0.2 ps and, thus, the exclusion will not affect the prediction error of the remaining clocks. Another test case investigated the impact of exclusion of one ground station that includes one AHM and three Cs clocks. In this case, the difference in time offsets increases to about 5 ps at the time of exclusion.

Besides evaluating the impacts of clock exclusions, which was handled by a controlled modification of the ensemble Kalman filter (the filter dimension was resized and the last filter outputs were used to reinitialize the filter), a second failure class was analyzed: the impact of clock feared events. These events disturb the measurement inputs of the filter and are processed by the filter in a regular manner.

The first feared event was a frequency step, which corresponds to a frequency change of 100 times compared to the specified clock stability (ADEV@15 min) at one of the satellite clocks. The simulations outline that the effect of this event at the Composite Clock is non-critical and the change in time offsets of the remaining clocks average 1 ps. Nevertheless, the estimate of the failed satellite clock changes about 4 ns 900 s after the event time and, thus, that prediction error will be distinctly affected.

The results were quite different for the case when a high-weighted AHM was disturbed by a frequency step. Here, every time offset including the one of failed AHM changes in average by about 10 ps at one step after the event time, and thus, the impact at the clock prediction error should be also non-critical. This behavior can be explained by the observation that, first, the Composite Clock reading changes only half of the step size, due to the assigned AHM weight of 50% and, second, half of the step size is distributed by the filter over a period of 24 h. So the change is propagated in a steady fashion to the time offsets.

Further, it was observed that the change in time offsets of the failed AHM was also about 10 ps and its behavior is similar to the remaining clocks. The argued reason for that is that the filter always estimates the difference of every clock to the Composite Clock and, thus, both steps (the one of the AHM and the one of the Composite Clock) difference up and results in half the step size of the corresponding time offset.

## ACKNOWLEDGMENTS

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