Improved Iono PHMI Calculation for SBAS Systems

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ABSTRACT

Besides the wide-area augmentation system (WAAS) in the US, an increasing number of space-based augmentation systems (SBAS) are being developed or planned, such as the European Geostationary Navigation Overlay System (EGNOS) in Europe, the Multi-functional Satellite Augmentation System (MSAS) system in Japan, and the future Ground-based Regional Augmentation System (GRAS) in Australia and the GPS and Geo Augmented Navigation (GAGAN) system covering the Indian subcontinent.

The integrity analysis in a given SBAS system includes calculating the probability of hazardous misleading information (PHMI) for the vertical ionospheric delays. We show that the built-in conservatism in the PHMI treatment that has been proposed for WAAS can be significantly reduced by using a more general model for the ionospheric process noise, *i.e.* we consider generalized models of the process noise, where the process noise is allowed to depend non-linearly on the chi-square statistic of the ionospheric measurement residuals. As a consequence, we obtain a more realistic dependence of PHMI on the ionospheric state, and the additional degrees of freedom of the process noise model can be used to optimize the system for availability under moderately disturbed ionospheric conditions.

INTRODUCTION

Since SBAS systems provide vertical ionospheric delays, the integrity analysis includes the calculation of the probability for hazardous misleading information PHMI for the ionospheric corrections [1].

Especially at the edges of the service volume the leading contribution to the grid ionospheric vertical error (GIVE) comes from mitigating the under-sampling threat [2][3]. Another contribution to the GIVE is the noise estimation.

In this work, we re-consider the part of the iono PHMI calculation which deals with the analysis and separation of the measurement noise from the process noise [1][7][8]. This is important since the user is mostly affected by the process noise, but the measurement noise can be much bigger than the process noise and in some cases can even mask the process noise.

Since the process noise is caused by the ionospheric de-correlation, a model for the ionospheric de-correlation is needed. By analyzing the variogram related to the ionospheric de-correlation [4], it has been found that the process noise for active ionospheric conditions can be modeled as a multiple of the noise for quiet ionospheric conditions,

$$\sigma_{process}^2 = w^2 \sigma_{0,process}^2 \quad (1)$$

where $\sigma_{0,process}$ denotes the process noise for nominal, *i.e.* quiet, ionospheric conditions. The state of the ionosphere is parameterized by the inflation factor w; a value of unity corresponds to nominal ionospheric conditions, values greater than unity indicate active ionospheric conditions. This inflation factor has to be over-bounded using the measurements.

While in previous work w^2 depended *linearly* on the χ^2-statistics of the de-trended and de-correlated fit residuals, here we propose to use a more general (polynomial) dependence of w^2 on the measurements. As a consequence lower values of the overbound of w^2 can be used while meeting the integrity requirements. This translates directly into lower SBAS grid ionospheric vertical errors.

IONOSPHERIC ESTIMATION

The plasma in the ionosphere influences the signals from the GPS satellites on their way to a user. Therefore SBAS systems broadcast ionospheric correction information.

To first order, slant ionospheric delays are proportional to the integral of the electron density along the ray path

$$I_S \propto f^{-2} \int_{\Gamma} n_e \mathrm{d}s \quad (2)$$

I_S Slant ionospheric delay
n_e Electron density
Γ Path from satellite to receiver
$\mathrm{d}s$ Line element
f Frequency of transmitted signal

The ionosphere is approximated by a single shell at a fixed height. Then the slant delays I_S are related to vertical ionospheric delays I_V by the so-called geometry function

$$I_S = G(elev) \cdot I_V \quad (3)$$

I_S Slant ionospheric delay [m]
I_V Vertical ionospheric delay [m]

defined as

$$G(elev) = \left(1 - \left(\frac{r_E \cos(elev)}{r_E + h_{Iono}}\right)^2\right)^{-1/2} \quad (4)$$

r_E Earth’s radius: 6371 km
h_{iono} Height of single layer ionosphere: 400 km
$elev$ Satellite elevation [rad]

SBAS systems provide vertical ionospheric delays at fixed grid points. These delays are computed from calibrated total electron content measurements of a network of reference stations. Since these measurements are in general not located at the fixed grid points, one way to obtain corrections at grid points is to perform a planar fit to the ionosphere around a given grid point using Kriging [1]. The model for the measurements is given by

$$I_V(x) = a_0 + a_1 x^{(east)} + a_2 x^{(north)} + r(x) + m(x) \quad (5)$$

x Position relative to the grid point
a_0, a_1, a_2 Fit parameters
$x^{(east)}$ East-component of distance
$x^{(north)}$ North-component of distance
$r(x)$ Process noise
$m(x)$ Measurement noise

By computing the variogram of $r(x)$ for different ionospheric conditions, it has been found that the fit residuals can be described by a random field with covariance

$$Cov(r(x_i), r(x_j)) = w^2 C_0(|x_i - x_j|) \quad (6)$$

where C_0 denotes the covariance under quiet ionospheric conditions:

$$C_0(d) = c\, e^{-d/a} \quad (7)$$

with parameters $c = 2\,\mathrm{m}^2$ and $a = 32000\,\mathrm{km}$ for the CONUS region [8]. For quiet ionospheric conditions the parameter w is 1; disturbed ionospheric conditions are modeled by $w > 1$: the de-correlation during storm times is a multiple of the de-correlation during quiet times.

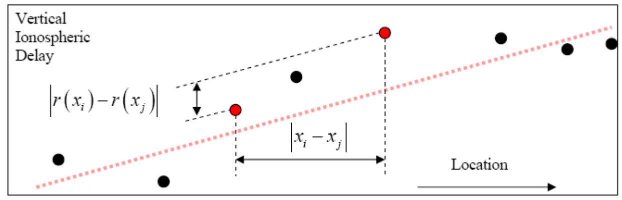

Fig 1
Vertical ionospheric delays are computed by performing a planar fit to the measurements.

When estimating the parameters (a_0, a_1, a_2) it is important to carefully separate process noise which is caused by ionospheric de-correlation from the measurement noise, since the measurement noise can mask the process noise but the user is most affected by the process noise.

PROBABILITY OF HAZARDOUS MISLEADING INFORMATION

The probability of hazardous misleading information is defined as the probability that the estimated ionospheric delay differs from the true ionospheric delay more than the error Δ provided by the SBAS system [1]:

$$P(HMI) = P(|I_{true} - I_{est}| > \Delta) \quad (8)$$

I_{true} True ionospheric delay
I_{est} Estimated ionospheric delay
Δ Estimated error bound

The probability allocated for ionospheric hazardous misleading information is 2.25×10^{-8} [5]. Thus we have to minimize Δ such that

$$P(HMI) < 2.25 \times 10^{-8} \quad (9)$$

As the covariance (6) used in the parameter estimation depends on the unknown parameter w, the following decomposition of PHMI is used

$$P(HMI) = \int_0^{\infty} P(HMI \mid w)\, p(w) \mathrm{d}u \quad (10)$$

where $p(w)$ is some probability density. Since $p(w)$ is in general unknown, we demand that

$$P(HMI \mid w) < 2.25 \times 10^{-8} \text{ for any } w > 0 \qquad (11)$$

As a starting point for the $P(HMI)$ analysis we using the expression for $P(HMI \mid w)$ derived, *e.g.*, in Eq. (4.25) in Ref. [8]. There $P(HMI \mid w)$ is given as a multi-dimensional integral with a Gaussian measure over the de-trended and de-correlated measurements:

$$P(HMI \mid w) = \int_{R^N} \mathrm{d}^N y \; 2Q\left(K \frac{w_0(y)}{w} \right) \times \frac{\exp\left\{ -\frac{1}{2} y^T S(w)^{-1} y \right\}}{(2\pi)^{N/2} |S(w)|^{1/2}} \qquad (12)$$

y	de-trended and de-correlated measurements
N	Number of measurements - 3
$w_0(y)$	Inflation factor
$S(w)$	Covariance of y
K	K-value, $K = 5.592$
$Q(x)$	Cumulative distribution function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \mathrm{d}t \, e^{-t^2/2}$$

Since three parameters a_0, a_1, a_2 are estimated there are $N =$ Number of measurements – 3 independent fit residuals, *cf.* Section 4.2.2 in [8].

After a change of variables

$$z = S(w)^{-1/2} y \qquad (13)$$

we obtain

$$P(HMI \mid u) = \int_{R^N} \mathrm{d}^N z \, 2Q\left(\frac{K \, w_0\left(S(u)^{1/2} z\right)}{w} \right) \times (2\pi)^{-N/2} \exp\left\{ -\frac{1}{2} z^T z \right\} \qquad (14)$$

In order to evaluate this integral we have to use a concrete representation of $S(u)$ and of the inflation factor $w_0(y)$. For the covariance of the reduced measurements we use

$$S(w) = s^2 1_N, \quad s^2 = \beta w^2 + (1 - \beta) \qquad (15)$$

where $\beta \in [0,1]$ determines the amount of measurement and process noise, the extreme cases being $\beta = 0$ for pure measurement noise, and $\beta = 1$ for pure process noise. Fig 1 shows a typical distribution of β determined from a WAAS-like system.

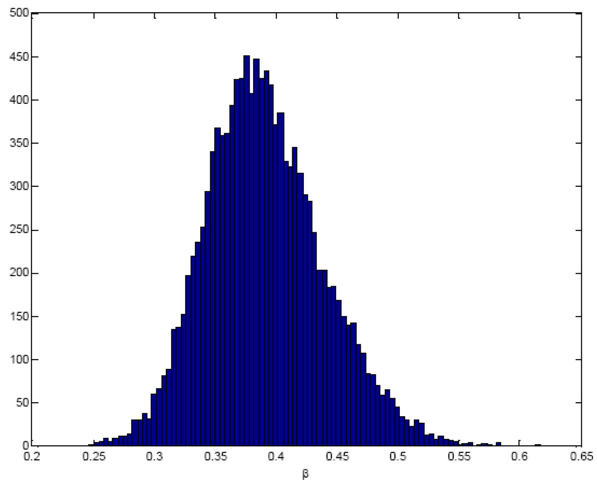

Fig 1
Distribution of process noise for a WAAS-like system.

Note that here we assume $S(w)$ to be proportional to the unit matrix. We leave the generalization of our results to a non-uniform $S(w)$ for future work.

For the inflation factor w_0 we consider polynomials of a positive definite quadratic form of the reduced measurements y :

$$w_0^2(y) = p\left(y^T R y\right) \qquad (16)$$

$p(x)$	Polynomial
R	Positive definite quadratic form

with

$$R = \alpha \mathbf{1}_N, \quad \alpha > 0 \qquad (17)$$

where $\mathbf{1}_N$ denotes the $N \times N$ unit matrix. While in [8] a *linear* polynomial, $p(x) = x$, was used, here we use polynomials of higher degree.

Since both matrices, S and R, are proportional to $\mathbf{1}_N$ the integrand depends on the radial coordinate $z = \sqrt{z^T z}$ only. Therefore, when switching to angular coordinates in z the angular integration can be performed right away; it is equal to the surface of the N-dimensional unit hypersphere. Hence, we are left with the following one-dimensional integral over the radial coordinate:

$$P(HMI \mid w) = \int_0^\infty \mathrm{d}z \, z^{N-1} \, 2\mathrm{Q}\left(\frac{K\sqrt{\mathrm{p}\left(\alpha s^2 z^2\right)}}{\mathrm{w}} \right) \times (2\pi)^{N/2} \exp\left\{ -\frac{1}{2} z^2 \right\} \frac{2\pi^{N/2}}{\Gamma(N/2)} \qquad (18)$$

This integral converges if the highest power of the polynomial $p(x)$ has a positive coefficient and can be evaluated numerically.

Given any polynomial $p(x)$ we are now in a position to evaluate $P(HMI \mid w)$ by numerical integration (18). By interval bisection, *e.g.*, we can then search in α for the critical value α_C depending on β and N, such that the requirement $PHMI < 2.25 \times 10^{-8}$ is fulfilled.

Since we use a polynomial for the inflation factor, there are additional degrees of freedom. Therefore we need to be able to assess the performance of a given polynomial. As a measure of performance we compute the 99% quantile of the inflation factor, w_c, under nominal conditions, *i.e.* when the N reduced measurements y are distributed according to a χ^2 distribution with N degrees of freedom. In the following we use a typical value $N = 30$.

Linear Polynomial

As a reference, first we use the linear polynomial

$$p(x) = x \qquad (19)$$

which has been used in [8]. An example of a $P(HMI \mid w)$ curve for a typical value of β is shown in Fig 2. For different values of β between 0.2 and 1.0, the 99% quantile of the inflation factor, w_c^2, ranges from 12.84 to 4.28, *cf.* Table 1. The values of α calculated by integrating (18) with a linear polynomial match the value of α found when using the determinant-like expression derived in (4.28) in Ref. [8].

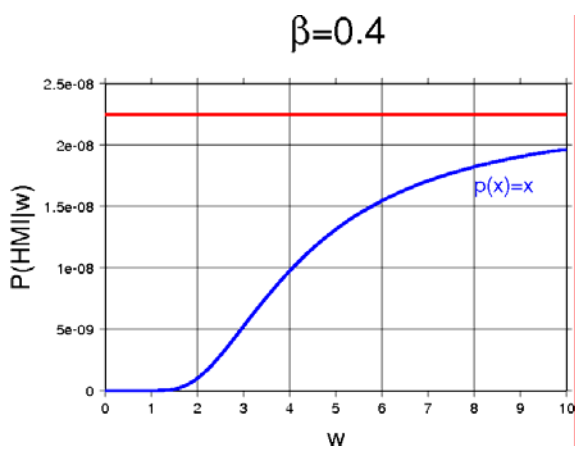

Fig 2

The bold blue curve is $P(HMI \mid w)$ for a linear polynomial. It approaches the limiting value at infinity; the red line indicates the iono $P(HMI)$ requirement.

Quintic Polynomial

Let us now consider a family of quintic polynomials

$$p(x) = x\left(1 - 2\gamma x + 2(\gamma x)^2 - (\gamma x)^3 + \frac{1}{5}(\gamma x)^4\right) \qquad (20)$$

parameterized by γ. Its first derivative is $(1-\gamma x)^4$. Clearly, when $\gamma = 0$ the linear polynomial (19) is recovered. This family of polynomials has been found to give the best results. However, an exhaustive search for the optimal polynomial remains to be done. For each parameter γ we obtain a different $P(HMI \mid w)$ curve and different values of w_c and α_C. As can be seen in Fig 3, the $P(HMI \mid u)$ curve for $\gamma \neq 0$ is qualitatively different from the curve for $\gamma = 0$: it has a maximum at finite values of w. We then search for a critical value of γ_C such that the reduction from $w_c\big|_{\gamma=0}$ to $w_c\big|_{\gamma=\gamma_c}$ becomes maximal. The resulting parameters are summarized in Table 1; for a typical value of $\beta = 0.4$ the achievable reduction is about 20%.

β	α_C	γ_C	$w_c^2\big\|_{\gamma=0}$	$w_c^2\big\|_{\gamma=\gamma_c}$	%
0.2	1.68	0.020	12.84	11.79	23
0.3	1.14	0.030	8.56	8.19	20
0.4	0.89	0.035	6.42	6.10	20
0.5	0.72	0.045	5.13	4.99	19
0.6	0.62	0.050	4.28	4.25	17
1.0	0.40	0.080	3.06	2.78	9

Table 1

Critical values for α and γ for different ratios of measurement to process noise parameterized by β for the polynomial (20) and $N = 30$. The last column displays the achieved reduction in the inflation factor relative to the method using a linear polynomial.

Fig 4 displays two optimal inflation factor functions for $\beta = 0.4$ as a function of χ^2, using $w_0^2(y) = p(\alpha\chi^2)$, *cf.* (16). Since at the evaluation point the quintic polynomial (in dark green) is lower than the linear polynomial (in blue), a reduction of the inflation factor is achieved. Note that the quintic polynomial function displays a threshold-like behavior: it is nearly constant for a range of χ^2 values and then grows rapidly.

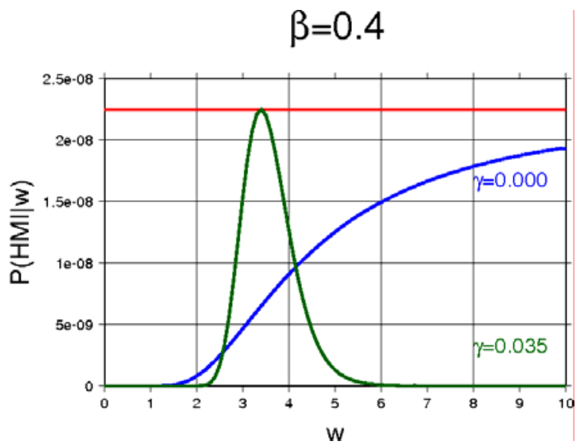

Fig 3

The dark green curve shows $P(HMI \mid w)$ determined according to (18) for the quintic polynomial (20) using $N = 30$, $K = 5.592$, $\gamma = 0.035$, and $\alpha = 0.89$; the blue curve is $P(HMI \mid w)$ for a linear polynomial; the red line indicates the iono $P(HMI)$ requirement.

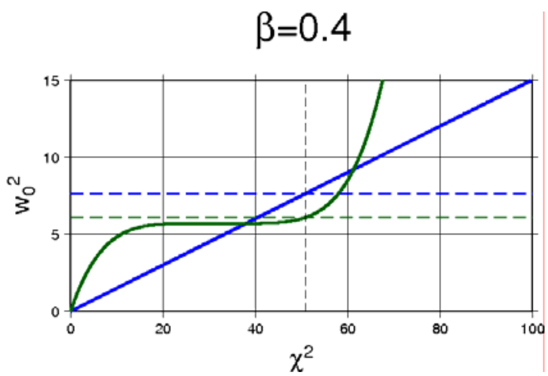

Fig 4

Shown is the inflation factor w_0^2 depending on χ^2. The blue curve corresponds to a linear polynomial, the dark green curve corresponds to the quintic polynomial (20) with $\gamma = 0.035$. The vertical dashed line denotes the evaluation point at $\mathrm{chi2inv}(30, 0.99)$.

CONCLUSIONS AND OUTLOOK

By using the generalized models for the ionospheric inflation factor proposed in this work, SBAS systems can be made to operate less conservatively, thus increasing their robustness and achievable availability.

When using a non-linear polynomial in a WAAS-like SBAS system, the lookup table $\alpha(N)$, which for a linear polynomial is one-dimensional, has to be replaced by two lookup tables: one table $\alpha(N,\beta)$ and another table $\gamma(N,\beta)$, both depending on N and on β. The second and third columns of Table 1 display $\alpha(30,\beta)$ and $\gamma(30,\beta)$ for the polynomial given in (20).

As a measure of performance we have computed the 99% quantile of the distribution of the inflation factor for nominal measurements. Depending on the ratio between measurement noise and process noise a reduction of ca. 20% in inflation factor is possible by using non-linear polynomials for the inflation factor.

These benefits should be most interesting for those SBAS systems operating in the low geographic latitude area, where the active ionospheric conditions will likely make these systems operate near the design threshold to unavailability.

Our analysis was performed for covariance matrices $S(w)$ which are a multiple of the identity matrix. In future, we will extend the results to the more realistic case of non-uniform covariance matrices not proportional to the identity matrix. Also a more systematic search for an optimal function $p(x)$ in the space of all polynomial (or even peacewise-defined) functions remains to be performed.

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