Fast analytical two-stream radiative transfer methods for horizontally homogeneous vegetation media

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Summary

This work reports on the two-stream radiative transfer in a horizontally homogeneous turbid vegetation medium assuming bi-Lambertian leaf scattering by planar model leaves. The deduced two-stream radiative transport equations are solved analytically for various leaf architectures considering leaf normal distribution (LND) functions from purely vertical to purely horizontal leaves. These transfer models are driven by radiative energy flux densities (EFDs) incident at the top of the vegetation (TOV) and separated into their diffuse and direct fractions, used as upper boundary conditions. The radiance field is treated as approximately isotropic, but its zenithal distribution can be varied by the so-called diffusivity factor, which allows together with the incident EFDs at TOV to take the sky conditions above the canopy into account. Simulations of the canopy reflectance and transmittance are performed in the UV/VIS as function of the solar zenith angle and the ratio of the direct and diffuse sky light above the canopy representing clear and overcast sky conditions. These computations demonstrate that the radiative transfer in vegetation is significantly influenced by this ratio and the LND of the leaves.

Zusammenfassung

Diese Arbeit berichtet über den Zweistrom-Strahlungstransfer eines horizontal homogenen und trüben Vegetationsmediums, wobei bi-Lambertsche Streuung an als eben betrachteten Modellblätter angenommen wird. Die abgeleiteten Zweistrom-Strahlungstransportgleichungen werden analytisch gelöst für verschiedene Blattarchitekturen, indem Blattnormalenverteilungsfunktionen (BNV) von rein vertikalen bis rein horizontale Blätter berücksichtigt werden. Diese Transfermodelle werden durch Strahlungsflussdichten angetrieben, die am Oberrand der Vegetation gegeben und in deren Direkt- und Diffusanteil aufgespalten sind, wobei letztere als obere Randbedingungen fungieren. Das Strahldichtefeld wird als nahezu isotrop angenommen, jedoch kann dessen Zenitwinkelabhängigkeit durch den sogenannten Diffusivitätsfaktor variiert werden, der es zusammen mit den am Oberrand einfallenden Strahlungsflussdichten erlaubt, unterschiedliche Bedingungen für das atmosphärische Himmelslicht zu berücksichtigen. Es wurden Simulationen der Reflektanz und Transmittanz des Gesamtmediums im UV/VIS als Funktion des Sonnenzenitwinkels sowie des Verhältnisses des Direkt- und Diffuslichts durchgeführt, stellvertretend für klare Tage und bewölkte Situationen. Diese Berechnungen zeigen, dass der Strahlungstransfer in Vegetation deutlich von diesem Verhältnis sowie von der BNV der Blätter abhängt.

1 Introduction

Extinction processes within vegetation take place on macroscopic leaves having sizes much larger than the wavelength of the transported radiation, in contrast to e.g. aerosols in the atmosphere. Therefore, the size of leaves can not be neglected in comparison to the scale height of a vegetation stand, and the radiative transfer depends on the geometrical structure of the canopy elements like the leaves' shape, area, orientation, number and

their spatial distribution as well as their optical properties. Ross (1981) and Myneni et al. (1989) gave an introduction to the radiative transfer within turbid vegetation media, which was the basis for the present work.
As one can imagine, vegetation is an irregular and spatially heterogeneous medium changing in time with shaded sites and gaps in the canopy. Nevertheless, several researchers assumed canopies to be horizontally homogeneous and treated two-stream approaches of the radiative transfer. Dickinson (1983) and Dickinson et al. (1987) introduced two-stream methods and benchmarked their accuracy within an error of 5 % in comparison to multi-stream models. Dickinson et al. (1990) suggested the two-stream transfer of radiation for modelling the climate of the Earth. Sellers (1985) used the two-stream approach to simulate reflectance, photosynthesis and transpiration of a vegetation canopy. Joseph et al. (1996) discussed the two-stream approximation for parameterisations of the solar radiation transport through vegetation as function of special leaf normal distributions (LNDs). Recent works deal, e.g., with the application of two-stream methods for simulating photosynthesis, stomatal conductance and leaf temperatures as well as the exchange of CO_2 and water vapour within canopies (Dai et al., 2004), or they were used in remote sensing techniques for climate modelling (Pinty et al., 2006, 2007).
In most of the works cited above standard analytical LNDs were considered (e.g. derived by fitting measurements) for which, however, the according G-functions (Ross-Nilson functions) were not calculated analytically in the most of the cases of LNDs. Therefore these were approximated (Goudriaan, 1977; Dickinson et al., 1990; de Ridder, 1997). Since the G-function determines the extinction coefficient of a vegetation medium, it plays the decisive role in the light transport. The recent work will present analytical expressions of G-functions for a big number of commonly used LNDs as well as analytical solutions of the two-stream radiative transfer for these standard LNDs from purely vertical to purely horizontal leaves.
Section 2 will give an overview on the radiative transfer in a horizontally homogeneous and time independent vegetation medium and will present the two-stream approximated transport equations and their analytical solutions. Sensitivity studies will then be discussed in section 3 with respect to the transport of diffuse radiation through deciduous canopies. Finally, the work is summarised in Section 4.

2 Radiative transfer in horizontally homogeneous turbid vegetation media and its two-stream solution

In radiative transport theories generally radiation transfer quantities (RTQs) are defined to describe the radiative interactions and, thus, the radiation field. These are: photon distribution function f_p, absorption density function k_a, scattering density function k_s, scattering function f_s, extinction density function k_e, emission density function f_e and phase function P. The independent variables usually are $(\mathbf{x}, \mathbf{y}, \mathbf{a} \,|\, \mathbf{b}, \lambda, t)$. The vector $\mathbf{x}$ is the location in space, $\mathbf{y}$ the direction of the radiation starting in $\mathbf{x}$, $\mathbf{a}$ the incident and $\mathbf{b}$ the exit direction of scattered radiation, λ the wavelength and t the time. A RTQ f is named isotropic if $f \neq f(\mathbf{y})$, homogeneous if $f \neq f(\mathbf{x})$ and spectral if $f = f(\lambda)$.
From the photon distribution function the radiance field I can be deduced by $I = ch\frac{c}{\lambda} f_p$, and from this RTQ the spectral *energy flux density* (EFD) or *irradiance*,

$$E(\mathbf{x}, \lambda, t) = \int_{S_1} I(\mathbf{x}, \mathbf{y}, \lambda, t) \frac{y_3}{\|\mathbf{y}\|} \, do(\mathbf{y}) = \left[\frac{\mathrm{W}}{\mathrm{m}^2 \, \mathrm{nm}} \right], \tag{1}$$

through a horizontal plane at $\mathbf{x}$, where y_3 is the third (vertical) component of the radiation direction $\mathbf{y}$, as well as the spectral *actinic flux density* (AFD),

$$A(\mathbf{x},\lambda,t) = \int\limits_{S_1} I(\mathbf{x},\mathbf{y},\lambda,t)\,do(\mathbf{y}) = \left[\frac{\mathrm{W}}{\mathrm{m}^2\,\mathrm{nm}}\right], \tag{2}$$

are calculated. S_1 is the unit sphere expressing the so-called integration over all solid angles. In this sense the unit sr^{-1} has to be replaced by the unit area m^{-2} so that $I = \left[\frac{\mathrm{W}}{\mathrm{m}^2\,\mathrm{nm}\,\mathrm{m}^2}\right]$ instead of $I = \left[\frac{\mathrm{W}}{\mathrm{m}^2\,\mathrm{nm}\,\mathrm{sr}}\right]$. Note that the solid angle Ω of an arbitrary surface F, intersecting any ray through the origin only once, can be defined (Fischer and Kaul, 1997) as its projection $\tilde{F}$ on S_1 leading to the number

$$\Omega(F) := A(\tilde{F}) \le 4\pi F_o$$

where $A(\cdot)$ means the area of a surface and $F_o = 1\,\mathrm{m}^2$ is the unit area. Thus, the integration over all solid angles means a surface integration over S_1 contributing the unit m^2 to the integrand. The unit sphere can be parameterised by $S_1 = \{\boldsymbol{\omega}\}$ using the vector

$$\boldsymbol{\omega}(\vartheta,\varphi) := C_o \begin{pmatrix} \cos\varphi\,\sin\vartheta \\ \sin\varphi\,\sin\vartheta \\ \cos\vartheta \end{pmatrix} \tag{3}$$

of the radiation direction with the spherical coordinates $(\vartheta,\varphi) \in [0,\pi]\times[0,2\pi]$ and the unit length $C_o = 1\,\mathrm{m}$. The parameterisation can be transformed using the so-called μ-transformation according to the map $\vartheta = h_\mu^{-1}(\mu) := \cos^{-1}(\mu)$, which leads to

$$\boldsymbol{\omega}(h_\mu^{-1}(\mu),\varphi) := C_o \begin{pmatrix} \cos\varphi\,\sqrt{1-\mu^2} \\ \sin\varphi\,\sqrt{1-\mu^2} \\ \mu \end{pmatrix} := \hat{\boldsymbol{\omega}}(\mu,\varphi) \tag{4}$$

with $\mu \in [-1,1]$ and $S_1 = \{\boldsymbol{\omega}\} \equiv \{\hat{\boldsymbol{\omega}}\}$. Then, the unit vector of the direct light can be written as $\mathbf{e}_D = \hat{\boldsymbol{\omega}}(\mu_D,\varphi_D)$ with the angles (μ_D,φ_D) of the direct beam where, especially, $\mu_D = \frac{1}{C_o}\langle\mathbf{e}_3,\mathbf{e}_D\rangle = \cos\vartheta_D < 0$ is the cosine of the solar zenith angle $\vartheta_D > \frac{\pi}{2}$ and $\langle\cdot,\cdot\rangle$ is the scalar product. With the help of (3) and (4) the upper and lower half space unit sphere $S_1^\pm$ can be parameterised by

$$S_1^\pm = \{\hat{\boldsymbol{\omega}}(\mu,\varphi) : (\mu,\varphi) \in [0,\pm 1]\times[0,2\pi]\}$$

and their surface areas are $A(S_1^\pm) = 2\pi F_o$.

To define the RTQs in vegetation a basic function is required: The dimensionless *leaf normal distribution function* (LND) $g_L(\mathbf{x},\mathbf{y}_L,t) \ge 0$ is a measure for the probability that the normal vector $\mathbf{y}_L$ of a planar leaf at $\mathbf{x}$ and time t is confined to the upper hemisphere. As probability density g_L is normalised with respect to S_1^+ by

$$1 = \frac{1}{A(S_1^+)}\int\limits_{S_1^+} g_L(\mathbf{x},\mathbf{y}_L,t)\,do(\mathbf{y}_L) = \frac{1}{2\pi}\int\limits_{[0,\frac{\pi}{2}]}\int\limits_{[0,2\pi]} g_L(\mathbf{x},\boldsymbol{\omega}(\vartheta_L,\varphi_L),t)\sin\vartheta_L\,d\varphi_L\,d\vartheta_L$$

to conserve the total probability of 1 $\forall(\mathbf{x},t)$. The LND is assumed to be homogeneous and independent of time, and its angular dependence is separated according to

$$g_L(\mathbf{y}_L{=}\boldsymbol{\omega}(\vartheta_L,\varphi_L)) = g_\vartheta(\vartheta_L)\,g_\varphi(\varphi_L) \qquad\qquad g_L(\mathbf{y}_L{=}\hat{\boldsymbol{\omega}}(\mu_L,\varphi_L)) = g_\mu(\mu_L)\,g_\varphi(\varphi_L)$$

in terms of the azimuth angle φ_L and the zenith angle ϑ_L of the leaf normals as well as its cosine $\mu_L = \cos\vartheta_L$. This introduction of the LND is common, see e.g. Ross (1981),

Table 1: Commonly used standard LNDs as function of ϑ_L and μ_L arranged in descending order of increasing fractions of horizontal leaves (vertical leaf normals) together with the according GFs after equation (6) as function of μ, which are drawn in Figure 1.

Notation	$g_\vartheta(\vartheta_L)$	$g_\mu(\mu_L)$	$G_\mu(\mu)$
horizontal	$\frac{1}{\sin\vartheta_L}\,\delta(\vartheta_L - \frac{\pi}{2})$	$\delta(\mu_L - 0)$	$\frac{2}{\pi}\sqrt{1-\mu^2}$
erectophile	$\frac{3}{4}(1-\cos 2\vartheta_L)$	$\frac{3}{2}(1-\mu_L^2)$	$\frac{3}{16}(3-\mu^2)$
extremophile	$\frac{15}{14}(1+\cos 4\vartheta_L)$	$\frac{15}{7}(4\mu_L^4 - 4\mu_L^2 + 1)$	$\frac{5}{28}(3-\mu^4)$
uniform	1	1	$\frac{1}{2}$
plagiophile	$\frac{15}{16}(1-\cos 4\vartheta_L)$	$\frac{15}{2}\mu_L^2(1-\mu_L^2)$	$\frac{5}{32}(3+\mu^4)$
spherical	$2\cos\vartheta_L$	$2\,\mu_L$	$\frac{4}{3\pi}\left[\frac{\pi}{2}\lvert\mu\rvert - \lvert\mu\rvert\cos^{-1}(\lvert\mu\rvert) + \sqrt{1-\mu^2}\right]$
planophile	$\frac{3}{2}(1+\cos 2\vartheta_L)\,^*$	$3\,\mu_L^2$	$\frac{3}{8}(1+\mu^2)$
spherical-3	$4\cos^3\vartheta_L$	$4\,\mu_L^3$	$\frac{8}{5\pi}\left[\frac{\pi}{2}\lvert\mu\rvert - \lvert\mu\rvert\cos^{-1}(\lvert\mu\rvert) + \frac{1}{3}\sqrt{1-\mu^2}\,(2+\mu^2)\right]$
spherical-4	$5\cos^4\vartheta_L$	$5\,\mu_L^4$	$\frac{5}{48}(3+6\mu^2-\mu^4)$
spherical-5	$6\cos^5\vartheta_L$	$6\,\mu_L^5$	$\frac{12}{7\pi}\left[\frac{\pi}{2}\lvert\mu\rvert - \lvert\mu\rvert\cos^{-1}(\lvert\mu\rvert) + \frac{2}{15}\sqrt{1-\mu^2}\,(4+\frac{9}{2}\mu^2-\mu^4)\right]$
spherical-6	$7\cos^6\vartheta_L$	$7\,\mu_L^6$	$\frac{7}{128}(5+15\mu^2-5\mu^4+\mu^6)$
vertical	$\frac{1}{\sin\vartheta_L}\,\delta(\vartheta_L - 0)$	$\delta(\mu_L - 1)$	$\lvert\mu\rvert$

* $= 3\cos^2\vartheta_L$ meaning *spherical-2*

Gerstl and Zardecki (1985), Myneni et al. (1987), Shultis and Myneni (1988), Joseph et al. (1996) and Dai and Sun (2006). Then, let the LND be distributed uniformly with respect to φ_L as expressed by $g_\varphi(\varphi_L) := 1$ leading to

$$\int\limits_{[0,\frac{\pi}{2}]} g_\vartheta(\vartheta_L)\sin\vartheta_L\,d\vartheta_L = \int\limits_{[0,1]} g_\mu(\mu_L)\,d\mu_L = 1$$

where it can be shown that $g_\mu(\mu_L) = g_\vartheta(\cos^{-1}(\mu_L))$. This assumption is also usually made by, e.g., Nilson (1971), Gerstl and Zardecki (1985), Shultis and Myneni (1988) as well as Dai and Sun (2006). Then idealised LNDs are defined: purely *horizontal* and *vertical* leaf normals, the *planophile* (mostly vertical normals), *erectophile* (mainly horizontally oriented normals), *extremophile* (showing maxima for both vertical and horizontal normals) and the *plagiophile* LND (has its maximum for leaf normals with a zenith angle of $\vartheta_L = 45°$ in contrast to the extremophile LND). In the case of a *uniform* LND all leaf orientations are equally probable. Table 1 presents analytical expressions for these LNDs, and the remaining LNDs *spherical(-1)*, *spherical-2* to *sphercial-6* (interpreting the *planophile* distribution as *spherical-2*) of rather horizontal leaves were introduced as prototype LNDs which with increasing exponent n in the terms $(n+1)\cos^n\vartheta_L$ of these *spherical-n* LNDs progressively approach to the purely vertical LND.

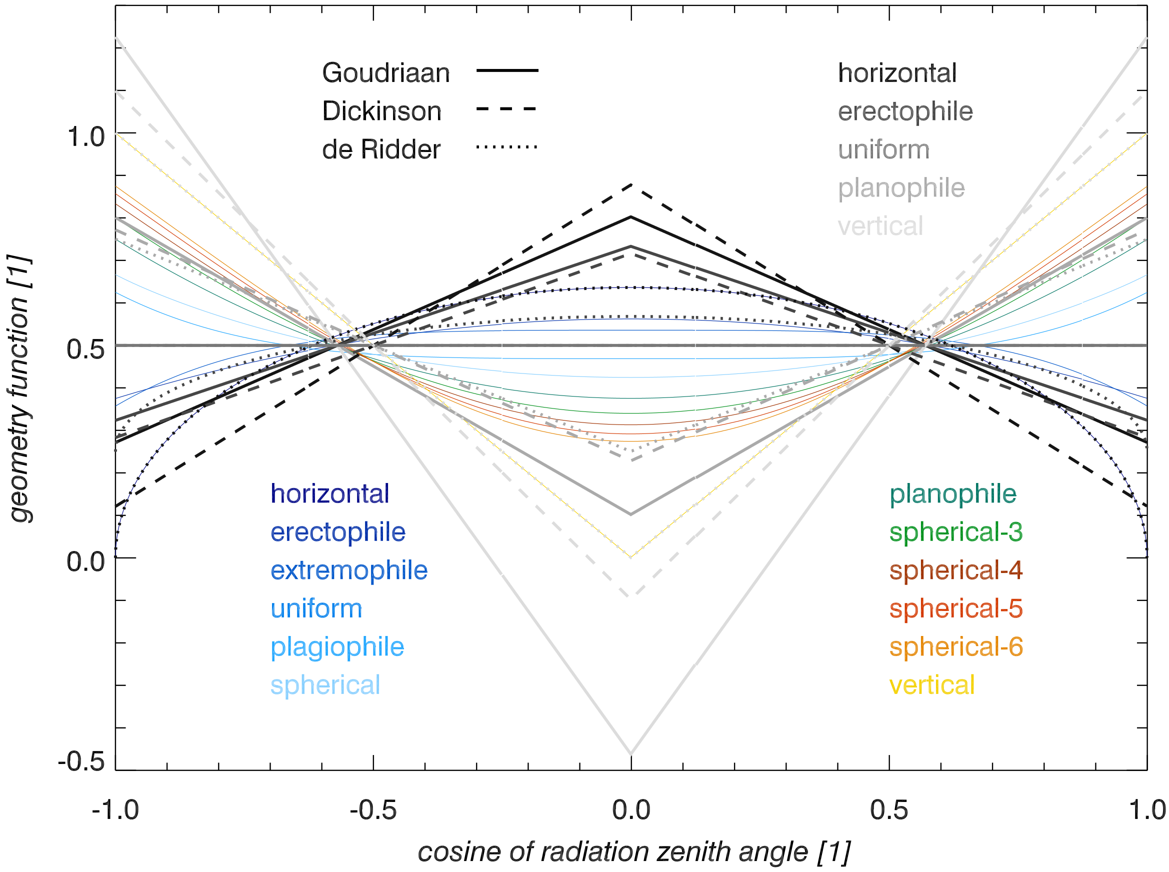

Figure 1: Coloured: Exact analytical expressions of the GFs after equation (6) as function of μ for the commonly used standard LNDs of Table 1. Black to light-grey: Goudriaan's (solid), Dickinson's (dashed) and de Ridder's (dotted) parameterisations of G_μ.

Using the LND $g_L \neq f(\mathbf{x}, t)$ the *geometry function* G (Ross-Nilson function) is defined by

$$G(\mathbf{y}) := \frac{1}{A(S_1^+)} \int_{S_1^+} |\, \mathbf{e_y} \cdot \mathbf{e}_{\mathbf{y}_L} | \, g_L(\mathbf{y}_L) \, do(\mathbf{y}_L) \tag{5}$$

with the unit vector $\mathbf{e_z} := \frac{\mathbf{z}}{\|\mathbf{z}\|}$ in direction of $\mathbf{z} \in \mathbb{R}^3$. The G-function (GF) is dimensionless as well as positive. It is the total probability that the leaves are oriented perpendicular to the radiation direction $\mathbf{y}$ and, thus, a measure for extinction. Using the parameterisations (4) one can demonstrate that the GF,

$$G(\mathbf{y} = \hat{\boldsymbol{\omega}}(\mu, \varphi)) =: G_\mu(\mu) \,, \tag{6}$$

is independent of the azimuth angle φ of the radiation field for all LNDs of Table 1 which also presents the according formulas of $G_\mu(\mu)$. Note that $G_\mu(\mu)$ can simply be deduced for delta distributions as LND or in the uniform case (Nilson, 1971; Gerstl and Zardecki, 1985; Verstraete, 1987; Shultis and Myneni, 1988; Pinty et al., 2006, e.g.), but, in all other cases the $|\, \mathbf{e_y} \cdot \mathbf{e}_{\mathbf{y}_L} |$ term under the integral sign in (5) leads to rather complicated expressions, so that $G_\mu(\mu)$ usually had to be computed numerically (Myneni et al., 1987, 1988b,d; Joseph et al., 1996; de Ridder, 1997; Pinty and Verstraete, 1998), or the *vertical* and *horizontal* LNDs were combined to approximate more realistic LNDs (Gerstl and Zardecki, 1985; Simmer and Gerstl, 1985). Further, analytical expressions for G were

approximated from measurement data (Goudriaan, 1977; Dickinson et al., 1990) and applied widely (Sellers, 1985; Kimes et al., 1987; Dickinson et al., 1987; Pinty et al., 1990; Liangrocapart and Petrou, 2002; Dai et al., 2004). The approach of de Ridder (1997) improved these approximations (Figure 1), which is exact for the *vertical*, *horizontal* and *uniform* LND.
Here, we present the exact analytical expressions of the GF for the standard LNDs (Table 1). These are plotted in Figure 1 together with Goudriaan's, Dickinson's and de Ridder's parameterisations. Firstly, one can learn from the exact expressions (coloured curves) that GF must satisfy $G \in [0, 1]$ and the curves of GF for the non-δ-distribution LNDs run between the GF for the two extreme LNDs of purely *horizontal* and purely *vertical* leaf normals represented by delta distributions, which are strongly different to the parameterised GFs of both Goudriaan and Dickinson (black to light-grey curves). Their GFs become negative for $|\mu| \to 0$ for purely *vertical* leaf normals whereas the parameterisation of Dickinson is closer to the exact GF (yellow graph). For purely *horizontal* leaf normals Goudriaan's approach overestimates the exact GF (darkest blue curve) less than Dickinson's. For *erectophile* and *planophile* leaf normal orientations the GFs of Dickinson are better approximations than Goudriaan's in comparison to the analytical GFs. On the other hand, de Ridder's GFs are more realistic and exact for the *horizontal*, *uniform* and *vertical* case.
It should be stressed that the GF has a significant effect on the transported radiation and dominates the exponentially decaying single scattering term, which the radiative transfer equation (10) later in this section demonstrates. Thus, small deviations in G can lead to large errors in the simulated flux densities.

A further basic quantity in order to define the RTQs and determine the radiative transfer in vegetation is the so-called *leaf area density* (LAD) $u_{_L}(\mathbf{x}, t) \geq 0$, which is a measure for the probability that a leaf exists spatio-temporally and specifies its total one-sided leaf area per volume of the host medium. Thus, $u_{_L}$ has the unit $\frac{\mathrm{m}^2}{\mathrm{m}^3} = \mathrm{m}^{-1}$. The larger the LAD the more probable are interactions between the incident radiation and the leaves. Therefore, the LAD is a measure for extinction. In the present work the LAD is assumed to be horizontally homogeneous and independent of time, so that $u_{_L}$ is only a function of the vertical coordinate x_3. From the LAD the *leaf area index* (LAI) L can be derived by

$$L = h_{_L}^{-1}(x_3) := \int\limits_{[x_3, H]} u_{_L}(s)\, ds \geq 0 \tag{7}$$

where $x_3 \in [0, H]$ with the height of the vegetation H. The dimensionless L is a strictly monotonic increasing function with decreasing altitude and can be regarded as the vertical distribution of the leaf area per horizontal unit area. It acts as independent variable in the radiative transfer equation for turbid vegetation media and is, thus, the analoguous quantity to the optical depth τ in the radiative transport theory for atmospheric media.

With the help of the previous introductions the RTQs for turbid vegetation media can be determined occuring in a respective radiative transfer equation (RTE). The definitions of the RTQs as given next follow the work of, e.g., Ross (1981), Shultis and Myneni (1988), Marshak (1989), Myneni et al. (1989) as well as Marshak and Davis (2005). All quantities are considered to be horizontally homogeneous and time independent.
The *extinction coefficient* is defined by

$$k_e(x_3, \mathbf{y}) := u_{_L}(x_3)\, G(\mathbf{y})$$

being anisotropic and independent of wavelength.

The *scattering function* f_s and *scattering coefficient* k_s are defined as

$$\begin{aligned} f_s(x_3, \mathbf{y} \,|\, \mathbf{y}', \lambda) &:= \frac{u_L(x_3)}{A(S_1^+)} \int\limits_{S_1^+} g_L(\mathbf{y}_L)\, \gamma_L(x_3, \mathbf{y} \,|\, \mathbf{y}', \mathbf{y}_L, \lambda)\, do(\mathbf{y}_L)\,, \\ k_s(x_3, \mathbf{y}, \lambda) &:= \frac{u_L(x_3)}{A(S_1)} \int\limits_{S_1} \Gamma(x_3, \mathbf{y} \,|\, \mathbf{y}', \lambda)\, do(\mathbf{y}') \end{aligned} \tag{8}$$

where $A(S_1) = 4\pi F_o$ is the surface area of S_1, and the vectors $\mathbf{a}$ and $\mathbf{b}$ mean the incident as well as the exit direction of scattered radiation in the expression $\mathbf{a} \,|\, \mathbf{b}$. Further, γ_L is the so-called *leaf scattering function* (LSF) representing a measure for the probability that radiation of the wavelength λ is scattered from direction $\mathbf{y}$ into direction $\mathbf{y}'$ by a leaf having the leaf normal direction $\mathbf{y}_L$. The scattering function and scattering coefficient are strictly related to each other,

$$\frac{1}{A(S_1)} \int\limits_{S_1} f_s(x_3, \mathbf{y} \,|\, \mathbf{y}', \lambda)\, do(\mathbf{y}') = k_s(x_3, \mathbf{y}, \lambda).$$

As a consequence, LSF γ_L determines Γ via the integral relation

$$\Gamma(x_3, \mathbf{y} \,|\, \mathbf{y}', \lambda) = \frac{1}{A(S_1^+)} \int\limits_{S_1^+} g_L(\mathbf{y}_L)\, \gamma_L(x_3, \mathbf{y} \,|\, \mathbf{y}', \mathbf{y}_L, \lambda)\, do(\mathbf{y}_L)\,, \tag{9}$$

averaging the LSF over the leaf normal orientations g_L. So, Γ is named *orientation-averaged leaf scattering function.* From (8) and (9) one can deduce $f_s = u_L\,\Gamma$.
Leaves are usually considered as Lambertian diffusers (Myneni et al., 1988a,c,d; Dai and Sun, 2006) and, thus, leaf scattering is described by the assumption of *bi-Lambertian scattering* (Liang and Strahler, 1995; Gobron et al., 1997; Tian et al., 2007, e.g.) in terms of

$$\gamma_L(\mathbf{y} \,|\, \mathbf{y}', \mathbf{y}_L, \lambda) := k\,|\alpha||\alpha'| \cdot \begin{cases} r_L(\lambda) & \vee \quad \alpha\,\alpha' < 0 \ \text{ reflectance} \\ t_L(\lambda) & \vee \quad \alpha\,\alpha' > 0 \ \text{ transmittance} \end{cases}$$

including the constant k, the hemispherical reflectance and transmittance of the individual leaves $r_L(\lambda)$ and $t_L(\lambda)$ as well as the two cosines: the incidence cosine α and the exit cosine α', defined by the scalar products

$$\alpha^{(\prime)}(\mathbf{y}^{(\prime)}, \mathbf{y}_L) := \langle \frac{\mathbf{y}^{(\prime)}}{\|\mathbf{y}^{(\prime)}\|}, \frac{\mathbf{y}_L}{\|\mathbf{y}_L\|} \rangle\,,$$

express the character of Lambertian scattering being the weaker the more the incident as well as exit rays are inclined in direction to the leaf plane. The factor k can be obtained taking energy conservation into account and this condition leads to the constraint

$$\frac{1}{A(S_1)} \int\limits_{S_1} \gamma_L(\mathbf{y} \,|\, \mathbf{y}', \mathbf{y}_L, \lambda)\, do(\mathbf{y}') = |\alpha(\mathbf{y}, \mathbf{y}_L)| \cdot (r_L(\lambda) + t_L(\lambda)).$$

Thereby, the amount of radiation scattered by a leaf (left side of the equation) has to be equal to the incident fraction $|\alpha|$ multiplied with the hemispherically scattered portions $r_L + t_L$ (right side) with the result of $k = 4$. Note that this simplyfied bi-Lambertian scattering approach makes no difference between reflectance or transmittance at the upper and lower face of the leaf elements, and r_L as well as t_L are considered to be independent of altitude x_3. Thus, Γ becomes independent of x_3, too.

Then, the single scattering albedo $\omega_o = \frac{k_s}{k_e}$, the absorption density k_a and the scattering phase function $P = \frac{f_s}{k_s}$ can be calculated for a vegetation medium,

$$\omega_o(\mathbf{y},\lambda) = \frac{G(\mathbf{y})^{-1}}{A(S_1)} \int\limits_{S_1} \Gamma(\mathbf{y}\,|\,\mathbf{y}',\lambda)\, do(\mathbf{y}') \qquad k_a(x_3,\mathbf{y},\lambda) = u_{{}_L}(x_3)\, G(\mathbf{y}) \Big(1 - \omega_o(\mathbf{y},\lambda)\Big)$$

$$P(\mathbf{y}\,|\,\mathbf{y}',\lambda) = \frac{\Gamma(\mathbf{y}\,|\,\mathbf{y}',\lambda)}{\omega_o(\mathbf{y},\lambda)\, G(\mathbf{y})} \qquad \Longrightarrow \qquad \frac{1}{A(S_1)} \int\limits_{S_1} P(\mathbf{y}\,|\,\mathbf{y}',\lambda)\, do(\mathbf{y}') = 1.$$

The previously defined RTQs can be utilised to derive a *radiative transfer equation* (RTE). We assume time independence, horizontal homogeneity and elastic scattering. Then, we consider bi-Lambertian scattering and split the total radiance field J into its direct and diffuse portions D and I. After substituting the independent vertical coordinate x_3 in a general RTE by L one obtains $J(L,\mathbf{y},\lambda) = I(L,\mathbf{y},\lambda) + D(L,\mathbf{y},\lambda)\,\delta(\mathbf{y} - \mathbf{e}_{{}_D})$ and

$$\begin{aligned} \frac{y_3}{\|\mathbf{y}\|} \frac{d}{dL} I(L,\mathbf{y},\lambda) = G(\mathbf{y})\, I(L,\mathbf{y},\lambda) - \frac{1}{A(S_1)} \int\limits_{S_1} \Gamma(\mathbf{y}'\,|\,\mathbf{y},\lambda)\, I(L,\mathbf{y}',\lambda)\, do(\mathbf{y}') \\ - \frac{1}{4\pi}\, \Gamma(\mathbf{e}_{{}_D}\,|\,\mathbf{y},\lambda)\, D_o(\lambda) \exp\left[\frac{G(\mathbf{e}_{{}_D})}{\frac{1}{C_o}\,\langle \mathbf{e}_3, \mathbf{e}_{{}_D}\rangle}\, L\right] \end{aligned} \tag{10}$$

where $\mathbf{e}_{{}_D}$ is the direction vector of the direct (solar) radiation intruding into the vegetation from the atmosphere above, and the factor $D_o(\lambda) = D(0,\mathbf{e}_{{}_D},\lambda)$ means the radiance at the top of the vegetation (TOV) with respect to the direction $\mathbf{e}_{{}_D}$ of the sun's light. The RTE (10) is fully determined via the functions $\gamma_{{}_L}$ and $g_{{}_L}$ as introduced above.

From equation (10) analytical two-stream equations can be deduced. Let

$$[f(\mathbf{y}^{(\prime)})]_{\pm}^{(\prime)} := \int\limits_{S_1^{\pm}} f(\mathbf{y}^{(\prime)})\, do(\mathbf{y}^{(\prime)}) \qquad \langle f(\mathbf{y}^{(\prime)})\rangle_{\pm}^{(\prime)} := \frac{[f(\mathbf{y}^{(\prime)})\, I(L,\mathbf{y}^{(\prime)},\lambda)]_{\pm}^{(\prime)}}{[I(L,\mathbf{y}^{(\prime)},\lambda)]_{\pm}^{(\prime)}} \tag{11}$$

be average operators for an arbitrary RTQ f, being a function of the radiation directions $\mathbf{y}^{(\prime)}$, with the diffuse radiance field I. The EFD and AFD from the equations (1) and (2) can be split into their upward (+) and downward (−) parts $E_\pm$ as well as $A_\pm$ by

$$E(L,\lambda) = \left[\frac{y_3}{\|\mathbf{y}\|}\, I(L,\mathbf{y},\lambda)\right]_+ - \left[-\frac{y_3}{\|\mathbf{y}\|}\, I(L,\mathbf{y},\lambda)\right]_- := E_+(L,\lambda) - E_-(L,\lambda)$$

as well as

$$A(L,\lambda) = [I(L,\mathbf{y},\lambda)]_+ + [I(L,\mathbf{y},\lambda)]_- := A_+(L,\lambda) + A_-(L,\lambda).$$

Introducing the upward and downward diffusivity factors,

$$U_\pm(L,\lambda) := \left\langle \pm \frac{y_3}{\|\mathbf{y}\|} \right\rangle_\pm^{-1} \in (1,\infty)\,,$$

one can relate the AFD and EFD by

$$A_\pm(L,\lambda) = U_\pm(L,\lambda)\, E_\pm(L,\lambda). \tag{12}$$

$U_\pm$ are functionals of the diffuse radiance distribution I.

The operators (11) can now be applied to the RTE (10). Since the diffuse radiance field I is unknown, we introduce the **two-stream assumption** that I let *approximately* be isotropic leading to

$$\langle f\rangle_{\pm}^{(\prime)} = \frac{[\,f\,I\,]_{\pm}^{(\prime)}}{[\,I\,]_{\pm}^{(\prime)}} \approx \frac{[\,f\,]_{\pm}^{(\prime)}}{2\pi F_o} \qquad\qquad U_+ \approx U_- := U(\lambda). \tag{13}$$

Further, it is assumed that the bi-Lambertian leaf scattering properties $r_{_L}$ and $t_{_L}$ are each equal for the both sides of a leaf. Finally, one can derive the following two-stream system of first order ordinary differential equations with constant coefficients for the vector field $\mathbf{E}(L,\lambda) = (E_+(L,\lambda), E_-(L,\lambda))$,

$$\frac{d}{dL}\mathbf{E}(L,\lambda) = \begin{pmatrix} \alpha_1(\lambda) & -\alpha_2(\lambda) \\ \alpha_2(\lambda) & -\alpha_1(\lambda) \end{pmatrix} \cdot \mathbf{E}(L,\lambda) + D(L,\lambda) \begin{pmatrix} -\alpha_3(\lambda) \\ \alpha_4(\lambda) \end{pmatrix} \tag{14}$$

with

$$\begin{aligned}
\alpha_1(\lambda) &= U\Big(\frac{1}{2} - \frac{B_+}{2\pi F_o}\Big) & \Longleftarrow \quad & B_+(\lambda) = \frac{1}{A(S_1)}\Big[[\Gamma(\mathbf{y}'\,|\,\mathbf{y},\lambda)]_+\Big]_+' \\
\alpha_2(\lambda) &= U\frac{B_-}{2\pi F_o} & \Longleftarrow \quad & B_-(\lambda) = \frac{1}{A(S_1)}\Big[[\Gamma(\mathbf{y}'\,|\,\mathbf{y},\lambda)]_+\Big]_-' \\
\alpha_3(\lambda) &= \frac{1}{A(S_1)}\,[\Gamma(\mathbf{e}_{_D}\,|\,\mathbf{y},\lambda)]_+ & & \\
\alpha_4(\lambda) &= \frac{1}{A(S_1)}\,[\Gamma(\mathbf{e}_{_D}\,|\,\mathbf{y},\lambda)]_- & & \\
D(L,\lambda) &= F_o\,D_o(\lambda)\exp\Big[G_{_D}\,L\Big] & \Longleftarrow \quad & G_{_D} = \frac{G_\mu(\mu_{_D})}{\mu_{_D}} \quad (\mu_{_D} < 0)
\end{aligned} \tag{15}$$

where the $\alpha_i(\lambda)$ and $G_{_D}$ can be calculated analytically for all LNDs of Table 1. Thus, a class of two-stream energy flux density radiative transfer models is defined for various canopies having leaf architectures of purely horizontal to purely vertical leaves. For the shortwave wavelengths the solution of (14) can be written as

$$\mathbf{E}(L,\lambda) = R_+\begin{pmatrix} 1 \\ C_+ \end{pmatrix}\exp(k\,L) + R_-\begin{pmatrix} 1 \\ C_- \end{pmatrix}\exp(-k\,L) + E_{_D}\begin{pmatrix} D_+ \\ D_- \end{pmatrix}\exp(G_{_D}\,L) \tag{16}$$

where k, $R_\pm$, $C_\pm$, $D_\pm$ and $E_{_D}$ are functions depending on the wavelength λ and given by

$$k = \sqrt{\alpha_1^2 - \alpha_2^2} \qquad E_{_D} = |\mu_{_D}|F_o\,D_o(\lambda) \qquad C_\pm = \frac{\alpha_2}{\alpha_1 \pm k}$$

$$D_+ = \frac{\alpha_2\,\alpha_4 + \alpha_3\,(\alpha_1 + G_{_D})}{|\mu_{_D}|(k^2 - G_{_D}^2)} \qquad D_- = \frac{\alpha_2\,\alpha_3 + \alpha_4\,(\alpha_1 - G_{_D})}{|\mu_{_D}|(k^2 - G_{_D}^2)}$$

$$F = (A_{_D} + A_d\,D_- - D_+)\,E_{_D}\exp(G_{_D}\,L_{_T}) \qquad K_\pm = (1 - A_d\,C_\pm)\exp(\pm k\,L_{_T})$$

$$R_- = \frac{C_+F + K_+(D_-E_{_D} - E_d)}{C_+K_- - C_-K_+} \qquad R_+ = \frac{1}{C_+}\,(E_d - R_-C_- - E_{_D}D_-).$$

The models' input parameters are: the spectral surface albedos $A_{_D}$ of the direct and A_d of the diffuse radiation reflected isotropically into the upper hemisphere, the total LAI $L_{_T} = h_{_L}^{-1}(0)$ of the canopy after equation (7), the direct and downward diffuse spectral EFDs $E_{_D}$ and E_d at TOV given as upper boundary conditions, which *enables one to consider various sky conditions above the canopy from clear to cloudy skies*, further the diffusivity factor U depending on the sky's radiation field, too, and finally the cosine $\mu_{_D}$ of the solar zenith angle of the sun's light as well as the leaf properties $r_{_L}$ and $t_{_L}$.

Using the two-stream solution (16) of the diffuse EFDs $E_\pm(L,\lambda)$ the corresponding diffuse AFDs $A_\pm(L,\lambda)$ can be calculated with the help of the formulas (12) and (13). Since the total downward radiation field not only consists of the diffuse portion, the direct part of light has to be regarded leading to the total downward EFD and AFD calculated via

$$\begin{aligned} E_{-,t}(L,\lambda) &= E_-(L,\lambda) + E_{_D}(\lambda)\exp(G_{_D}\, L)\,, \\ A_{-,t}(L,\lambda) &= U(\lambda)\, E_-(L,\lambda) + \frac{E_{_D}(\lambda)}{|\mu_{_D}|}\exp(G_{_D}\, L) \end{aligned}$$

at an arbitrary altitude $x_3 = h_{_L}(L)$ within the canopy as function of the wavelength. In order to characterise the fraction of radiation being reflected from or transmitted through the vegetation medium the canopy reflectance (CR) R_c and transmittance (CT) T_c can be defined,

$$R_c(\lambda) := \frac{E_+(0,\lambda)}{E_{-,t}(0,\lambda)} = \frac{E_+(0,\lambda)}{E_d(\lambda)+E_{_D}(\lambda)} \qquad T_c(\lambda) := \frac{E_{-,t}(L_{_T},\lambda)}{E_{-,t}(0,\lambda)} = \frac{E_{-,t}(L_{_T},\lambda)}{E_d(\lambda)+E_{_D}(\lambda)}\,, \tag{17}$$

and will be investigated as function of the models' input parameters in the following.
Note that equation (16) is the solution for a horizontally homogeneous vegetation medium being homogeneous in the vertical direction, too. For vertically inhomogeneous media (e.g. if $r_{_L}$ and $t_{_L}$ depend on altitude representing sunlit and shaded leaves) one can just consider a certain number of such homogeneous layers where the according solutions have to be coupled fulfilling continuity conditions at the layer boundaries.

3 Diffuse light transport in vegetation

Most of the authors assume completely isotropic radiance fields (Dickinson, 1983; Sellers, 1985). Such models can not deal with anisotropic ones within or above the medium, e.g., the sky conditions above the canopy. Due to the diffusivity factor $U(\lambda)$ occuring in (15) as well as the direct and diffuse EFDs $E_{_D}$ and E_d as upper boundary condition, our two-stream methods can treat such situations which are defined for various LNDs (Table 1) and, thus, close the gap namely that "effective methods to deal with cases as [...] the distribution of incident sky radiation being anisotropic, or the leaves in the canopy not being horizontal" are missing (Dai and Sun, 2006). Furthermore, we generalise the treatment of the boundary conditions improving the limitations stated by Dai and Sun (2006) that two-stream models could only describe isotropic overcast sky conditions.
In the following we will present a sensitivity study of our radiative transfer schemes as function of the main input parameters: $L_{_T}$, $\mu_{_D}$, $E_{_D}$ and E_d at TOV, $r_{_L}$ and $t_{_L}$, $A_{_D}$ and A_d as well as U in order to demonstrate the transport of diffuse radiation through vegetation media for clear and overcast sky conditions.
Before doing this we have to note something about the diffusivity factor being important. U of the atmosphere above the canopy is generally a function of solar zenith angle (SZA) and wavelength. It depends on the vertical structure of the atmosphere containing gases (Rayleigh scattering and absorption), aerosols (Ruggaber et al., 1993) and clouds reducing direct light. It is influenced by the surface albedo (Cotte et al., 2004), too. Since the presented radiative transfer models can deal with the incident direct and diffuse radiation $E_{_D}$ and E_d at TOV, which are related by the ratio

$$r_{_{Dd}} = \frac{E_{_D}}{E_d},$$

in order to describe realistic sky situations above the vegetation medium, we are able to distinguish between clear sky ($r_{_{Dd}} > 1$) as well as cloudy sky ($r_{_{Dd}} < 1$) conditions.

Therefor, U has to be determined at the bottom of the atmosphere. Following the works of Landgraf (1998), Webb et al. (2002) and Kazadzis et al. (2004) one can conclude: For cloudless skies characterised by $r_{Dd} \sim 2$ the diffusivity factor shows values of ~ 2 ranging from 1.5 for small to 3.0 for large UV wavelengths. However, at overcast sky conditions, represented by $r_{Dd} \sim 0.2$ or even lower values, U is only a slight function of the wavelength with values of ~ 1.7. Therefore, we will use $U(r_{Dd} = 2.0) = 2.0$ or 2.3 for clear sky and $U(r_{Dd} = 0.2) = 1.7$ for cloudy sky conditions. Note that this U, given for the atmosphere above, is also assumed to be valid within the canopy in order to incorporate the sky situations.
To study the transport of diffuse radiation we computed CR and CT after (17) for ultraviolet (UV) and visible (VIS) wavelengths where the input parameters r_L, t_L, A_D and A_d were fixed to the representative values

$$\begin{aligned} r_L &= t_L = 0.05\,, \\ A_D &= A_d = 0.1 \end{aligned} \tag{18}$$

within this spectral region following Bowker et al. (1985), Liangrocapart and Petrou (2002) and Tian et al. (2007). The computations were performed considering the ratio of direct and diffuse radiation r_{Dd} as well as an incident energy of $E_D + E_d = 1$. Then, the simulated upward and downward energy flux densities $E_+(L = 0)$ and $E_{-,t}(L_T)$ are equivalent to R_c and T_c as one can deduce from (17).
Figure 2 shows CR (*top*) and CT (*bottom*) as function of the solar zenith angle in the cases of cloudless (solid) and overcast (dashed) sky conditions above a canopy having a total LAI of $L_T = 5.5$ representing deciduous canopies as oak forests (Rauner, 1976). The SZA ranges from $\vartheta_D = 90°$ in the morning and evening (with $\mu_D = 0$) to $\vartheta_D = 180°$ at noon (with $\mu_D = -1$). First, the simulations demonstrate that the CR/CT calculated by the RT models of the various LNDs vary stronger for clear skies (larger r_{Dd}), since the transport of direct light is strongly dominated by variations in the GFs (Table 1) being significantly different for the several LNDs (Figure 1). The lower r_{Dd} (overcast skies), the smaller these differences of the LND models, and the *vertical* model represents an asymptotic case. Second, the CR/CT decreases/increases with rising or setting sun and CT changes over two orders of magnitude. Third, comparing clear and cloudy sky conditions over the day the simulated CT of all models is always larger on a clouded day in the morning and evening ($\mu_D \to 0$). Around noon ($\mu_D \to -1$) CT is only larger for the models with nearly horizontal leaves (near-vertical models) as in the case of a deciduous forest. For such vegetation canopies *the portion of radiation reaching the bottom is always larger over the day for overcast relative to clear atmospheres above the canopy.* However, LNDs of nearly vertical leaves can transmit a larger fraction of incident radiation on clear sky days around noontime. This is due to the fact that direct light is transported more effectively in downward directions by such distributed leaves for high positions of the sun. Moreover, the simulations demonstrate that nearly vertical/horizontal leaves contribute less/more to the CT at midday but more/less at the beginning as well as the end of the day, and vice versa in the case of the CR. The limiting case, that all models of the LND produce similar CRs/CTs under both clear and overcast sky conditions, occurs for $\mu_D \sim -0.57$ (a SZA of $\sim 125\,°$). This is the same value for the cosine of the radiation angle, at which the GFs for the various LNDs are approximately equal (Figure 1).
The (*top*) and (*bottom*) plates of Figure 3 depict the CRs and CTs as function of r_{Dd} for two choices of the diffusivity factor, a mean daily SZA for mid-latitudes of $140\,°$ and $L_T = 5.5$ being representative for a typical deciduous forest as before. We distinguish clear and cloudy sky conditions by setting the following parameter ranges:

- $U \sim 2.3$ and $r_{Dd} > 1$ for clear skies
- $U \sim 1.7$ and $r_{Dd} \ll 1$ for cloudy skies.

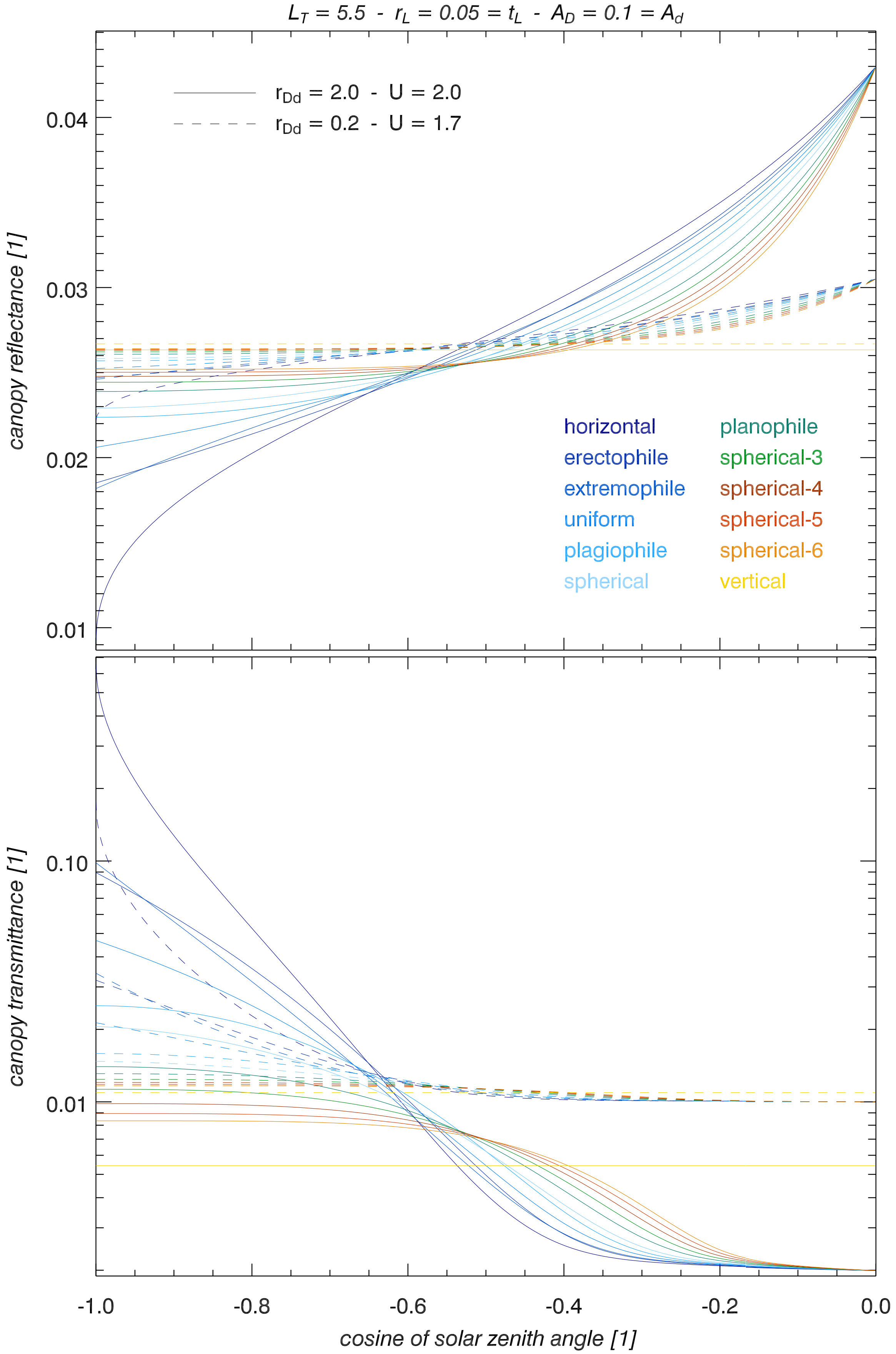

Figure 2: (*top/bottom*) CR/CT in the UV/VIS for the LNDs of Table 1 as function of μ_D with constant parameters after (18) and $L_T = 5.5$ for deciduous forests. The solid lines represent clear sky and the dashed ones cloudy sky conditions, see the main text.

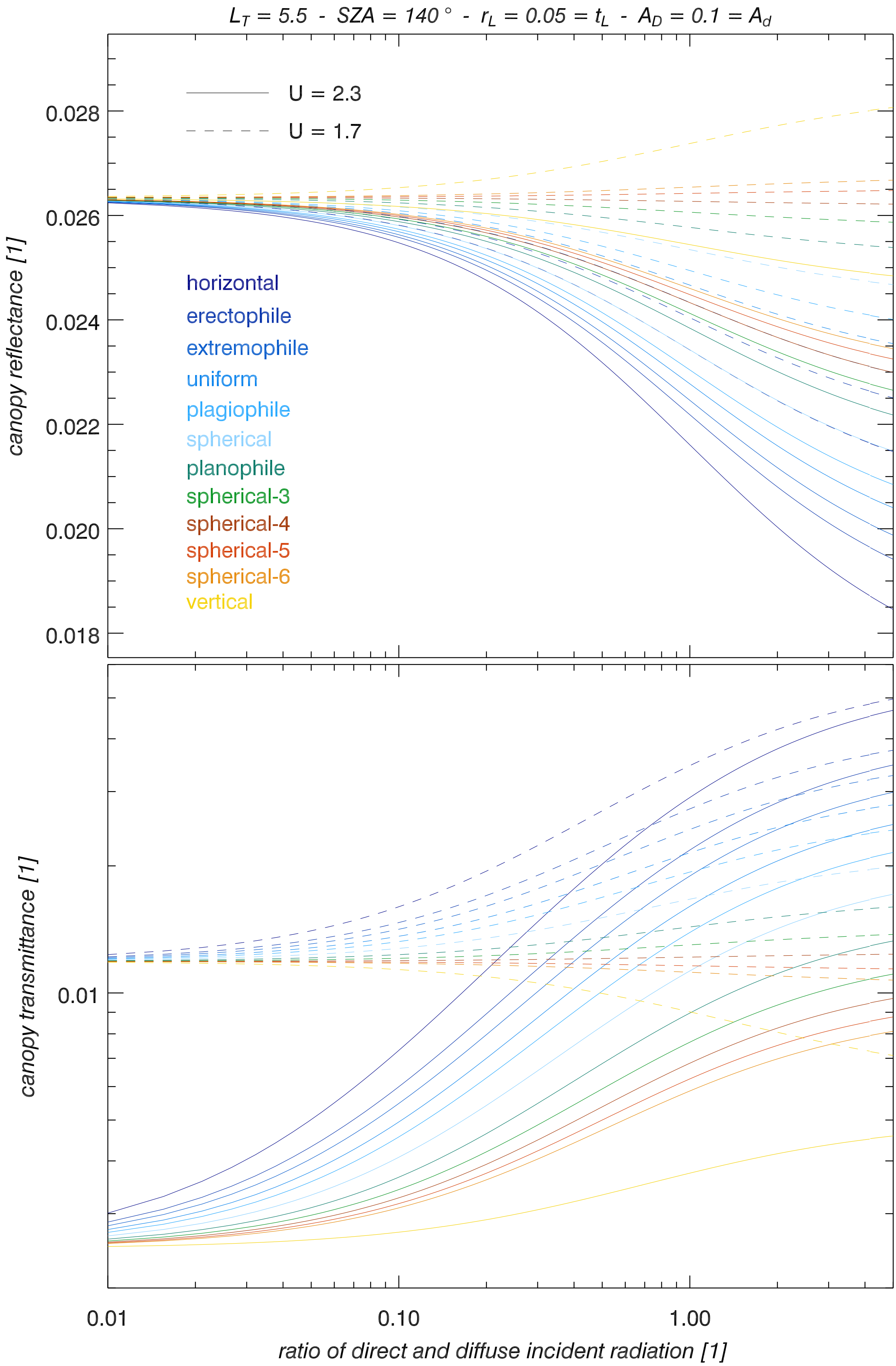

Figure 3: (*top/bottom*) CR/CT in the UV/VIS for the LNDs of Table 1 as function of r_{Dd} with constant parameters after (18) for a SZA of 140 ° as well as $L_T = 5.5$ for deciduous forests. Two diffusivity factors were chosen for clear sky (solid) and cloudy sky (dashed) conditions.

Compare the solid curves for large values of r_{Dd} (clear sky) with the dashed ones for a low r_{Dd} (overcast): The CR is always larger under cloudy sky conditions independent on the LND model. While the CT only shows this behaviour for nearly *vertical* LNDs (horizontal leaves), the nearly *horizontal* LND models can lead to higher CTs for clear atmospheres due to the enhanced transmittance of direct light. As mentioned above the deviations of the LND models are the smaller for overcast situations the larger the diffuse fraction of radiation is, and vice versa, they increase for clear skies, the larger the portion of the direct light component is.

4 Conclusions and outlook

The presented work reported on two-stream radiative transfer in a horizontally homogeneous turbid vegetation medium. Bi-Lambertian leaf scattering was assumed with leaf optical properties (leaf reflectance and transmittance) being equal for both sides of the planar model leaves. Their distributions were described by leaf normal distribution (LND) functions which were adopted to be time and spatially independent. Two-stream radiative transport equations were derived and solved analytically for various leaf architectures considering LNDs from purely vertical to purely horizontal leaves. The according models were driven by radiative energy flux densities, incident at the top of the vegetation, separating them into their diffuse and direct portions to take the sky conditions above the canopy into account. At the bottom of the vegetation a typical soil albedo was considered. Further, the presented formulation of the two-stream solution allows one to consider also anisotropic radiance fields by the introduction of the diffusivity factor. Thus, more realistic situations of vegetation media fully coupled with the overlying atmosphere can be treated more satisfactorily than previously possible.

The presented radiative transfer models for the various LNDs were used to calculate canopy reflectances and transmittances. It turned out that the radiation regime depends significantly on the LND and the sky conditions above the canopy, i.e. the ratio of direct and diffuse light. For deciduous forests with predominantly horizontal leaves the canopy transmittance was simulated to be larger on overcast than on clear sky days.

The LNDs as well as the leaf optical properties were considered to be independent of space, because from an experimental point of view vertically resolved canopy properties are rarely available. If necessary, however, the presented two-stream theory can be extended to approximately account for, e.g., vertically variable LNDs or leaf optical properties to distinguish sun and shaded leaves. Moreover, specular leaf scattering can be taken into account. Considering vertical structures the vegetation medium must be layered homogeneously, and the analytical solutions for each of these layers have to be coupled fulfilling continuity conditions at the layers' interfaces. It should be emphasised that, as a further step, the two-stream approach can also be applied in the case of such layered media without to forfeit significantly their computational rapidness. Such more complex analytical models can be used, e.g., for calculations of photosynthesis and photolysis processes within deciduous forests as basis for simulations of atmospheric transport processes within the vegetation or, in a broader context, for climate modelling.

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